MICROMECHANICAL SIMULATION OF EARTHQUAKE-INDUCED FRACTURE IN STEEL STRUCTURES

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Report No. 145

July 2004
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Abstract

Characterized by large-strain low-cycle conditions, earthquake-induced fractures are quite different from high-cycle fatigue fractures that have been extensively studied in bridges and mechanical components. Cyclic loading of structures due to earthquakes involves many fewer cycles (typically less than ten cycles) than conventional low-cycle fatigue and strains that are well in excess of yield. Such conditions can be termed as ultra-low-cycle fatigue (ULCF). Traditional techniques to predict fracture under ULCF are not nearly as well developed as those for other limit states. Computationally intensive micromechanical models, which aim to capture fundamental fracture initiation mechanisms of void growth and coalescence, show great promise in predicting failure due to inelastic fracture and ULCF commonly seen during earthquakes. This research develops and applies such models for simulating inelastic earthquake-induced fractures with an emphasis on ductile crack initiation due to monotonic and cyclic loading. The research includes complementary computational (finite element) and experimental studies that utilize state-of-the-art fracture and micromechanical models to analyze failures. Over two hundred tests (and complementary finite element analyses) are carried out on seven different varieties of steel. The experiments include monotonic and cyclic stress-strain tests for materials, standard fracture tests, smooth notched fracture tests, as well as pull-plate tests that resemble structural configurations. Based on the simulation results and tests, models to simulate ductile crack initiation under ULCF are developed and validated. The study (1) validates monotonic micromechanical models for structural steels and makes recommendations for their use (2) generates material toughness data for a variety of structural steels (3) proposes new mechanisms for ULCF (4) develops new micromechanical models for ULCF based on these mechanisms (5) uses experiments similar to structural details to validate the micromechanical models and (6) comments on the limitations of such approaches and makes recommendations for future research.
Acknowledgements

This report is based upon research supported by the National Science Foundation under the US Japan Cooperative Research for Urban Earthquake Disaster Mitigation initiative (Grant No. CMS 9988902). Additional support was provided by the Steel Structures Development Center of the Nippon Steel Corporation (Futtsu, Japan), which provided steel materials, machining services, and fracture data, and by donations of steel material from the Garry Steel Company (Oakland, CA) and the ATLSS Engineering Research Center (Bethlehem, PA).

The advice and guidance provided by Robert Dodds (University of Illinois), Wei-Ming Chi (GE Research Labs), Reiner Dauskardt (Stanford University), Jerry Hajjar (University of Minnesota), Ryoichi Kanno and Takahiko Suzuki (Nippon Steel Corporation), Koji Morita and Yukihiro Harada (Chiba University) are gratefully acknowledged. Also acknowledged are the technical and/or administrative assistance provided by Robert Brown, Racquel Hagen, Karlheinz Merkle, and Robert Jones (Stanford University), Roger Ferch (Herrick Steel Corporation), Chris Rothe (Applied Process Equipment), and Edward Foreman (FTI Anamet).

Any opinions, findings and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation or other contributors.
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Chapter 1

Introduction

1.1 Motivation for the study

Modern earthquake resistant design relies on structural ductility to absorb seismic energy. This is especially true in steel structures, such as moment resisting frames, where the predominant mode of seismic energy dissipation is large inelastic cyclic deformations in the plastic hinge region. Until the 1994 Northridge earthquake, the ductility of steel was almost unquestioned; it was assumed that steel moment connections would be able to deform plastically to large rotations absorbing significant amounts of energy. Though the steel material is very ductile, (with ductility ratios as large as 50-100) moment connections and other structural details have much less ductility due to heterogeneity in weldments. The stress risers, coupled with the lower toughness in the weld and Heat Affected Zone (HAZ) areas, can lead to fractures at much lower ductility ratios. Such brittle fractures were found in many moment frames inspected after the Northridge earthquake. This was a startling discovery, which brought into question the common assumptions about the ductility of steel, and prompted several studies to investigate fracture in steel moment frames.

Studies conducted after the Northridge earthquake (such as the SAC Steel Project) resulted in many improvements to the connection details, which included modifying the connection configurations, e.g. removal of backing bars and minimizing defects through better quality control during welding. Other ideas included new connection configurations such as the Reduced Beam Section (RBS) connection design, which reduce the overall loading demand on the weld area. Tougher, notch-toughness rated materials were prescribed to increase toughness of the weld and base metals. These
measures have been shown to successfully prevent the brittle cracking of the type observed during Northridge; however, the improved connections are still susceptible to ductile crack initiation at unacceptably low hinge rotations (Stojadinovic et al, 2000).

Conventional fracture mechanics approaches, such as the J-integral or the Crack-Tip Opening Displacement (CTOD) are based on the assumptions of well-constrained or small-scale yielding. An earlier study by Chi et al (2000) demonstrated that conventional fracture mechanics is a reliable tool to examine the fracture process in steel moment frame connections in pre-Northridge type connections where the yielding is limited in extent. However, it is questionable to apply conventional fracture mechanics to model fracture under large-scale yielding observed in post-Northridge connections. Moreover, the failure under earthquake type loading is characterized by cyclic loading. Thus, it is strictly not a fracture problem, but a fracture fatigue interaction problem. Interestingly, the fatigue experienced by structures during earthquakes causes failure in less than ten or twenty cycles, which is very different from conventional high or low cycle fatigue. This can be termed as Ultra Low Cycle Fatigue (ULCF).

Conventional fracture mechanics based approaches for fracture and fatigue have limited use in predicting fracture under large-scale plasticity and cyclic inelastic deformations. Especially in the situation of ULCF, earthquake engineers have no choice but to rely on semi-empirical models and experimental data to predict performance. The obvious drawback of such an empirical approach is that the data needs to be calibrated on a case-by-case basis and cannot be generalized easily.

1.2 Objectives and scope of the study

Pseudo-continuum models that capture the fundamental micromechanical mechanisms of fracture have shown promise in simulating failure due to monotonic ductile fracture (Hancock and Mackenzie, 1976; Panontin and Sheppard, 1995). However, the use of these models has not been widespread due to the associated computational expense and lack of verification data. With the rate of advances in computing technology, one can
envision situations where it is possible to model critical areas in large structures down to microstructural levels. Without the obstacle of computing capabilities, the governing gap in knowledge will be the physical models that capture the fracture and fatigue phenomena at that scale. Thus, newer micromechanical based models need to be developed for ductile fracture and ultra-low cycle fatigue which need to be calibrated and verified according to standardized guidelines. Once validated, these models can be used by the structural engineering and earthquake engineering community to solve practical problems. This study makes an effort in that direction by –

- Calibrating existing micromechanical models for ductile crack initiation for a variety of structural steels (a total of seven steel varieties are investigated in this study). As most of the earlier studies of micromechanical models focused on pressure vessel steels or similar materials used for mechanical engineering applications, this aspect of the research answers a need to investigate whether these models are suitable for implementation with common structural steel materials.

- Unifying different approaches and models from different studies into a consolidated method for calibration and application to practical structural engineering problems.

- Using test data, finite element simulations and microstructural data to develop and extend micromechanical models for ULCF.

- Verifying the applicability of the monotonic and the new cyclic micromechanical models to a variety of stress and strain conditions by using different experiments, which are representative of realistic situations found in structures.

The current state of structural engineering to design and fracture resistance largely relies on simplified analytical models whose results are interpreted based on empirical approaches. The applied fracture and fatigue mechanics approaches are often dependent
on phenomenological approaches, which are limited by many case-dependent assumptions. This study envisions that in the long term, such approaches will be replaced by approaches that are more fundamental, and can be generally applied to evaluate fracture and ULCF in structures and materials. Figure 1.1 shows a schematic of how performance assessment (in the context of fracture) may be conducted in the future.

There are many mechanisms that can cause fracture and fatigue of structures. One of the more prevalent mechanisms that causes ductile crack initiation as well as ULCF, is void growth and coalescence. There are other mechanisms that may occur, depending on material toughness, loading and constraint conditions, but this investigation and the models developed as part of it, will focus on the void growth and coalescence mechanism to predict fracture and ULCF. The other mechanisms will be briefly discussed, but will not be investigated in detail.

Though primarily aimed at structural and earthquake engineering, the results from this study can be generalized to other situations, because they are based on fundamental micromechanisms, and relatively free from ad-hoc assumptions. Moreover, this study takes a step in the earthquake engineering community’s stated goal of migrating from an empirical prescriptive approach to one that is performance-based.

1.3 Organization and Outline

Chapter 2 of this thesis provides important background for the dissertation research by discussing the generally accepted concepts, models and methods for predicting fracture and fatigue in structures. The chapter begins by introducing the variety of micromechanisms that cause fracture and fatigue (e.g. ductile versus brittle fracture, or high-cycle versus low-cycle fatigue etc.) Existing approaches to quantify fracture and fatigue, such as the J-integral for fracture and the $\Delta K$ for fatigue, are then presented. Finally, the existing micromechanical models for ductile crack initiation such as the Stress Modified Critical Strain (SMCS) and the Void Growth Model (VGM) are introduced. The discussions make clear distinctions between the micromechanisms of
fracture and the macro-fracture behavior, and clearly outline the assumptions involved in each model. Concepts of the micromechanisms are introduced to justify why traditional fracture mechanics approaches work in certain situations, but not in others.

Chapter 3 introduces two new models to simulate ULCF in steel structures, based on the void growth and coalescence mechanism. This chapter outlines the theoretical basis for these models, as related to test data, which justifies the choices for damage mechanisms. The two proposed models are extensions of the monotonic micromechanical models introduced in Chapter 2. Common features between the new ULCF models and monotonic models are that they both (1) capture the void growth and shrinkage mechanism under cyclic loading and (2) incorporate a damage mechanism to model cyclic damage accumulation. One model is an extension of the SMCS model and is termed the DSPS (Degraded Significant Plastic Strain) model, while the other is termed CVGM (or the Cyclic VGM). A new strain measure, termed the significant plastic strain is defined and used in the DSPS model.

Chapter 4 provides summarizes the testing and analysis component of the investigation for monotonic loading cases. Over two-hundred tests were conducted for seven different varieties of steel (including the monotonic and cyclic experiments). The discussion introduces these steel varieties and reports their basic material characteristics such as the stress-strain data, Charpy V-Notch data, chemical and grain size data. The chapter then describes in detail the calibration process for the SMCS and the VGM models under monotonic loading using circumferentially notched round bars. The testing configurations and procedures, finite element models, meshes and analysis techniques are outlined in detail. Experimental results (failure displacements and toughness indices) for all the steels are then presented. The chapter then discusses a unified approach for applying the length scale parameter in the micromechanical models. After the calibration of all the parameters of the micromechanical model parameters, the results of the verification tests which include ASTM type standard fracture mechanics tests, as well new blunt notched compact tension specimens, are presented. Next, the chapter presents a step by step procedure describing the use of these models to predict failure under different conditions.
Conventional fracture mechanics approaches are compared with the micromechanical models as well as with the experimental observations. The chapter concludes by summarizing the applicability and accuracy of these models for different situations.

Chapter 5 describes the testing and analysis component for the cyclic loading or ULCF component of the investigation. It begins by revisiting the DSPS and CVGM models introduced in Chapter 3 and outlining a detailed procedure to calibrate the model parameters. Similar to Chapter 4, this chapter describes the testing procedures (based on notched round bars), and finite element models, which are used in the calibration process. The chapter outlines procedures for verifying the calibrated DSPS and CVGM using different test configurations and results of the verification tests are compared to the failure predictions made by the DSPS and CVGM models. An unexpected phenomenon observed during some of the cyclic experiments (relating to intergranular separation and force degradation in steel) is discussed. These observations suggest that mechanisms other than void growth and coalescence (which are a focus of this study) can exist and be important in certain situations. The chapter concludes by comparing the models and their abilities to predict ULCF in steels, as well as on the trends in the relative ULCF fatigue resistance of the seven steels are evaluated in the research.

Chapter 6 demonstrates the use of the ULCF models in situations that are similar to those found in civil engineering structures. A series of monotonic and cyclic pull-plate type experiments are conducted, with details representative of (1) bolted connections (2) RBS connections. The experimental results from the different tests are then presented and compared with predictions from the micromechanical models. For the monotonic tests, the SMCS and the VGM models are used, while the DSPS and the CVGM models are used for the ULCF situations. The chapter thus illustrates the use of these models for simulating fracture in structural components.

Chapter 7 presents a summary and conclusion to the research. It is divided into two sections. The first section summarizes the work done in this study, reviewing the scope of the analytical and experimental investigations and the key findings if the study –
including the new models, the further development of the existing models, and ability to simulate fractures in structural components at the micromechanical level. Recommendations about the use of these models are made along with the areas of suggested use and possible improvements. The final section discusses many of the new questions that this research has raised, with suggestions on how these might be addressed in future research.
Application of micromechanical *initiation* models to structural components through detailed finite element meshes. Different models simulating different mechanisms and their interactions can be active simultaneously.

**Micromechanical models that are –**
1. **Microstructure based**
2. **Mechanism Specific**
3. **Virtually free from assumptions**

1. Accurate, assumption-free evaluation of performance
2. Better connection guidelines
3. Safer structures

*Figure 1.1 The future use of micromechanical models for performance based earthquake engineering*
Chapter 2

Background and Current State of Research in Fracture and Fatigue Mechanics

This chapter provides an overview of the current state of research in the areas of fracture and fatigue mechanics as they are related to this research. It first looks at the fundamental mechanisms underlying these phenomena, and then at the approaches and mathematical methods that have been adopted to simulate and apply them, identifying areas that need to be investigated in more detail. The fracture mechanisms discussed here are typically observed in metals such as low carbon structural steels. Depending upon the material properties, geometry and loading configuration, often entirely different modes of fracture are observed, ranging from ductile tearing to extremely brittle and sudden fracture. Also of interest is the process of transition from ductile crack initiation to brittle fracture. Similar variations in mechanisms are observed in fatigue processes, where mechanisms that cause high-cycle fatigue are different from those causing low-cycle fatigue.

The chapter begins by introducing the mechanisms of fracture in structural steel materials, including void growth, cleavage and intragranular fracture. The chapter then describes some of the mechanisms associated with the different stages of fatigue. Following this, the chapter looks at traditional models that are available to deal with fracture and fatigue, and outlines some of their limitations associated with large scale yielding and stage III fatigue. Some novel micromechanical models for ductile crack initiation are then introduced, which include the Void Growth Model (VGM) and the Stress Modified Critical Strain (SMCS) model. The discussion then highlights the key areas where the current state of knowledge can be improved such as for modeling ultra-
low cycle fatigue (or fracture fatigue interaction), which is common in earthquake situations.

Models for simulating fracture can be broadly categorized into traditional fracture mechanics approaches, and non-traditional micromechanical methods. Traditional approaches have been in use for the past forty or fifty years and are somewhat phenomenological in nature. They are more suited to simplified analysis and do not take full advantage of advances in computational technology. On the other hand, the newer micromechanical models aim to model the fundamental mechanisms of fracture and fatigue by utilizing modern computational power to simulate localized stresses and strains.

Some of the concepts discussed in this chapter are standard textbook material, and the reader is referred to popular textbooks for further background, e.g. Anderson (1995) for fracture and Suresh (1998) for fatigue. We also reference papers from which ideas have been borrowed for the purpose of this discussion. The aim of the chapter is to introduce and combine existing concepts and ideas of fracture and fatigue mechanics and present them in a perspective to facilitate development of newer micromechanical models.

2.1 Mechanisms of Fracture in structural steel materials

Depending on material microstructure, stress conditions and temperature, steels can fracture in a variety of modes, ranging from stable ductile tearing to unstable, brittle and sudden failure. These modes of fracture are caused by alternative mechanisms which are active under different conditions. In order to develop models for fracture, it is necessary to first understand these mechanisms in detail. This main focus of this thesis is on the mechanism of microvoid growth and coalescence. Other mechanisms such as cleavage, transition or inter or intragranular fracture mechanisms are reviewed for completeness and because in certain cases, these phenomena cannot be avoided.
2.1.1 Microvoid growth and coalescence

Mild steel, commonly used in structural engineering applications, typically exhibits ductile fracture accompanied by large scale plasticity. The stages observed during this type of fracture are observed to be those of microvoid nucleation, growth and coalescence, shown schematically in Figure 2.1.

1) Void Nucleation: Most steels contain secondary particles or inclusions such as carbides which sit in the steel matrix. When sufficient stress is applied to the interfacial bonds between these particles and the matrix, these bonds break, and a void nucleates around the secondary particle. In materials where the second phase particles are well-bonded to the matrix, void nucleation typically controls the fracture properties. In materials where this interfacial bond is not so strong, other mechanisms to be described (such as growth and coalescence), control the behavior. Argon (1975) and Goods et al (1979) provide models and a deeper understanding of the nucleation issue.

2) Growth and Coalescence: After nucleation, plastic strain and hydrostatic stress cause the voids to grow. Initially, the voids grow independent of one another, but upon further growth, neighboring voids interact and eventually, plastic strain is concentrated along a certain plane of voids. At this point local necking instabilities cause the voids to grow suddenly forming the macroscopic fracture surface. In many commonly used steels, this step governs the fracture process, and the stress and strain fields governing growth and coalescence are important for predicting fracture. This mechanism will be discussed in detail in this thesis, and numerous references can be found throughout.

Figure 2.2 shows a scanning electron micrograph of a surface that has fractured due to void growth and coalescence. The dimples on the surface show the locations of the voids that have coalesced. This type of fracture shows a high resistance to fracture, and is associated with stable crack behavior, meaning that the we need to continually increase
loading to propagate the crack further. This is because the crack propagation is accompanied by crack tip blunting and consequent weakening of the stress field ahead of the crack tip. See Figure 2.3. This means that as the crack grows, it gets more difficult to fracture the material due to microvoid coalescence. Thus, this type of fracture is very stable, i.e. sudden and brittle crack propagation will likely not be encountered under most loading conditions.

Microvoid coalescence is often observed in situations such as tensile tests which typically exhibit cup-cone fractures – see Figure 2.4. Due to the high hydrostatic stresses in the center of the specimen, a penny crack forms, which has a dimpled appearance when viewed under a microscope. The penny shaped crack locally alters the stress and strain distribution such that plastic strain is highly concentrated on planes at 45 degrees to the loading axis. This concentration of strain in these deformation bands provides sufficient plasticity to nucleate and coalesce voids around smaller, more numerous inclusions in the matrix (as compared to the larger and widely spaced inclusions which initiated the penny shaped crack). The fracture surface in the deformation bands is much smoother due to the smaller voids and forms the “shear lips” of the cup cone fracture.

Much of the existing literature often tends to present an idealized view of the void growth and coalescence process. The view presented is that the voids grow out in a direction transverse to the direction of the remotely applied plastic strain, and two adjacent voids coalesce such that the macroscopic crack surface is normal to the axis of straining. Rice and Tracey in 1969 have suggested in their paper that void growth is often not equiaxial and depends strongly on the level of triaxiality of the stress state. The triaxiality $T$ can be defined as the ratio of the mean stress to the effective or the von Mises stress –

$$T = \frac{\sigma_m}{\sigma_e}$$

(2.1)

Where $\sigma_m$ is the mean normal stress, and $\sigma_e$ is the effective stress. A weakly triaxial stress (triaxiality less than 1) state would cause the voids to grow asymmetrically into a prolate shape such that the major axis is aligned along the direction of remote plastic strain. On the other hand, situations with large triaxiality (triaxiality greater than 3) would
cause the void to grow into an oblate shape with the larger dimension being normal to the axis of straining. Researchers such as Becker et al. (1989) have identified this issue. However, the failure usually depends on the onset of shear localization in the ligament between the voids. The criterion for shear localization is typically a combination of the net stretching of the ligament and the transverse size of the voids. Rice and Tracey (1969) noted that for a given load history, the axial elongation of a void is practically independent of the average volume expansion – leading to the approximation that the failure state can be quantified effectively using the void volume fraction (an idea also used by Gurson in his model). The average void radius that shows up in Rice and Tracey’s Void Growth Model (VGM) and is the basis for the Stress Modified Critical Strain (SMCS) model (to be discussed in subsequent sections and chapters) is a good measure of the void volume increase. This justifies the usability of the average void volume based models such as the VGM and SMCS even in low triaxiality situations, i.e., with triaxiality ratios as low as 0.9.

2.1.2 Cleavage Fracture

Cleavage fracture occurs due to the sudden separation of material along crystallographic planes. It is typically very brittle and sudden but is often preceded by large scale ductile crack growth; further details on this transition are described in the next section. Cleavage occurs along planes which the lowest packing density, so as to break the fewest bonds in the process, e.g. in body centered cubic materials (BCC), the preferred cleavage planes are the \{100\} planes. In polycrystalline materials, the process is transgranular, occurring within grains as shown in Figure 2.5.

Cleavage involves breaking of bonds, which requires that the local stress exceed the cohesive strength of the material. In order for this to happen, there must be a local discontinuity ahead of the macroscopic crack that is sufficient to exceed the bond strength. Sharp microcracks ahead of the macrocrack tip often provide this type of stress raiser. Figure 2.6 schematically illustrates the mechanism of cleavage nucleation. It shows a microcrack around a secondary phase particle such as a carbide. Once the crack
nucleates, two things can happen. Either the crack remains sharp carrying with it the stress discontinuity ahead of it, causing quick propagation and failure. The other alternative is that the crack is arrested at a grain boundary or because of a steep stress gradient or a low externally applied stress intensity.

If the crack propagates, it will have to travel through different grains where the orientations of the fracture planes are different. This crack front will have to rotate or “twist” as it moves forward. Figure 2.7 shows such a process (Anderson, 1995). The crack is growing from grain A to grain B in this situation. The crack has to rotate because the cleavage planes in the two grains are different. The merging of cleavage planes over different grains cause so-called “river” patterns which can be seen in under a microscope – Figure 2.8.

It is striking to note how smooth the cleavage fracture surface (Figure 2.8) looks as compared to the fracture surface due to microvoid coalescence. The shiny, faceted appearance of the fracture surface is the defining characteristic of cleavage fracture. This type of fracture shows a falling resistance curve, which causes sudden crack propagation, meaning that the crack can be self-propelled, with the rate strain energy released by the propagation exceeding the rate of strain energy required to rupture the material. Refer Anderson (1995) for more details.

2.1.3 The Ductile Brittle Transition

Though microvoid coalescence and cleavage fracture are two different and seemingly unrelated mechanisms, very often fracture begins through the ductile mechanism of microvoid growth and coalescence, and suddenly transitions into a brittle cleavage mode. Evidence of this is seen in the resulting fracture surface which starts out as a ductile dimpled surface that transitions to a brittle shiny surface. Such behavior is observed in situations of intermediate temperatures, as well as intermediate constraint.
The mechanism for ductile to brittle transition is fairly straightforward in terms of the underlying physics. The crack initially grows by ductile tearing, and samples more and more material as it travels. Eventually a critical particle is sampled and cleavage occurs. This process is highly statistical in nature because the toughness of the material is strongly controlled by the distance of the nearest critical particle from the initial crack front. (Heerens and Read, 1988). The location of the critical particle is random, and hence the fracture process is random as well. There is a very strong correlation between the distance of the crack front to the nearest critical particle and the fracture toughness. Looking at a fractograph of a surface ruptured in this manner shows cleavage type shiny facets and river patterns with some spots or patches of ductile tearing.

2.1.4 Intergranular Fracture

Though most metals do not fail along grain boundaries, there are situations when cracks can form and propagate along grain boundaries. Referring to Figure 2.9, this mode is called intergranular fracture. Brittle phases that are deposited on grain boundaries through a variety of metallurgical processes can be a trigger for such behavior. Other situations that weaken the grain boundaries include environment assisted cracking, intergranular corrosion and hydrogen embrittlement. The failure surface due to such a mechanism is shown in Figure 2.10.

2.2 Mechanisms of Fatigue in Structural Steel Materials

Fatigue can be defined as crack propagation and failure due to the application of cyclic stresses. Cyclic loading is found to have a very detrimental effect on the strength and ductility of structures and mechanical components. Similar to fracture, fatigue can be controlled by many different mechanisms, based on the stress conditions, loading history and material microstructure. Fatigue is broadly classified between high-cycle fatigue and low-cycle. In the former case, small amplitudes of stress and strain cause failure of the component only after a few hundred thousand cycles. In the latter, the stress and strain amplitudes are much larger, and failure is achieved within tens or hundreds of cycles.
These definitions are often not mechanism-based, rather they are but phenomenological, This is an important distinction to consider during the development of micromechanical models for fatigue.

Fatigue mechanisms are typically dependent on the range of stress applied to the crack tip. Based on the stress range (or stress intensity factor range, as described in Section 2.5), fatigue is characterized as being Stage I, II or III. Figure 2.11 shows a schematic log-log plot of stress range versus the growth rate (expressed in crack advance per cycle). Each of these stages has different mechanisms. As a crack grows in length, the stress intensity factor usually increases, and the crack moves from Stages I to III, eventually fracturing when the rate of propagation becomes infinite.

2.2.1 Stage I Fatigue

Figure 2.11 shows a “fatigue threshold” for the stress or stress intensity range above which the curve bends over and the crack growth rate increases substantially. Around this level of stress intensity, crack tip plasticity is very limited, such that it does not extend into neighboring grains. As a result, large plastic deformation takes place along highly localized slip bands known as persistent slip bands, and fatigue damage accumulates on crystallographic planes along these slip bands. The damage concentration in these bands causes the zigzag crystallographic crack growth. Since microstructure affects slip it has a strong effect on Stage I fatigue.

2.2.2 Stage II Fatigue

Stage II fatigue corresponds to the middle part of the curve during which the crack growth rate increases uniformly with increase in stress intensity plotted on the log-scale (Paris and Erdogan, 1963). In this type of fatigue also known as Paris type fatigue, the commonly accepted mechanism (Laird, 1979) is illustrated in Figure 2.12. The crack tip blunts as load is applied, and the crack expands due to the formation of a stretch zone ahead of the crack tip. When the loading direction is reversed, the crack tip folds inwards
as the material is squeezed into the crack tip sharpening it. In the next cycle, the crack tip blunts again causing crack extension. Such a mechanism often causes striations on the fracture surface, which are small ridges perpendicular to the direction of crack propagation. These are caused because the re-sharpening of the crack is not able to recover all the deformation experienced during blunting. This mechanism is often called the slip and decohesion mechanism.

Fatigue crack growth due to the slip and decohesion is not strongly influenced by material microstructure. Thus, for example two steel samples (both iron alloys), or two aluminum alloys with totally different microstructure will likely exhibit very similar fatigue properties. A possible explanation for this is that cyclic flow properties, which are often similar for different alloys of the same metal, and insensitive to microstructure, govern this type of fracture.

2.2.3 Stage III Fatigue

When the stress intensity is very large, there is an interaction between fracture and fatigue mechanisms. There is a combination of fracture processes such as microvoid coalescence and cleavage, as well as typical fatigue mechanisms such as blunting and re-sharpening of the cracks. As the stress intensity increases, the relative contribution of the fatigue mechanisms decreases and the mode switches mainly to fracture.

Stage III fatigue typically causes failure in very few cycles (of the order of ten), and is very strongly influenced by microstructure, because the contributing fracture processes such as microvoid coalescence and cleavage are sensitive to microstructure. We will refer to this type of fatigue as Ultra Low Cycle Fatigue (ULCF) to distinguish it from the low cycle fatigue which typically involves a few hundred cycles to failure.

Sometimes in literature, the term describing the type of fracture or fatigue is used synonymously with the mechanism of fracture or fatigue. As an example, ductile fracture has come to be associated with microvoid coalescence, while brittle fracture is associated
with cleavage fracture. However, in the broad sense, the words ductile and brittle refer to
the total magnitude of yielding in the specimen or structure before fracture, which are not
necessarily indicative of the fracture process. So, for example it is possible to have large
scale yielding in the specimen before cleavage occurs, and conversely it is possible to
originate fracture based on microvoid coalescence due to large local yielding with very
little overall yielding. In a similar way, the words high cycle fatigue and low cycle
fatigue are often used as different types of fatigue, whereas it is likely that the
mechanisms in these cases might be very similar. These issues will be revisited in detail
in a subsequent section that deals with current fatigue models.

2.3 Traditional approaches dealing with fracture

Fracture mechanics is a relatively recent field in engineering, and is still developing.
Fracture mechanics began with the development of the energy release rate concept by
Griffith in 1920 (refer section 2.3.2), and many approaches in the following years built
upon this concept. The concept is that a crack in a solid will grow when the strain energy
released by the crack extension exceeds the energy required to rupture the material to
grow the crack. Fracture mechanics, to a large extent has been aimed towards quantifying
the demand energy release rate as accurately as possible under different conditions. As a
result, the fracture toughness of a material has become synonymous with some form of
“capacity” energy release rate.

This type of fracture mechanics (which we shall refer to as traditional fracture mechanics
henceforth) does not aim to capture the fundamental mechanisms of fracture which
introduces some serious limitations and disadvantages –

1) It assumes that the capacity energy release rate is a fundamental material
property, whereas the capacity energy release rate is more related to the
micromechanism of fracture, e.g. cleavage fracture might show totally different
values as compared to microvoid coalescence. Because the micromechanisms are
not explicitly considered, defining the material toughness in this way will show
dependence on constraint conditions (plane-stress vs. plane-strain), loading rates and temperature.

2) It assumes that the energy release rate relates uniquely to the stress state. In reality, the stress and strain state governs fracture, and not the energy release rate. As a result, traditional fracture mechanics is an indirect means of quantifying fracture toughness, and if the one-to-one correspondence between energy release-rate and stress state is lost, then traditional fracture mechanics is unable to capture the fracture process.

3) Traditional fracture mechanics is single parameter fracture mechanics, and aims to quantify the entire stress strain field in terms of one parameter. Obviously, this cannot always remain true, especially in situations of large scale yielding.

Because of these reasons, there are significant limitations on the applicability of traditional fracture mechanics. As an example, when there is large-scale plasticity, the energy release rate no longer uniquely corresponds to the stress-state, and it cannot be reliably used for predicting fracture.

Another problem, though somewhat less discussed, is that determination of energy release rate itself might be an issue, especially if the crack tip stress field experiences unloading, due to crack growth or load relaxation (low-cycle fatigue). Traditional fracture mechanics approaches are strictly valid for linear elastic or nonlinear elastic behavior of materials. Therefore incremental plastic behavior (unsymmetrical response to loading or unloading), which is more representative of true behavior of metals, can invalidate the estimate of the energy release rates. However, experimental techniques to measure the characteristic toughness (energy release rate) are designed to minimize such errors. In this section, we begin by stating the problem of stress singularity, and then discuss the different approaches used to deal with the fracture problem.
2.3.1 Early work and the stress singularity problem

It was widely recognized by the beginning of the twentieth century that fracture would occur due to the presence of flaws or cracks that concentrate the stress in a material. Inglis (1913) mathematically studied the stress concentration effect of such flaws and derived an expression for the stress concentration due to a long elliptical flaw. See Figure 2.13 and Equation (2.2), which shows the stress concentration solution in a plate with an elliptical hole.

\[ \sigma_A = \sigma \left(1 + 2\sqrt{\frac{a}{\rho}}\right) \]  \hspace{1cm} (2.2)

Where \( a \) is the long dimension of the ellipse as shown in Figure 2.13, and \( \rho \) is the radius of curvature, which can be expressed by the ratio –

\[ \rho = \frac{b^2}{a} \]  \hspace{1cm} (2.3)

Where \( a \) and \( b \) are the major and minor radii, \( \sigma \) is the far-field stress, and \( \sigma_A \) is the maximum stress at the flaw tip. The expression in parentheses represents the stress concentration factor for such geometry.

If the radius of curvature of such a flaw were to be zero, in other words the crack was to be infinitely sharp, then the stress at the crack tip would rise to infinity, according to Equation 2.1. Of course, it is not possible to have such a situation, because no material can sustain infinite stresses. This presents a difficulty to characterizing fracture behavior, since it is impossible to define a critical “fracture stress” at the crack tip. This led Griffith (1920), to postulate the concept of the energy release rate, which forms the basis of all traditional fracture mechanics. The problem, as described earlier, is that this an indirect approach to fracture (because it is the stress field and not the energy release rate as such which governs fracture), and this approach is valid only as long as the energy release rate maintains a unique correspondence with the local crack tip stress field.
Griffith started by looking at a sharp crack of length $2a$ in a continuum as shown in Figure 2.15. The system contains a potential energy given by –

$$\Pi = U - P\Delta$$  \hspace{1cm} (2.4)

Where $U$ is the strain energy and $P$ and $\Delta$ represent the force and displacement applied to the system causing the external work done. $\Pi$ is the potential energy of the system. If the crack were to extend by a very small area, $dA$ then the energy in the system would be lowered (due to relaxation of stresses). Using stress analysis, it is possible to show that the change in energy is proportional to the square of crack length $a$, such that –

$$\Pi = \Pi_0 - \frac{\pi \sigma^2 a^2}{E}$$  \hspace{1cm} (2.5)

On the other hand, as the crack length increases, more and more energy is required to create the crack by rupturing the material. Assuming perfectly elastic behavior until rupture, this energy should be directly proportional to the crack length (by some constant of proportionality relating to the surface energy of the crack). Thus, we have a relationship of the following type –

$$W_{\text{crack}} = w.a$$  \hspace{1cm} (2.6)

In the neighborhood of zero crack length, a small crack extension does not release appreciable energy at all, however as the crack length grows, the amount of strain energy released per unit crack extension increases. This energy release rate is given by (note that

$$G = \frac{d\Pi}{dA} = \frac{\pi \sigma^2 a}{E}$$  \hspace{1cm} (2.7)

Where $A$ is the crack area which is 2 times the crack length times the unit width of the plate continuum). Initially, a small extension of the crack will require more energy to
create than the strain energy released by this extension. Thus crack extension is not feasible. As the crack grows, or the stress state becomes more severe, a point comes where the strain energy released by the crack extension is sufficient to cause the extension of the crack. This will happen when the energy release per unit crack extension is equal to the energy required for unit crack extension. So the critical energy release rate is given by –

\[ G_c = w \]  

(2.8)

The left hand side of this Equation is the demand parameter, calculated based on the remotely applied stress and the crack length, whereas the right hand side of the Equation is considered an inherent material property.

Definition of toughness by the energy release rate is the basic underlying principle of traditional fracture mechanics. The evolution of traditional fracture mechanics has typically involved finding better methods of predicting either the demand or the capacity energy release rate, or adapting existing methods to different situations of loading, material or geometry. This is obviously a problematic approach, because the correlation of the far-field parameters with the near-tip stresses may not always be exact. On the other hand, the computational power required to characterize the near tip stresses and strains has become accessible to researchers only recently.

Today, with advances in computational technology, we are in a position to characterize stress and strain fields at the crack tip, thus solving the problem directly, as opposed to the manner in which Inglis and Griffith attempted. The major gap in knowledge is the models that are required to use these stress and strain fields to predict failure in engineering problems. The following sections will look at traditional fracture mechanics, and then at some of these existing novel micromechanical models, finally discussing some limitations and issues.
2.3.2 Linear Elastic Fracture Mechanics

Linear elastic fracture mechanics, as the name suggests implies linear elastic behavior of the material during the fracture process. Based on this assumption, it is possible to determine the stress conditions around a crack. Figure 2.14 shows the solution for an infinite plate with a crack of length $2a$. A far field transverse stress $\sigma$ is applied to the plate, and the stress and strain situation is analyzed. It should be noted here that we will limit our discussion to Mode I type loading (in-plane opening), as opposed to Mode II and III (in-plane sliding and out-of-plane tearing respectively). Theoretical elasticity derivations show that any of the stress components can be represented in general by the expression in Equation (2.9)

$$\sigma_{ij} = \frac{K_I}{\sqrt{2\pi r}} f_{ij}(\theta) + \text{2nd term} + \text{3rd term} + \cdots + \text{nth term}$$  \hspace{1cm} (2.9)

where $K_I$ denotes the stress-intensity factor for Mode I crack opening – which is the amplitude of the stress-field singularity, $r$ is the distance ahead of the crack tip, and $f$ is an angular function that scales the singularity for the stress component under consideration. For example, the stress component $\sigma_{22}$ would be described by the angular function $f_{22}$ –

$$f_{22}(\theta) = \cos \frac{\theta}{2} \left(1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2}\right)$$  \hspace{1cm} (2.10)

The stress intensity factor can be related to the far field loading conditions through analytical elasticity or some computational procedure such as the finite element method. Stress intensity factors for various commonly observed geometries have been determined and are often found in texts relating to fracture mechanics, e.g. see Anderson (1995). The stress intensity factor, for a linear elastic situation can be uniquely related to the energy release rate according by the Equations (2.11) and (2.12) –
\[
G = \frac{(1 - \nu^2)K_I^2}{E} \text{ for plane strain} \tag{2.11}
\]

\[
G = \frac{K_I^2}{E} \text{ for plane stress} \tag{2.12}
\]

Equation (2.9) has many higher-order terms which are not explicitly stated, which consider the effect of far field stresses, or geometric effects. They do not contain the singularity term, and hence do not significantly affect the stress field very near the crack tip which is dominated by the leading inverse square root \(1/\sqrt{r}\) singularity.

Linear elastic fracture mechanics assume linear elastic behavior of the material, and works best in the most brittle cases where behavior remains elastic until the point of fracture. In many cases, such as fracture of metals, LEFM is found to be reasonably accurate even when there is localized yielding prior to fracture. Moreover, one can be curious about the dependence of fracture on far field stresses and geometric configuration, i.e., is it justifiable to use a single parameter approach to consider contributions of all these effects. It is useful to discuss the applicability of LEFM in such situations to provide a better insight into the working and limitations of LEFM.

As discussed earlier, in the region very close to the crack tip, the stress state is governed almost totally by the singular term of Equation (2.9). Geometry effects are typically seen in the other terms. This means that there is always a region around the crack tip, where the singular term characterizes the stresses with some accuracy (e.g. the singularity term contributes 90% of the stresses near the crack tip). We can refer to this region as the K-dominance zone. If the plasticity in the sample (the process zone where fracture actually occurs), is buried deep inside this region, then the stresses on the boundary of the K-dominance zone will not be greatly affected by this behavior. This means that the stresses at this boundary will be the same (or nearly the same) for two different situations (say a structure and a lab specimen), as long as the stress intensity factor is similar for both of them. Moreover, if the stresses at the boundary uniquely correspond to a given value of a stress intensity factor, it is reasonable to assume that all the stresses and strains inside this
boundary will uniquely correspond to that stress intensity factor, whether they are elastic or not.

As the plastic zone grows in size it influences the stresses at the K-dominance zone boundary, which then start to depend on the specimen geometry. In this case, if a structure and a specimen have the same stress intensity factor, it could very well be possible that the stresses are totally different, because the plastic zone extends well outside the K-dominance zone, which means that the K-dominance zone is no longer a valid measure of the cracking behavior. Refer Anderson (1995), section 2.10 for a detailed description of this problem.

Linear elastic fracture mechanics is assumed to be valid when there is “well contained” yielding. Typically, this is true if the specimen dimensions are about fifty times (Anderson, 1995) the dimension of the plastic zone. According the ASTM standards, LEFM is a valid approach for fracture characterization if the dimensions of the specimen are larger than prescribed the formula in Equation (2.13) –

\[
Specimen\ Size \geq 2.5 \left( \frac{K_I}{\sigma_{yield}} \right)^2
\]  

(2.13)

If the plastic region becomes larger than the admissible size, then the correspondence between the stress intensity factor and the local stress field in the fracture “process zone” is no longer unique. This creates problems for the applicability of linear elastic fracture mechanics to situations with large scale plasticity. Elastic plastic fracture mechanics aimed to improve upon this shortcoming of LEFM and is discussed in the following section.

2.3.3 Elastic Plastic Fracture Mechanics

One could imagine that if we developed a model that predicts the energy release rate more accurately than LEFM, then it would approximate the stress-strain fields more accurately, which would increase the dominance zone of the energy release rate based model.
One of the most popular approaches for this purpose was developed by Rice (1968), where a path independent contour integral was proposed as the energy release rate parameter. This contour integral, known as the J-integral is calculated as follows. An arbitrary contour beginning on the bottom surface of the crack (as shown in Figure 2.15) and traveling counterclockwise around the crack tip to reach the top crack face is chosen. The contour integral of Equation (2.14) is then evaluated along this path (Refer Rice’s paper for mathematical derivation). The directions $X_1$ and $X_2$ shown in Figure 2.15 are defined parallel and perpendicular to the crack propagation direction, and they show up accordingly in the J-integral Equation.

$$J = -\frac{d\Pi}{da} = -\int_A \frac{dW}{da} dA + \int_T \frac{du_i}{da} ds$$

(2.14)

Simplifying further, we can obtain –

$$J = -\frac{d\Pi}{da} = \int_T \frac{du_i}{\partial x_i} ds$$

(2.15)

In these Equations, the $W$ represents the strain energy density, the vector $\tilde{T}$ is the traction vector on the boundary of the contour, $\tilde{u}$ is the displacement vector, and $x_1, x_2$ are the direction coordinates, and $ds$ is the incremental distance along the contour. This quantity, $J$, or the J-integral is an estimate of the energy release rate in a nonlinear elastic material. This means that this integral exactly quantifies the energy release rate due to infinitesimal crack extension taking into consideration the nonlinear behavior of the material. Thus, this estimate of energy release rate is no longer limited by the linear elasticity assumption.

The J-integral, taking into consideration the nonlinear effects, can characterize fracture behavior in situations of elastic-plastic behavior, long after the LEFM assumptions have broken down. The most important reason for this is that the J-dominance zone is typically much larger than the K-dominance zone. As an example, we can reference the HRR
singularity stress field, which can be mathematically related to the J-integral. This stress field, proposed simultaneously by Hutchinson, Rice and Rosengren (1968) which captures the nonlinear stress-strain relationship by using the Ramberg-Osgood relationship in the stress field derivation. The stress field shows a unique dependence on the J value, for hardening materials. This proves that the J-integral can be used as a stress intensity parameter in certain cases.

The J-integral has a larger zone of influence as compared to the K-dominance zone, and as a result is more applicable in more situations than LEFM. However, it has some limitations. An important limitation, similar to the LEFM approach is due to the fact that the energy release rate is an indirect approach to fracture (i.e., it does not address the problem directly through the stress field). When the plastic zone is very large, it can typically destroy even the J-dominance zone. The primary reason for this is that the J integral predicts a singular stress field similar to the K-field, however the order of the singularity is different. However, crack tip blunting that always occurs during plastification and destroys the singularity.

Moreover, the three-dimensional constraint issue is also raised here, i.e., it is observed that thicker geometries are more prone to fracture as compared to thinner geometries (due to larger stress constraint), and the J-integral, being two-dimensional by definition, cannot pick this effect up. Here, we can reference the J-Q theory proposed by O’Dowd and Shih (1991-1992), which aims to characterize material toughness by a dual parameter failure envelope called the J-Q locus, where the J represents the J-integral, while the Q represents a stress constraint ratio derived from the HRR singularity field. As expected, the critical J value decreases with increasing Q. There are difficulties in the application of this approach, because the J-Q locus often shows geometry dependence which makes it difficult to categorize as a fundamental material property.

Another issue with the J-integral is its application to situations with a growing crack. If the crack is growing, and the “crack growth resistance curve” needs to be computed, then the crack leaves behind a plastic “wake” as it moves forward (see Figure 2.16). This area
represents the region where the material has been plastically loaded but then unloads elastically unloaded as the crack moves through. The J-integral assumes all the material to be nonlinear elastic (a deformation plasticity type approach), but the unloading violates this assumption, and the energy release rate estimate is questionable (especially in situations of large crack growth).

Despite the limitations and assumptions, the J-integral remains one of the most popular and widely used approaches in fracture mechanics because it can be successfully applied to many practical situations.

Before the J-integral was formally developed and applied by Rice, another approach, known as the crack tip opening displacement approach was being explored in Britain. Irwin in 1961 had shown that crack tip plasticity had a similar effect on stress field as lengthening the crack. Wells (1961) identified that this effect could be used to relate the crack tip opening to the stress field. He proposed a toughness criterion where fracture would occur if the crack tip opening displacement (CTOD) would exceed a critical value. A number of alternative definitions of CTOD are possible and one known as the 90 degree intercept definition is shown in Figure 2.17.

A number of later studies have shown that the J-integral and the CTOD approaches are in fact analogous, and the crack tip opening displacement can be related mathematically to the J-integral. A convenient derivation can be based on Dugdale’s analysis (1960), and is a part of most fracture mechanics textbooks, e.g., Saxena (1997). Thus, the CTOD is a legitimate stress intensity parameter.

Equation (2.16) shows such a relationship. As can be seen from the Equation, the relationship depends on the constraint condition through the parameter m. For plane strain, this parameter is 2.0, and for plane stress it is 1.0.

\[ \text{CTOD} = \frac{J}{m\sigma_y} \]  

\[(2.16)\]
2.3.4 Summary of traditional approaches dealing with fracture

Section 2.3 covers many of the basic ideas and evolution of traditional fracture mechanics, which has been an invaluable tool for predicting fracture nearly half a century, before high-performance computers became widespread. However, it does not take full advantage of modern computing to characterize fundamental stress-strain states at the crack tip. This simplicity has caused it to overlook some of the micromechanisms that cause fracture. As a result, it has limitations in characterizing failure under large scale yielding conditions. Newer micromechanical models that are computationally intensive can often improve fracture predictions by characterizing crack tip stress and strain conditions under some of the conditions that violate assumptions basic to traditional fracture mechanics. The next few sections will introduce some such models, and present the scope for their improvement and application.

2.4 Micromechanical Models for Predicting Fracture in Metals and Metallic Alloys

The energy release rate concept, which offered a brilliant solution to the stress-singularity problem in the early part of the century, also introduced limitations in traditional fracture mechanics. A direct stress and strain field approach would provide a far more realistic simulation of fracture under situations such as large scale yielding. Such an approach would aim to capture the stress-strain interaction and their influence on the microstructural features, which cause fracture. This would require modeling the stresses and strains at a length scale that will be set by the dimensions of the microstructural features that cause fracture, e.g. the grain size. Such modeling is extremely computationally intensive, and could not be carried out in the past. Today, one can model the stress and strain fields at the microstructural level using extremely detailed finite element analyses, and use this data to predict fracture due to a variety of mechanisms. However, this requires the development of reliable and advanced models that can take advantage of the computational ability to determine the stress and strain fields at these microstructural scales.
There has been some research in this direction, but much of it is of an exploratory nature. Rigorous verification and development of such models has not been carried out, especially where fatigue or cyclic loading is involved. In the following section, some models that have been proposed in this direction are introduced, and two of them – the Void Growth Model (VGM) and the Stress Modified Critical Strain model (SMCS), is studied in detail. The concepts underlying these models are found to be very suitable for extension to cyclic loading or low cycle fatigue.

These micromechanical models are divided into two categories, one dealing with fracture due to the microvoid growth and coalescence mechanism (“ductile” crack initiation), and the other category dealing with all other mechanisms, such as cleavage. This distinction is made because the primary thrust of this study is looking at ductile crack initiation in metals caused by the microvoid growth and coalescence mechanism. The other models are briefly discussed because in some of the tests of this study, mechanisms other than microvoid coalescence are observed. Though these are not investigated in detail, a brief explanation is provided with suitable references that discuss this behavior further for each.

2.4.1 Micromechanical models predicting fracture due to void growth and coalescence

The following model development is based on the assumption that ductile fracture is caused by microvoid nucleation, growth and coalescence. In materials where second phase particles are well-bonded to the metal matrix, void nucleation is the governing step, because fracture occurs very soon after the voids nucleate. However, in many cases, the void nucleation may occur very easily, and the processes of growth and coalescence would control fracture. In most ductile or tough materials, void growth and coalescence generally dominates.

A popular void nucleation model was proposed by Argon et. al. (1975), who hypothesized that the interfacial stress at a cylindrical particle was roughly equal to the
sum of the hydrostatic and effective (von Mises) stresses. Following this assumption, the critical stress criterion for void nucleation can be written as –

\[ \sigma_c = \sigma_e + \sigma_m \]  

(2.17)

Where \( \sigma_m \), \( \sigma_e \) are the mean and the effective stresses, and \( \sigma_c \) is the critical nucleation stress. Other empirical relationships such as one by Beremin et. al. (1981) argue that the effective stress term in Argon’s Equation should be replaced by a fitting parameter multiplied by the difference between the effective stress and the material’s yield stress.

McClintock (1968) and Rice and Tracey (1969) derived Equations for the growth of single cylindrical and spherical voids in an elastic perfectly plastic material under plastic strain applied in the presence of a triaxial stress state. Their findings were essentially similar, identifying that the void growth rate is exponentially related to the triaxiality of the stress state. According to Rice and Tracey’s result for a spherical void, the void growth rate can be determined as –

\[ \frac{dR}{R} = 0.283 \exp \left( \frac{1.5 \sigma_m}{\sigma_y} \right) d\varepsilon_p \]  

(2.18)

Where \( R \) is the instantaneous void diameter, \( \sigma_y \) is the yield stress, and \( d\varepsilon_p \) is the equivalent plastic strain defined by Equation (2.19) and \( d\varepsilon_p \) is its incremental form defined by Equation (2.20)

\[ \varepsilon_p = \sqrt{\frac{2}{3} \varepsilon_p^e \varepsilon_p^e} \]  

(2.19)

\[ d\varepsilon_p = \sqrt{\frac{2}{3} d\varepsilon_p^e \cdot d\varepsilon_p^e} \]  

(2.20)

Integrating Equation (2.18), the total void growth during a plastic loading excursion can be calculated as –
\[
\ln \left( \frac{R}{R_0} \right) = 0.283 \int_0^{\varepsilon_p} \exp \left( \frac{1.5 \sigma_m}{\sigma_e} \right) d\varepsilon_p
\] (2.21)

The left hand side of this Equation can be interpreted as a critical void size (or critical void ratio of final to initial void size), and the right hand side would represent the imposed demand as a function of the stress and strain history.

Rice and Tracey’s Void Growth Model, or VGM as we will refer to it henceforth, is fairly basic and has a few limitations. First, it does not account for multiple void interaction behavior leading up necking instability between voids, and second, the analysis is limited to elastic perfectly plastic materials. To account for hardening behavior in the Equation, D’Escata and Devaux (1979) replaced the yield stress by the effective or von Mises stress \( \sigma_e \). This changes the form of Equation (2.21) to (2.22) –

\[
\ln \left( \frac{R}{R_0} \right) = 0.283 \int_0^{\varepsilon_p} \exp \left( \frac{1.5 \sigma_m}{\sigma_e} \right) d\varepsilon_p
\] (2.22)

The ratio of the mean stress to the effective stress is termed as triaxiality, \( T = \frac{\sigma_m}{\sigma_e} \) (which is a measure of the constraint of the stress state). While Rice and Tracey did not propose a failure criterion within the scope of their paper, it is easy to see how a failure criterion could be applied to this model. One could postulate that ductile crack initiation occurred when the voids grow under the applied plastic strain until they reach a critical void size or a critical void ratio. Thus the failure criterion could be considered as –

\[
\ln \left( \frac{R}{R_0} \right) = \ln \left( \frac{R}{R_0} \right)_{\text{critical}} = 0.283 \int_0^{\varepsilon_p} \exp(1.5T) d\varepsilon_p
\] (2.23)

For predicting fracture using this model, a FEM analysis would be conducted which records the triaxiality and plastic strain, and incrementally calculates the integral on the right hand side till it side reaches its critical value (the left hand side), thus predicting fracture. The coefficient 0.283 on the right hand side of Equation (2.23) was obtained.
from a curve fitting procedure by Rice and Tracey. Denoting this by a constant $c$, we could simplify Equation (2.23) –

$$
\eta = \ln \left( \frac{R}{R_0} \right)_{\text{critical}} = \frac{\varepsilon_p}{c} = \int_{0}^{\varepsilon_p} \exp(1.5T) \, d\varepsilon_p \quad (2.24)
$$

Where $\eta$ is the material capacity in terms of void growth normalized by a coefficient controlling void growth rate. This is assumed to be a material property, and can be calibrated using tensile notched bar tests and used in finite element analyses for failure predictions. This forms the focus of Chapter 4.

Such a failure criterion does not explicitly account for void to void interactions, since it deals with only one void. However the effect of interaction of voids is included through the critical void size, which might be indicative of inter-void necking instabilities.

Rice and Tracey’s VGM includes an explicit term for integrating the triaxiality with respect to plastic strain. However in many realistic situations the material does not deform a great amount such that the triaxiality (which is geometry dependent), remains largely unchanged even as the plastic strain increased rapidly. Figure 2.18 shows a typical plot of the plastic strain versus triaxiality plot. As can be seen from Figure 2.18, for many situations, the triaxiality is largely independent of the plastic strain, and consequently, the terms inside the integral in Equation (2.24) can be represented as a product, where the VGM model reduces to the following –

$$
\ln \left( \frac{R}{R_0} \right)_{\text{critical}} = C \cdot \exp(1.5T) \cdot \varepsilon_p^{\text{critical}} \quad (2.25)
$$

This leads to the critical plastic strain being expressed as –

$$
\varepsilon_p^{\text{critical}} = \frac{\ln \left( \frac{R}{R_0} \right)_{\text{monotonic}}^{\text{critical}}}{C \cdot \exp(1.5T)} = \frac{\ln \left( \frac{R}{R_0} \right)_{\text{monotonic}}^{\text{critical}}}{C} \cdot \exp(-1.5T) \quad (2.26)
$$
Recognizing that both terms on the right hand side are basic material coefficients, these can be combined into the single material parameter $\alpha$, which is given as:

$$\alpha = \frac{\ln \left( \frac{R}{R_0} \right)_{\text{critical}}}{C}$$

(2.27)

This forms the basis of the SMCS model. This is based on an explanation by Hancock (1980) who concluded that the single-material point ductile tearing depends upon the interactions of triaxiality and plastic strain. As the triaxiality increases, the critical value of plastic strain reduces, as is illustrated in the SMCS Equation (2.28) –

$$\varepsilon_p^{\text{critical}} = \alpha \cdot \exp(-1.5T)$$

(2.28)

In either the SMCS or the VGM, the ductile fracture initiation is quantified as global behavior and not single point behavior, and therefore a “length-scale” parameter needs to be included to collect multiple single-material point failures, i.e. did we sample enough voids in our analysis to be able to apply the model. Thus a length scale parameter is introduced which completes the definition of these models. The model states that the occurrence of the ductile fracture initiation is triggered once the VGM or the SMCS reach their critical condition over the characteristic length, $l^*$. The choice and determination of this length scale is based on microstructural measurements as well as finite element analyses. This aspect of the models is explained in detail in Chapter 4.

Thus the ductile fracture criteria can be described in Equations (2.29) and (2.30). As deformation increases, the criterion is satisfied over a progressively increasing distance, until it is satisfied all over the characteristic length and fracture is predicted to occur. In case of the SMCS model, the simplified fracture initiation criterion called Strain Modified Critical Strain Criterion (SMCS) is simply the difference between the imposed plastic strain $\varepsilon_p$, and the critical plastic strain given by Equation (2.28),
\[
SMCS = \varepsilon_p - \alpha \exp\left(-1.5 \frac{\sigma_m}{\sigma_c}\right) > 0 \quad \text{for } r \leq \tau^* \quad (2.29)
\]

Similarly for the VGM, we can envision a failure criterion where –

\[
\int_0^{\varepsilon_{\text{ref}}} \exp\left(-1.5 \frac{\sigma_m}{\sigma_c}\right) d\varepsilon_p - \eta > 0 \quad \text{for } r \leq \tau^* \quad (2.30)
\]

As indicated earlier, the SMCS model is based on the assumption that the triaxiality does not change appreciably with respect to increasing plastic strain, and hence the critical plastic strain depends only on the current level of triaxiality, and ignores history effects, which are not typically important unless large geometry changes are present. For many structural engineering type problems – this model seems to provide the simplicity of a direct model not requiring integration as well provides good correlation between tests and analytical predictions.

For situations with large geometry changes in very ductile materials, the triaxiality often changes appreciably, and as a result, the VGM might be the more appropriate model, though it is slightly more cumbersome to use due to the numerical integration of the stress-strain histories. For most situations, we recommend using the SMCS due to its simplicity, but if large triaxiality changes or deformations are present, the VGM should be used.

Other popular models that have been studied in this regard are Gurson’s (1977) void growth model – which consists of voids enclosed in a base metal matrix that obeys the von Mises yield criterion and Hill plasticity. The model is especially useful for predicting material response under situations of porous metal plasticity (i.e. how a metal behaves when voids nucleate within it – in terms of stress strain behavior). However, it has been found to have many disadvantages with respect to fracture prediction – these include the indirect approach to fracture, a large number of parameters to calibrate and special numerical techniques to deal with the implementation of the model for crack propagation.
type problems. For further reading about the Gurson model, we direct the reader to Gurson (1977), Mear (1993), and for an application – Skallerud and Zhang (2001). This model is primarily created for the monotonic case, and might not be able to pick up complicated residual stress type situations that might arise in cyclic loading, as a result, we do not use this model in our investigations.

It must be noted that the Gurson model is more suited for application to crack propagation type problems – because it is possible to embed cell elements with length scales into the FEM mesh, and make these elements extinct as the void ratio reaches a critical value. Studies by Dodds, Ruggieri and Panontin (1997) investigate the use of the Gurson model to propagate cracks and compute J-Resistance curves for metals.

2.5 Traditional Approaches for Dealing with Fatigue Processes

In this section, we revisit some of the fatigue mechanisms that have been discussed previously, and discuss some of the models popularly used to capture these mechanisms. We identify the problems and limitations of these models, and lay the groundwork for developing newer models in Chapter 3.

As with monotonic fracture, the nomenclature of the fatigue processes does not often reflect the mechanisms, and hence there is potential confusion as to why some models work for certain situations and not others. Sometimes, we need to apply different models for such situations just because of the way the models are defined (and incapable of relating directly to the stress-strain situation at the crack tip), and not because there is a change in mechanism. For example, if a very large specimen is constructed from a ductile metal, a K-test might still be valid, though the mechanism causing crack initiation is ductile tearing at the crack tip. Using a J-test for a smaller specimen does not indicate a different “type” of fracture as regards the fundamental process, but only as far as the correspondence between far-field and near-tip parameters is concerned. For a fundamental model such as micromechanical models, specimen size should not matter,
because there is no issue of categorizing fracture as “ductile” or “brittle”, as long as a specific mechanism is identified.

As discussed earlier, there seem to be three predominant micromechanisms of fatigue crack propagation in mode I type of loading, depending on the stress range applied. For very small stress ranges, the plastic zone is so small, that it does not even extend into neighboring grains. As a result, crack growth occurs due to damage of crystallographic planes. It occurs at typically 45 degrees to the crack direction (due to maximum shear stresses in that direction), and in a zigzag fashion. This is Stage I fatigue, and it is very sensitive to microstructure. As a result, macroscopic models are not very common for this type of fatigue. Also, it is not of great interest, because the fatigue life at these stress levels is likely tens of millions of cycles.

The second and most commonly observed mechanism (in most mechanical and aerospace applications) is the sharpening and blunting of the crack tip, which causes striations on the fatigue surface. The crack grows a little bit each time the material ahead of the crack is squeezed, where the fracture mechanism is the slip and decohesion. This type of fatigue largely constitutes Stage II fatigue.

Growth in Stage II is directly related to the range of shear strain at the crack tip. Thus, the crack growth rate is related to the “local” strain range. To develop a model which captures the fatigue propagation mechanism, one would need to determine a far field stress or strain parameter, which would correspond uniquely to the local strain range. If the crack loading is elastic, then the stress range has a fairly unique correspondence with the crack tip strain range for a given geometry. This is the basis for the applicability of the stress life approach, where the fatigue life of a component is related inversely to the stress range that it is being subjected to, by the means of S-N curves as shown in Figure 2.19.

S-N curves cannot be generalized for different geometries, because the relationship between local strain and global stress is specimen dependent. However, for small-scale
yielding type conditions, the local strain range relates well to $\Delta K$, thus enabling the use of the stress intensity factor as a fatigue parameter. This is the basis for the Paris Law (1963) which related the crack growth rate to the stress intensity factor range –

$$\frac{da}{dN} = C(\Delta K)^m$$  \hspace{1cm} (2.31)

Where $\frac{da}{dN}$ refers to the crack extension rate per cycle, and $C$ and $m$ are material constants. This region of stress intensity factor corresponds to the straight line on the log-log plot shown in Figure 2.11, and the fatigue is said to be in the Paris regime, where the stress intensity factor can directly be related to the crack growth rate. This type of fatigue mechanism typically falls in the category of high cycle fatigue.

Now if we look at the situation where the crack loading is elasto-plastic (at a global level), the stress range loses unique correspondence with the strain range at the crack tip. In such cases, it might be more useful to use a strain-life approach which relates the global strain amplitude to component life in terms of cycles. Curves similar to S-N curves can be obtained. These methods are established and are parts of standard texts on the subject, see Suresh (1998). As expected, the $\Delta K$ approach does not work for such situations because the stress has lost correspondence with the local strain range. Replacing the $\Delta K$, by the $\Delta J$ or the range of the J-integral (calculated based upon the cyclic stress-strain curve of the material), seems to provide much better correlation with experiments.

One must, however not conclude that the micromechanism of fatigue is different, but that its relationship with our macro parameters is not maintained anymore. This argument is supported by the fact that a $\Delta J$ approach works fairly well in characterizing fatigue under such situations, because it is better related to the local strain field. Equation (2.32) quantifies the $\Delta J$ approach. In form, it is very similar to the Paris $\Delta K$ approach –
\[
\frac{da}{dN} = C(\Delta J)^m
\]  

(2.32)

This is analogous to the case where crack tip plasticity invalidates the K-field, but not the J-field, the fracture mechanism remaining the same in each case. This type of fatigue is called low-cycle fatigue because of the short life span (100s or 1000s of cycles).

Based on the above discussion, we can conjecture that if we have to develop a micromechanical model for low-cycle fatigue, it might not be different from one for high cycle fatigue, since the mechanism is similar. Thus, we see that in most cases (excepting some where void formation or cleavage occurs at relatively lower strains or stresses) similar mechanisms dominate fatigue crack growth if the stress levels are such that plastic zone size is bigger than a few grain sizes, but the strains are not so severe as to be in the range of ductile crack initiation through void growth or coalescence. The macromechanical models often differ, because of their correspondence to crack tip strains.

Models for low-cycle of fatigue are well established. These include the stress and strain life models, as well as the $\Delta J$ and $\Delta K$ type models. Further refinements have been made to these models by various researchers, e.g., the load ratio dependence of Paris Law was successfully explained by crack closure theories, and elaborate rules, such as rainflow counting schemes were introduced to incorporate these models into variable amplitude loading.

Stage III fatigue is often studied the least. This is largely due to the fact that the perception in the mechanical engineering and aerospace communities is that if the local strains are so large that less than 10-15 cycles will produce fracture then it is not even a fatigue problem anymore, it is more of a fracture problem. Stage III fatigue is encountered in earthquake engineering and high-strain low-cycle fatigue. In the larger framework of the science of fatigue, it might be more logical to call it Ultra Low Cycle Fatigue (ULCF) which combines the mechanisms of fracture and fatigue. This brings us
to the interesting question of how fracture and fatigue are traditionally defined and treated in the analytical sense.

Fatigue is more often dealt with as a propagation problem, i.e. the key question in fatigue is to quantify the amount of crack growth in each cycle – as long as the stresses are above the threshold, the crack will always grow with each cycle. In fracture, the key question is whether a crack will grow at all. The explanation for such definitions is that fatigue typically corresponds to thousands or millions of cycles, and hence the rate of growth per cycle is more important than if the crack grows at all. The idea is that the blunting and re-sharpening mechanisms, as well as dislocation slip are always active, and cause crack very small extension, and this becomes significant when they cycles are large in number. When the strain fields are very large (as in fracture) and the cycles are very small, the growth per cycle (due to blunting and re-sharpening mechanism), is not as important as crack growth due to other mechanisms such as void growth. And hence, there is the question of whether a crack will grow at a given cycle or not.

A different approach to look at this issue would be to say that conventionally accepted definitions of fatigue refer to mechanisms where the crack grows by dislocation slip and material transfer. Such processes always occur during cyclic loading, and the extent per cycle of such growth is very small. If the cyclicity of the loading is halted, then at current stress levels, the crack will not grow further and cyclicity is necessary for crack growth. Conventional approaches defining fracture typically require separation of material due to some phenomenon such as void growth or cleavage, under static, monotonic stresses. Cyclicity is not necessary for such crack growth.

Referring to Figure 2.11, on the far right of stage III fracture, the crack growth rates seem to go to infinity, because in this region, cyclicity is not necessary for crack extension – it is more of a fracture problem. At such large strains, mechanisms of void growth and coalescence are likely much more dominant as compared to the slip mechanisms.
Thus, if the strains are so large that the crack growth occurs even without cyclicity, then the problem is more of a fracture problem as opposed to fatigue, and the component has already completed its life in the fatigue sense. As a result of this, the phrase low cycle fatigue is used for situations where failure requires around 100-10000 cycles, and the mechanisms are essentially similar to the Paris type fatigue. The only difference being that one has to use delta-J or strain based models, as opposed to delta-K or stress based models.

Also, this suggests that the problem of ultra-low cycle fatigue – where the fracture toughness or strain of materials reduces due to cycling, has not been studied. In other words, the damage accumulation due to cyclicity and its interaction with fracture mechanisms such as void coalescence and cleavage has not been studied in detail.

This problem is of certain interest to the metal forming community, who deal with the issue based more on empirical grounds. There have been few studies, one as late as 2001 (by Skallerud and Zhang), that aims to capture the fatigue-fracture transition by using a delta-J model, then followed by a Gurson-type monotonic model. This study shows that as the crack gets longer, the local strain fields get larger and larger, and the contribution of the static failure modes increases, and the crack growth per cycle is dominated by the void growth and coalescence mechanisms. To incorporate static void growth failure, the study uses cells of virgin Gurson material ahead of the crack tip, and cell extinction. What it does not consider, as a result of this, is the damage effects on the void growth and coalescence mechanism itself. It recognizes that the Gurson model, in its current state is not suited to capture this mechanism, because according to them, void shapes may change considerably during compressive loading. They also indicate that other studies to this end are topics of future investigation. Our study aims to fill this gap.

Another issue is multiaxial fatigue, which has been studied in detail for the slip type mechanism. This is not of immediate importance to us or our study, but included the sake of completeness. Brown and Miller’s (1973) critical plane model, later discussed by Fatemi and Socie (1988), is an important contribution in that direction. According to
Brown and Miller, the fatigue crack will propagate along the plane of maximum shear strain, but a secondary effect is that the tensile strain across this maximum shear strain plane affects the dislocation mobility enhancing the decohesion process. A law involving the combination of the maximum shear strain and the tensile strain provides better correlation with crack propagation, especially in brittle materials where the crack growth might be controlled by the maximum principal strain. However, according to Fatemi and Socie – “shear based critical plane approaches are appropriate for situations where mode II failure mechanism (shear mode) is predominant. But since combinations of materials and loading conditions result in different failure modes, it is unlikely that shear based approaches with fixed parameters will be applicable in all multiaxial fatigue situations”.

So, it is important to point out here that the critical plane type models are geared towards mechanisms of slip and decohesion, and not towards the more static or monotonic modes of fracture such as void growth.

Considering all these points, one could anticipate the scope of the models that we aim to propose for our studies. Our models for ULCF would aim to solve the mode I type fatigue models in the extreme stage III of fatigue, where fracture could be caused even due to static or monotonic mechanisms, without requiring cyclicity. Such models would look at the fatigue problem as a accelerated fracture problem, where an equivalent fracture capacity of the material is degraded based on the cyclicity.

Considering all the different mechanisms of fatigue, it might not be the best idea to have micromechanical models that merge into each other, i.e., an ultra-low cycle fatigue model that is a specific case of a more general fracture model which includes low cycle as well as high cycle fatigue. This is because the mechanisms are so different, and practically independent of each other. The better idea would be to have models all active at the same time, and depending upon the current loading conditions, one of them would drive the fatigue/fracture process.
2.6 Summary of Current State of Research and Future Direction of Work

This chapter sums up the current state of research in the fields of fracture and fatigue, especially as applicable to the disciplines of structural engineering and earthquake engineering. The initial part of the chapter presents the different mechanisms leading to different types of fracture and fatigue processes. In the process, it aims to clarify commonly held misconceptions about the different failure modes, often due to somewhat inaccurate terminologies.

The significant models – such as the void growth and coalescence model are presented to introduce the reader to the fundamental processes of fracture and fatigue, as well as to uncouple discussion of the mechanism from some more general definitions of fracture and fatigue such as “ductile” or “brittle” fracture.

Traditional approaches for dealing with the fracture and fatigue problems are then introduced, and in each case an attempt is made to explain why each of the popular approaches work even if they might not be able to capture fundamental behavior or micromechanisms directly, e.g., in the case of linear elastic fracture mechanics and the stress intensity factor we invoke the argument of K-dominance to clarify this point. The description of the traditional approaches and their limitations makes the case for non-traditional approaches such as fundamental micromechanical models. Micromechanical models for the different types of fracture and fatigue processes are introduced.

It is recognized that many different types of failure processes, such as fracture with limited yielding, or fatigue with a large number of cycles (in the range of hundreds of thousands) can be accurately modeled using the currently available traditional and non-traditional approaches. However for situations with large scale yielding and ultra-low cycle fatigue (typically less than 10-15 cycles), the mechanisms are not very clearly understood and there is a need for more fundamental research not only in terms of the models but also in terms of identifying the basic mechanisms that cause ultra-low cycle
fatigue. We aim to address this by developing simple and fundamental models, which can be easily calibrated, all while maintaining transparency in their methods and logic.

The current state of computational power is not completely utilized by the current state of computational models, which are geared more towards simplistic analysis. In the future, the progress will be in the direction of computationally intensive models that simulate material behavior at the grain or lower level, and which can be calibrated based on microstructure. This study is a small step in that direction.
Figure 2.1 Mechanism of void nucleation growth and coalescence
Figure 2.2 Scanning Electron Micrograph of surface fractured due to microvoid coalescence, showing dimpled surface shown here for an AW50 sample
Figure 2.3. Mechanism for ductile crack growth

(a) Before loading

(b) Crack tip blunts as voids grow

(b) Voids coalesce with growing crack

Nucleating Particles
Figure 2.4. Cup-cone fractures observed in tensile notched bar tensile tests
Figure 2.5. Transgranular cleavage fracture
(rupture surface lies within grains)

Figure 2.6. Cleavage nucleation due to microcrack
Figure 2.7. River pattern formation during cleavage fracture

Figure 2.8. River patterns and shiny facets seen in cleavage fracture surface (Anderson, 1995)
Figure 2.9. Intergranular fracture (Anderson, 1995)

Figure 2.10. Intergranular fracture surface showing the exposed grain boundaries (Anderson, 1995)
Figure 2.11 Stages of fatigue crack growth

Figure 2.12. Slip and decohesion mechanism of Stage II fatigue crack growth (Anderson 1995)
Figure 2.13 Inglis’ solution for an elliptical flaw

\[ \sigma_A = \sigma \left( 1 + 2 \sqrt{\frac{a}{\rho}} \right) \]

Figure 2.14. Sharp crack in infinite elastic plate - LEFM

\[ \sigma_{ij} = \frac{K_I}{\sqrt{2\pi r}} f_{ij}(\theta) \]
Figure 2.15. Arbitrary contour around crack tip for calculating the J-integral

Figure 2.16. Advancing crack trailed by plastic wake
Figure 2.17 Definition of Crack Tip Opening Displacement (CTOD)

Figure 2.18 Schematic plot of triaxiality versus plastic strain
Figure 2.19 S-N Curve showing inverse relation between applied stress range and the number of cycles to failure
Chapter 3

New models to simulate ultra-low cycle fatigue (ULCF) failure in metals

The area of research concerning Ultra-Low Cycle Fatigue (ULCF) is of great importance to structural and earthquake engineers. This chapter describes some new models that we have developed to better simulate this phenomenon.

As introduced in Chapter 2, ULCF (or extreme Stage III fatigue) involves extremely large plastic strains and very few (of the order of 10 to 20) cycles to failure. The mechanisms controlling ULCF are more similar to monotonic fracture mechanisms such as void growth and coalescence, as compared to fatigue mechanisms such as slip and decohesion. Thus, the mechanisms that cause ULCF are distinct from those that cause traditional low-cycle fatigue which involves cycles in the range of several hundred to a thousand. Generally speaking, ULCF is not very relevant to mechanical or aerospace applications, where components are controlled by low-cycle or high-cycle Stage II fatigue. For this reason, ULCF has received considerably less attention and is not as well developed as low-cycle and high-cycle fatigue.

Modern earthquake resistant steel structures are designed to absorb seismic energy by sustaining large inelastic deformations under cyclic loads. Structural damage is considered to be an essential mechanism for absorbing energy to ensure life safety under large-magnitude events. Moreover, a structure (or its components, such as steel connections), typically sustain less than ten large strain cycles during an earthquake. As a result, there are important practical applications of improved ULCF models to improve earthquake engineering research and practice.
The two models presented in this Chapter are named the Degraded Significant Plastic Strain (DSPS) model and the Cyclic Void Growth Model (CVGM). These models capture the process of ULCF by simulating cyclic accumulation of the void growth and coalescence together with the progressive damage to the intervoid material. The DSPS model couples some features of the SMCS with a newly developed strain measure which we will call the significant plastic strain. The DSPS model degenerates to the SMCS model under conditions of monotonic loading. For this reason, we can also refer to it as a Cyclic SMCS model.

The CVGM model is an extension of the void growth model to cyclic loading, which aims to capture the void growth process by taking into account the varying triaxiality similar to the VGM. The calibration and experimental verification of these models is discussed in detail in Chapter 5, and a practical application is examined in Chapter 6.

3.1 The mechanism of Ultra Low Cycle Fatigue (ULCF)

Before introducing the ULCF models, it is useful to review the current state of understanding of the ULCF process. As described in Chapter 2, traditional low-cycle fatigue refers to situations where hundreds of cycles cause failure. The crack propagation mechanisms in such situations are somewhat similar to high cycle Paris-type fatigue, though the correspondence between far-field stress parameters (such as $\Delta K$) and crack tip deformations is not unique due to a larger plastic zone. The nonlinear energy parameter $\Delta J$ based on the cyclic stress-strain curve is found to work better in such situations, not because the mechanism of fracture is substantially different from high-cycle fatigue, but because the correlation between crack tip strains and the far-field parameter ($\Delta J$) is much stronger.

The ULCF problem under investigation involves many fewer cycles and much larger strains which drive crack propagation by fracture initiation mechanisms. This means that cyclicity is not required for crack propagation, though fracture occurs at seemingly less severe stress and strain states due to accumulated damage effects from cyclicity. As
discussed previously, such phenomena have been investigated in far less detail than the traditional low-cycle fatigue processes, primarily because of their lack of significance to the mechanical engineering, aerospace and materials science communities. A brief reference is made to this issue by Skallerud and Zhang (2001), who note the lack of research in characterizing this type of damage or failure mode.

It is well established that under monotonic loading, void growth and coalescence leads to ductile crack initiation. The increasing void size intensifies the situation of stress and strain between adjacent voids until one of the following occurs –

1. The necked material between the voids ruptures due to dislocation and flow.
2. Shear localizes on a plane between the main voids due to nucleation of secondary voids in the necked region (discussed in Chapter 2)
3. The reducing area of the necked region, coupled with increasing constraint elevates the local stress above the critical cleavage stress causing the necked material to fracture.

For monotonic loading, these phenomena are assumed to be captured by defining a critical void ratio (equivalent to void volume fraction), which is considered to be uniquely related to the state of stress and strain in the ligament between the voids. The model for monotonic loading implicitly assumes that the localized combination of stresses and strains that causes coalescence is independent of the void shape, or locations of individual voids.

For cyclic loading, the situation is similar but for two key differences. Firstly, the voids encounter alternating excursions of positive and negative mean stress, which cause alternate expansion and shrinkage or squeezing of the voids that have already nucleated. Secondly, due to the cyclicity of loading, damage accumulates differently in the necked material. We define this damage by assuming that the loading cyclicity affects the unique correspondence between the void volume fraction and the stress/strain condition that triggers void coalescence. In other words, we assume a smaller void volume fraction (as
compared to the monotonic case) might trigger coalescence due to damage accumulation. This can happen due to a variety of mechanisms. Figure 3.1 shows some of these mechanisms, which are described below –

- The compressive mean stresses on the voids cause the void shapes to become oblate, as opposed to spherical. This shape change has the potential to cause stress raisers by reducing the ligament length as well as increasing the void curvature – Figure 3.1 (a). These increased stresses in turn may exceed the local material cleavage strength causing cleavage fracture of the necked material leading to void coalescence. This is supported by scanning electron micrographs which show pockets of cleavage fracture between the voids after cyclic loading (Figure 3.2).

- The reversed cyclic plastic strain might also cause secondary voids to nucleate on slip bands between the primary voids – Figure 3.1 (b), at void volume fractions lower than that of the monotonic case. The resulting bands of localized deformation between the primary voids will lead to void coalescence at a void volume fraction that is smaller than that required to cause a similar situation in the monotonic case.

- Various researchers (Tvergaard; 1981, Koplik and Needleman, 1988) have reported that apart from the void size (or volume fraction), the localization between the voids is also sensitive to the Ramberg-Osgood hardening parameter $n$. A larger hardening modulus indicates a steeper stress-strain curve for the metal. (Figure 3.3). A totally flat stress-strain curve would allow localization to take place very early on in the void growth process, whereas a steep curve would hinder the localization. Due to the shape change of the of nucleated voids and cyclicity of the loading, a given void fraction similar under cyclic loading may correspond to a larger strain range as compared to the same conditions under monotonic loading, thereby reducing the effective hardening modulus (as typically the material hardening response flattens out.
at larger strains). This sounds contradictory in terms of the far field strain fields, but it is difficult to ascertain the local strain response in the neck region as it might be dependent on a number of non-quantifiable factors such as inclusion spacing, shape and size. The implication is that for a similar void volume fraction, a cycled material might experience flatter stress-strain response hastening the coalescence process.

Figure 3.4 compares fractographs from a monotonic failure – Figure 3.4 (a) to a ULCF failure – Figure 3.4 (b) for the same material after five cycles. It is very interesting to note that the monotonic failure has much deeper dimples, indicating a larger critical void ratio. In contrast, the cyclic failure shows shallow dimples with pockets of cleavage type fracture. This supports our postulated mechanisms, that under cyclic loading smaller void ratios lead to failure due to cyclically accumulated damage.

In all these, it is assumed that the number of cycles are so few in number that the slip and decohesion type mechanisms, which are associated with conventional types of fatigue are of little significance in causing damage. Rather, the two most important processes that the ULCF model needs to capture are the growth of voids under reversed cyclic loading, and damage accumulation due to the cyclicity. Once these are accounted for, we can envision a model that degrades the critical void ratio based on a damage mechanism to predict failure under ULCF. As will be seen in the subsequent section, many difficulties arise in developing such a model, and a combination of mathematics, intuition and experimental as well as micromechanical data is used to propose a solution. The result is the Degraded Significant Plastic Strain (DSPS) model and the Cyclic Void Growth Model (CVGM) which we will describe in detail in Sections 3.2 and 3.4, respectively.

### 3.2 Degraded Significant Plastic Strain (DSPS) Model for ULCF

As discussed in the previous section, the problem of ULCF can be broken down into two smaller problems. The first problem is to determine the size of the void under reversed cyclic loading. Once the void size has been determined with reasonable confidence, this
is compared to a critical void size. The second important problem is to factor in the
cyclicity of the loading as the resulting damage accumulation affects the critical void
size.

### 3.2.1 Capturing the void growth under reversed cyclic loading

In Chapter 2, we discussed the growth of a void under a triaxial stress field based on Rice
and Tracey’s equation. Revisiting this, the void growth rate is given by the following
equation –

\[
\frac{dR}{R} = C \exp(1.5T) \cdot d\varepsilon_p 
\]

(3.1)

This leads to the monotonic failure criterion given by equation (3.2) which forms the
basis for the SMCS and the VGM models.

\[
\ln \left( \frac{R}{R_0} \right)_{\text{monotonic}}^{\text{critical}} = C \int_0^{\varepsilon_p^{\text{critical}}} \exp(1.5T) \cdot d\varepsilon_p
\]

(3.2)

For cyclic loading, we need to consider a more generalized loading history, which
includes the possibilities of negative as well as positive triaxialities. This is not accounted
for in equation (3.2), which was originally developed for monotonic tension only. To
account for cycling, the triaxiality term \(T\) in the exponent needs to be modified to its
absolute value, because mathematically, the sign of the triaxiality is not important for
void growth, but the magnitude is. If the triaxiality is negative, the void will shrink under
plastic straining, and if it is positive, the void will grow. This is mentioned briefly in Rice
and Tracey’s 1969 paper. Thus, to estimate the void fraction due to cyclic loading, we
need to integrate the more complicated equation –

\[
\frac{dR}{R} = \text{sign}(T) \cdot C \cdot \exp(|1.5T|) \cdot d\varepsilon_p
\]

(3.3)
The \( \text{sign}(T) \) term takes into account the sign of the triaxiality, such that the void radius would increase if the triaxiality is positive, or the mean stress is tensile, and the void radius would decrease if the triaxiality is negative and the mean stress is compressive in nature.

Looking at equation (3.3), we observe that the voids grow during intervals of positive triaxiality and shrink during intervals of negative triaxiality. Figure 3.5 shows a schematic plot of the equivalent plastic strain versus the triaxiality at the critical location in a typical specimen, such as the notched round bar during reversed cyclic loading. The figure illustrates that when triaxiality is positive, the voids are assumed to grow, and when it is negative they are assumed to shrink.

3.2.2 Issues with the equivalent plastic strain definition and development of the new significant plastic strain measure

It is interesting to note that in even under reversed cyclic loading, the equivalent plastic strain continues to increase monotonically. This is because the equivalent plastic strain is a cumulative measure based on the equations (3.4) and (3.5) which give the expressions for the equivalent plastic strain and the incremental equivalent plastic strain respectively.

\[
\varepsilon_p = \sqrt{\frac{2}{3} \varepsilon_p^{ij} \varepsilon_p^{ij}} \quad (3.4)
\]

\[
d\varepsilon_p = \sqrt{\frac{2}{3} d\varepsilon_p^{ij} d\varepsilon_p^{ij}} \quad (3.5)
\]

One concern about applying the concept of equivalent plastic strain as a damage measure for the ULCF modeling the void growth type under cyclic loading is that the equivalent plastic strain was originally envisioned as a measure of the total shearing deformation in applications to metal plasticity with isochoric deformations. As per this definition, the equivalent plastic strain is an ever-increasing function, irrespective of whether the
loading is tensile or compressive. Thus, the equivalent plastic strain is not a very powerful measure of void growth and shrinkage under cyclic loading. As an example, if a material is subjected to many small amplitude plastic cycles, the equivalent plastic strain will monotonically increase to large values while the void size might remain small due to the small strain amplitude causing it to lose correlation with void growth under cyclic loading.

Another issue is that in multiaxial stress and strain space, it is not trivial to distinguish a tensile versus compressive cycle of loading. For the purposes of this study, we will define excursions where the mean stress is tensile to be tensile or positive excursions, and those where the mean stress is compressive to be negative or compressive excursions. According to this, the tensile cycles correspond to void growth and the compressive cycles correspond to void shrinkage. This concepts underlie our definition of a new strain measure, termed the significant plastic strain, which has closer physical relation to the void growth process.

At any loading stage, we define the tensile equivalent plastic strain (at a given material point) $\varepsilon^*_t$, as the integration of equivalent plastic strain when the triaxiality was positive and the compressive equivalent plastic strain $\varepsilon^*_c$ as the integration of equivalent plastic strain when the triaxiality was negative. The difference between these is defined as the significant plastic strain –

$$
\varepsilon^* = \varepsilon_t - \varepsilon_c
$$

The difference in strain quantities is considered to be significant because it is an indicator of how much the material has been “stretched”, somewhat analogous to a residual displacement in a uniaxial tension coupon. This quantity relates well to the critical plastic strain in the monotonic SMCS model (since that too relates to the material “stretch”) and is a measure of void growth.
Like the stress and strain quantities in the SMCS and VGM models, the significant plastic strain is a local quantity, to be evaluated at each point within the material. In tests of simple notched tensile bars, the significant plastic strain increases during an excursion of global tension, and decreases during an excursion of global compression. However, for more complicated geometries or loading conditions, the global and local loadings might not be correlated. In any case, the definition of the significant plastic strain is general, and quantifies the void size at local points in the material. Figure 3.6 schematically plots the significant plastic strain versus the triaxiality at the center of notched tensile bar (which can be obtained from finite element analysis). We can compare this plot to the one from figure 3.5, in which the equivalent plastic strain was always increasing. It is also illustrative to plot the significant plastic strain versus analysis step and compare it to the equivalent plastic strain (see Figure 3.7) which again contrasts the monotonically increasing equivalent plastic strain with the significant plastic strain. A restriction on the significant plastic strain is that it can never take on negative values because it would correspond to a negative void size, which does not make physical sense. As a result, the significant plastic strain reaches a lower limit of zero and maintains that value until it is again raised above zero by a tensile excursion of the equivalent plastic strain. In other words, if a large compressive excursion follows a small tensile excursion, the significant plastic strain will decrease to a value of zero, and remain at that value as long as the triaxiality is negative. When the triaxiality becomes positive again, the significant plastic strain will begin to increase at the rate of the equivalent plastic strain.

Based on equation (3.3), we can write an integral to compute the void size under cyclic loading as -

\[
\ln \left( \frac{R}{R_0} \right)_{cyclic} = C_1 \int_{tensile-cycles} \exp(1.5T) \, d\varepsilon_i - C_2 \int_{compressive-cycles} \exp(1.5T) \, d\varepsilon_c \quad (3.7)
\]

Where the first term on the right hand side evaluates the cumulative void growth over all the cycles of positive mean stress (during the cycles of $\varepsilon_i$). The negative sign on the second term considers the effect of negative triaxialities on the void size, i.e. the second
integral explicitly calculates the void shrinkage, which occurs during cycles of $\varepsilon_c$. The two constants $C_1$ and $C_2$ are used to allow for different rates of growth and shrinkage. For the purposes of this study, we decided to set $C = C_1 = C_2$, given the lack of data to distinguish between the two. Moreover, as in the SMCS model, we can use the assumption that the triaxiality does not change much in magnitude during each excursion to move the exponential terms outside of the integral. This simplifies to equation (3.8) and subsequently to Equation (3.9) and (3.10).

\[
\ln \left( \frac{R}{R_0} \right)_{\text{cyclic}} = C \exp\left( 1.5 T \right) \varepsilon_i - C \exp\left( 1.5 T \right) \varepsilon_c \quad (3.8)
\]

\[
\ln \left( \frac{R}{R_0} \right)_{\text{cyclic}} = C \exp\left( 1.5 T \right) (\varepsilon_i - \varepsilon_c) \quad (3.9)
\]

\[
\ln \left( \frac{R}{R_0} \right)_{\text{cyclic}} = C \exp\left( 1.5 T \right) \varepsilon^* \quad (3.10)
\]

Thus, it can be seen that with suitable assumptions (not very different from those for the SMCS model) the significant plastic strain can be related to the void size. It is striking to note the similarity between this equation and the equation on which the SMCS is based, i.e.

\[
\ln \left( \frac{R}{R_0} \right)_{\text{monotonic}} = C \exp(1.5 T) \varepsilon_{p,\text{critical}} \quad (3.11)
\]

where the only difference is that in the SMCS model the equivalent plastic strain is used in place of the significant plastic strain. For monotonic loading, all of the significant plastic strain is tensile, and Equation (3.10) degenerates to Equation (3.11), thus demonstrating that the significant plastic strain is simply a more general version of the equivalent plastic strain which aims to quantify the void growth under cyclic loading.
3.2.3 Simulation of the damage process for the DSPS model

Having derived a methodology for estimating void growth under cyclic loading, we need to define the critical condition or critical void ratio at which ductile crack initiation due to ULCF occurs in the material. Based on our discussion earlier in this chapter, the critical void ratio under cyclic loading is presumed to be smaller than the critical void ratio under monotonic loading due to accumulated damage. Mathematically, we can write this as –

\[
\ln \left( \frac{R}{R_0} \right)_{cyclic}^{critical} = f(D) \ln \left( \frac{R}{R_0} \right)_{monotonic}^{critical}
\]  
(3.12)

Where \( f(D) \) is a damage function for the cyclic damage accumulation process and \( D \) is the damage variable which controls the damage process. The constraint on the damage function is that it must equal unity when the damage variable is equal to zero, and decrease as the damage increases. Thus, the critical value of the void ratio under cyclic loading can range from zero to a maximum of the critical value under monotonic loading.

Combining equations (3.10) and (3.12), we can predict failure to occur at a point where the void ratio exceeds the critical value. Mathematically, the situation at failure is –

\[
\ln \left( \frac{R}{R_0} \right)_{cyclic}^{critical} = C \exp \left( |1.5T| \right) \varepsilon_{critical}^\ast 
\]  
(3.13)

\[
f(D) \ln \left( \frac{R}{R_0} \right)_{monotonic}^{critical} = C \exp \left( |1.5T| \right) \varepsilon_{critical}^\ast 
\]  
(3.14)

Rearranging, the value of the critical value of significant plastic strain as –

\[
\varepsilon_{critical}^\ast = f(D) \frac{\ln \left( \frac{R}{R_0} \right)_{monotonic}^{critical}}{C \exp \left( |1.5T| \right)}
\]  
(3.15)
Referring back to Chapter 2, equation (2.23) we recall that for the monotonic case, the critical equivalent plastic strain can be expressed as equation (3.16), which is the basis of the SMCS model –

\[
\varepsilon_p^{\text{critical}} = \frac{\ln \left( \frac{R}{R_0} \right)_{\text{monotonic}}^{\text{critical}}}{C \cdot \exp(1.5T)}
\]  

(3.16)

Combining equations (3.15) and (3.16), the critical value of the significant plastic strain as a diminished version of the critical equivalent plastic strain under the identical triaxiality situation –

\[
\varepsilon_{\text{critical}}^* = f(D) \varepsilon_p^{\text{critical}}
\]  

(3.17)

### 3.2.4 Application of the DSPS Model

Sections 3.2.1 through 3.2.3 formulate the DSPS model, represented by equation (3.17) is the final equation representing it, which indicates that failure will occur once the significant plastic strain exceeds its critical value. The exact nature of damage function and the damage variable are discussed in Section 3.2.5. These are determined from cyclic notched bar tests and detailed finite element analyses that require cyclic plasticity. Chapter 5 outlines this process in detail. The following discussion assumes that these have already been determined, and qualitatively describes how the DSPS model would be used to predict ULCF.

To predict ULCF failure, detailed finite element analyses need to be run on the specimen for the entire cyclic load history, evaluating the damage function \( f(D) \) at the beginning of each “tensile” or positive cycle, From the damage function, one can determine the critical significant plastic strain for that particular excursion. For this, a monotonic analysis is run ahead of time to find the critical equivalent plastic strain to failure at a given point. This
critical equivalent plastic strain at the local point is then degraded based on the damage function at the beginning of that particular tensile excursion using equation (3.17).

Once the critical significant plastic strain is determined, the actual significant plastic strain is monitored during the entire excursion and checked against the critical value. If it exceeds the critical value during the cycle, then failure is predicted to occur at that point in the loading. If the loading reverses, i.e., we enter a compressive excursion, failure is assumed not to occur. The damage function is calculated again at the beginning of the next reversal to enter the tensile (positive triaxiality) excursion, and a prediction of the critical significant plastic strain is made. Again the actual significant plastic strain is compared during the tensile cycle to this critical value to make predictions of failure. Chapter 5 discusses this entire process in detail with examples.

The process is described above for a single material point, and the failure thus determined is a single material point failure. As in the case of the SMCS model, ductile crack initiation involves a length scale which is set by the characteristic length parameter $l^*$. The critical significant plastic strain condition must be exceeded over a minimum distance equal to the characteristic length to initiate material failure. For situations such as notched round bars, where the failure criterion is satisfied suddenly over a large material volume, the characteristic length is less important than in situations with sharper strain gradients. Lacking data to prove otherwise, we assume that the characteristic length for the cyclic condition is identical to the characteristic length for monotonic condition. A detailed study to explore possible differences between the two would be quite involved and probably not justified given the large uncertainty in other aspects of the model, such as the characteristic length estimate for the monotonic situation itself.

3.2.5 The damage function $f(D)$ and the damage variable $D$

A key aspect of the DSPS model that has not been discussed as yet is the choice of the damage variable and the damage function that reduces the critical void ratio under cyclic
loading. The numerous different damaging mechanisms that are listed in earlier part of this chapter need to be included in one damage index, which needs to be determined.

There are a couple of different approaches to solving this problem. One approach is to have a combined index which ideally aims to include each of the different damage effects explicitly. A major difficulty in such an approach would be to isolate each mechanism of damage, and postulate a stress or strain quantity that might drive that mechanism. One would then have to attach weights to the different mechanisms. Even if such a model were to be constructed, its calibration would be a very cumbersome and require large data sets.

The second approach is to assume that the different stress and strain quantities driving the different damage mechanisms are correlated – since most of the damage mechanisms deal with deformations and shape change of the voids, loss of hardening slope, as well as plastic straining in the ligaments. One can also visualize that these mechanisms might all maintain a correspondence with the local state of strain around the voids. The cumulative equivalent plastic strain appears to be a measurable quantity that would be related to the state of deformation and consequently damage in the void ligament area. Based on this reasoning, we choose the cumulative equivalent plastic strain as a damage variable. A number of factors, such as the inclusion spacing or the initial void shape may change the nature of the relationship between the equivalent plastic strain, which is somewhat of a far field measure, and the local plastic strain. As a result, the precise influence of the equivalent plastic strain on the damage cannot be directly quantified, but is assumed to be a material property for each material. Using cumulative equivalent plastic strain as a damage index, an exponential function for damage accumulation is assumed as given by equation (3.18) –

\[ f(D) = f(\varepsilon_p) = \exp(-\lambda \cdot \varepsilon_p) \]  

(3.18)

The function has a value of unity when the damage variable or the equivalent plastic strain is zero, meaning that the material still has its full capacity in terms of critical void ratio. As the damage increases, the capacity of the material decays exponentially. The
parameter $\lambda$ considered a material dependent damageability coefficient. The cumulative equivalent plastic strain is calculated at the beginning of each tensile excursion for a given point, and is substituted into equation (3.18) to calculate the current state of damage. This current state of damage is then substituted into equation (3.17) to make a prediction about the point of failure as described earlier.

Many other choices for the damage function were considered, such as multi-linear, polynomial as well as a shifted-exponential. These choices typically include more parameters, and sometimes show a better fit to experimental data for some steels. However there is the choice to make between simplicity and ease of calibration on the one hand, and close fit to data on the other. It is always possible to envision a model with numerous parameters that fits the data very well, but such models raise issues about calibration, as well as reliability when extended to different materials or situations. Considering these factors, and in light of the available data, we chose to model the damage function as shown in equation (3.18) with the single damageability parameter $\lambda$, obtained from curve-fitting the model to the test data (a procedure that will be described in detail in Chapter 5). Combining equations (3.18) and (3.17), we get the complete form of the DSPS model which is shown in equation (3.19) –

$$
\varepsilon_{critical}^* = \exp(-\lambda_{DSPS} \varepsilon_p) \cdot \varepsilon_{critical}^{critical}
$$

(3.19)

As can be seen from the equation, the DSPS model is basically an extension to the SMCS model, where the critical value of equivalent plastic strain that shows up on the right hand side of equation (3.19) is obtained from the SMCS model. Thus, it is important to have the SMCS parameter $\alpha$ from the monotonic tests before using the DSPS model. Once the SMCS parameter $\alpha$ is determined, the only other parameter, i.e. the damageability parameter $\lambda$ can be determined from cyclic tests of notched round bars. The characteristic length $l^*$ is assumed to be identical for monotonic and cyclic situations, and the estimates used in Chapter 2 are used for the DSPS model as well. The calibration aspects of this models will be dealt with in Chapter 5.
3.3 Issues with the DSPS model and alternate approaches to ULCF

The DSPS model provides a fairly simple and straightforward means of predicting failure under ULCF type conditions seen in earthquake loading. Though the simplicity offers ease of calibration and usage, it makes some assumptions which compromise the reliability of the model in certain situations. In this section, we look at some such issues with the DSPS model, and indicate possible directions for improvement.

The Rice and Tracey equation relates the instantaneous rate of void growth or shrinkage to the current state of triaxiality. For large changes in void diameter, or in situations where the void shape changes appreciably, this relationship might not maintain the same nature, e.g. necking behavior between growing voids might produce a response that is different from the response under compressive loading. As a result, the void shrinkage in compression might not bear the exact same relationship with the triaxiality as the void expansion. Revisiting equation (3.7), we can account for this by using different coefficients for the void expansion and void shrinkage terms in the integral –

\[
\ln\left(\frac{R}{R_0}\right)_{\text{cyclic}} = C_1 \int_{\text{tensile-cycles}} \exp\left(|1.5T|\right) \, d\varepsilon_t - C_2 \int_{\text{compressive-cycles}} \exp\left(|1.5T|\right) \, d\varepsilon_c \quad (3.20)
\]

Again, we can make the assumption about the triaxiality not varying appreciably during one cycle of plastic strain, and simplify equation (3.20) to equation –

\[
\ln\left(\frac{R}{R_0}\right)_{\text{cyclic}} = C_1 \left[ \exp\left(|1.5T|\right) \varepsilon_t - \beta \exp\left(|1.5T|\right) \varepsilon_c \right] \quad (3.21)
\]

Where \( \beta = \frac{C_2}{C_1} \); which leads to the equation –

\[
\ln\left(\frac{R}{R_0}\right)_{\text{cyclic}} = C_1 \exp\left(|1.5T|\right) \left(\varepsilon_t - \beta \varepsilon_c\right) \quad (3.22)
\]
We can redefine the significant plastic strain to be the weighted term as seen in equation (3.22), i.e. –

\[ \varepsilon^* = \varepsilon_i - \beta \cdot \varepsilon_c \] (3.23)

Substituting this into equation (3.22), and noting that $C_i = C$, the coefficient for expansion of the voids, leads to the form of the equation which is identical to basic DSPS equation (3.10), except that the significant plastic strain is now defined differently with the addition of the parameter $\beta$. This is an opportunity to enhance the abilities of this. However, adding parameters to the model will require more extensive calibration testing. Note that one can recover the standard DSPS model (with the original definition of significant plastic strain) by setting $\beta = 1$.

The other, more significant issue with the DSPS model is that like the SMCS model, it implicitly assumes that the triaxiality does not change significantly with increasing plastic strains. Furthermore, the DSPS model assumes that the magnitude of triaxiality is approximately equal for cycles of negative and positive mean stress, and that the triaxiality is a function only of the specimen geometry. As a matter of fact, the only place where the triaxiality ratio enters into the DSPS formulation is during the calculation of the critical equivalent plastic strain for the monotonic loading condition, which is then degraded to account for the accumulated cyclic damage. Moreover, it assumes that the triaxiality at failure in the cyclic situation is similar to the triaxiality at failure in the monotonic situation.

All these assumptions might be invalidated especially for very ductile materials that show large changes in triaxiality due to large deformations before failure. To overcome this limitation, we developed a variant to the DSPS model, which accounts explicitly for all the triaxiality changes and aims to track the void growth and coalescence process as accurately as possible. This model – which we will refer to as the CVGM (cyclic void growth model) is analogous to the Void Growth Model (VGM), just as the DSPS is analogous to the SMCS model for monotonic loading.
3.4 Cyclic Void Growth (CVGM) model for ULCF

Simplifying equation (3.7), we can decompose it into tensile and compressive parts as follows –

\[
\ln \left( \frac{R}{R_0} \right)_{\text{cyclic}}^{\text{critical}} = \sum_{\text{tensile-cycles}} C_1 \int_{\varepsilon_i}^{\varepsilon_2} \exp\left(1.5T\right) d\varepsilon_t - \sum_{\text{compressive-cycles}} C_2 \int_{\varepsilon_i}^{\varepsilon_2} \exp\left(1.5T\right) d\varepsilon_c
\] (3.24)

This illustrates the fact that we are in fact dealing with a summation of integrals for tension and compression loadings. This form of the equation also provides the flexibility to control the void growth and shrinkage rates independently of each other using the two different constants \(C_1\) and \(C_2\). We can simplify equation (3.24) to –

\[
\ln \left( \frac{R}{R_0} \right)_{\text{cyclic}}^{\text{critical}} = \sum_{\text{tensile-cycles}} \int_{\varepsilon_i}^{\varepsilon_2} \exp\left(1.5T\right) d\varepsilon_t - \sum_{\text{compressive-cycles}} \frac{C_2}{C_1} \int_{\varepsilon_i}^{\varepsilon_2} \exp\left(1.5T\right) d\varepsilon_c
\] (3.25)

As in the DSPS, we can substitute \(\beta = \frac{C_2}{C_1}\), and the critical void ratio to be a degraded function of the critical void ratio under monotonic conditions. This changes the form of equation (3.25) to –

\[
f(D)\eta = \sum_{\text{tensile-cycles}} \int_{\varepsilon_i}^{\varepsilon_2} \exp\left(1.5T\right) d\varepsilon_t - \sum_{\text{compressive-cycles}} \beta \int_{\varepsilon_i}^{\varepsilon_2} \exp\left(1.5T\right) d\varepsilon_c
\] (3.26)

Where \(\eta\) is the toughness parameter for the VGM, and \(f(D)\) is the damage function similar to that presented for the DSPS model. Setting \(\beta = 1\), we recover the simplified form –

\[
f(D)\eta = \sum_{\text{tensile-cycles}} \int_{\varepsilon_i}^{\varepsilon_2} \exp\left(1.5T\right) d\varepsilon_t - \sum_{\text{compressive-cycles}} \int_{\varepsilon_i}^{\varepsilon_2} \exp\left(1.5T\right) d\varepsilon_c
\] (3.27)
The methodology and use of this model is identical to the DSPS model except that (a) the $\eta$ parameter from the VGM is used as the capacity, and (b) the integral terms on the right hand side are preserved as integrals to account for changes in the triaxiality during the plastic loading. The $\eta$ parameter can be calibrated from monotonic tests, as described in Chapter 4, and the parameter of the damage function $f(D)$ can be obtained from calibration to test data as described in Chapter 5. The calibration parameter is different from that used for the DSPS model as seen in following equation (3.19), i.e. –

$$\eta_{\text{cyclic}} = \exp(-\lambda_{\text{CVGM}} \varepsilon_p) \eta_{\text{monotonic}}$$

(3.28)

Leading to –

$$\exp(-\lambda_{\text{CVGM}} \varepsilon_p) \eta_{\text{monotonic}} = \sum_{\text{tensile-cycles } \varepsilon_t} \int \exp(|1.5T|) d\varepsilon_t - \sum_{\text{compressive-cycles } \varepsilon_c} \int \exp(|1.5T|) d\varepsilon_c$$

(3.29)

This method aims to be more exact than the DSPS, but involves the calculation of the integrals on the right hand side of equation (3.27), which is numerically more intensive. On the other hand, it does not require a monotonic analysis to be run before the cyclic analysis. This is because the $\eta$ is not a function of triaxiality, and is a constant for all the points in the material, unlike the critical equivalent plastic strain $\varepsilon_p^{\text{critical}}$, which is a function of the triaxiality as defined by the SMCS model. As a result, the $\eta$ calibrated for the material can be directly used in equation (3.29) to predict failure.

### 3.5 Summary

This chapter begins by reintroducing the phenomenon of ultra-low cycle fatigue (ULCF) as distinguished from low cycle fatigue. Different mechanisms for ULCF are described based on tests and fractographs. These mechanisms consider void shape change to accelerate cleavage, early localization of deformation due to nucleation of secondary particles, and the change in effective hardening properties.

To accurately capture ULCF behavior, we need to quantify the void growth process under cyclic loading accompanied by damage accumulation. The void growth and shrinkage is a function of the absolute value of triaxiality and the incremental plastic strain, such that the voids grow if the triaxiality is positive and shrink if it is negative. This important
distinction between the effect of positive and negative triaxiality is not made by any of the existing models, which were developed for monotonic loading. The two models that are presented in the chapter make this distinction by explicitly accounting for void shrinkage during cyclic loading. The first model, i.e. the DSPS model captures this process through a newly introduced strain variable known as the significant plastic strain. The significant plastic strain is introduced as an alternative to equivalent plastic strain because it increases and decreases based on triaxiality, unlike the standard plastic strain, which is an ever increasing quantity. The DSPS model uses the significant plastic strain as the quantity that drives failure under ULCF type loading, and a critical value of significant plastic strain is determined as a degraded value of the critical equivalent plastic strain under monotonic loading for the same situation.

The DSPS lacks the ability to capture large changes in triaxiality because it is based on an assumption similar to the SMCS, i.e. that triaxiality does not change appreciably with deformations. Thus, it is valid as long as the deformations are small enough not to affect the triaxiality. For more ductile materials, where the deformations might be larger, we recommend the CVGM, which is an extension of the Rice and Tracey’s void growth model. This involves explicit integration of the stress and strain histories to calculate void growth. The critical void ratio is assumed to be a diminished version of the critical void ratio under monotonic conditions. Based on the damage mechanisms, the quantity controlling damage for each of the models is proposed to be equivalent plastic strain at the beginning of each tensile excursion, and the damage function is assumed to be of an exponentially decaying nature, based on its fit with the test data (which will be discussed in Chapter 5). Table 3.1 illustrates the monotonic and cyclic models and their features.
<table>
<thead>
<tr>
<th>Model features</th>
<th>Monotonic Model</th>
<th>Cyclic Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Captures void growth (and/or shrinkage), assuming constant triaxiality</td>
<td>SMCS $\varepsilon_p - \alpha \exp \left(-1.5 \frac{\sigma_m}{\sigma_p}\right) &gt; 0$</td>
<td>$\varepsilon_{\text{critical}}^* = \exp(-\lambda_{\text{DSPS}} \varepsilon_p) \cdot \varepsilon_p^{\text{critical}}$</td>
</tr>
<tr>
<td>Captures void growth (and/or shrinkage), accounting for variable triaxiality</td>
<td>VGM $\eta = \frac{\ln \left( \frac{R}{R_0} \right)_{\text{critical}}}{\varepsilon} = \int_0^{\varepsilon_p} \exp(1.5T) d\varepsilon_p$</td>
<td>CVGM $\exp(-\lambda_{\text{CVGM}} \varepsilon_p) \eta_{\text{monotonic}} = \sum_{\text{tensile-cycles}} \int_{\varepsilon_i}^{\varepsilon_f} \exp(1.5T) d\varepsilon - \sum_{\text{compressive-cycles}} \int_{\varepsilon_i}^{\varepsilon_f} \exp(1.5T) d\varepsilon$</td>
</tr>
</tbody>
</table>

Table 3.1 Features of monotonic and cyclic (ULCF) models
Figure 3.1(a) Cyclic loading causing damage in terms of sharper void curvature and smaller inter-void ligament for the same void volume fraction.

Figure 3.1(b) Secondary voids nucleating between primary voids due to cyclic plastic strains to cause localized deformation.
Figure 3.2 Fractograph of AW50 steel after reversed cyclic loading, showing cleavage type fracture between adjacent voids

Figure 3.3 Increasing the Ramberg-Osgood parameter makes the stress-strain curve steeper
(a) Fractograph of fracture surface after monotonic loading (note the deep dimples indicating large void ration at coalescence)

(b) Fractograph of fracture surface after ULCF (note the shallower dimples and presence of brittle, cleavage type pockets)

Figure 3.4 Comparison of monotonic and ULCF fracture surfaces for AW50 illustrating the degradation in critical void ratio
Figure 3.5 Triaxiality versus Plastic Strain for Reversed Cyclic Loading

Figure 3.6 Schematic plot of triaxiality versus significant plastic strain
Figure 3.7 Equivalent plastic strain and significant plastic strain at the center of notched round bar
Chapter 4

Micromechanical simulation of monotonic tests and analyses of metals

This chapter provides a detailed description of the tests and analyses to simulate the ductile fracture of steels under monotonic loading. The chapter begins with a brief introduction to the different steels tested, with a description of test results related to the basic material categorization, i.e., tensile stress-strain tests, Charpy V-Notch tests, grain size tests and spectrochemical tests. This is followed by a description of the calibration of the VGM and SMCS criteria for predicting failure under monotonic loading. A detailed calibration procedure outlined, including the tests, complementary analyses as well as the issues involved. All of the test results and data are presented as appropriate. After the basic parameters of these models are calibrated, the models are verified using the sharp-cracked ASTM type fracture tests, as well as specially devised blunt notched tests that permit cyclic loading. Analytical predictions of fracture (based on VGM and SMCS) and experimental observations demonstrate the efficiency of these models as fracture predictive tools. For some of the ductile varieties of steel, where fracture is accompanied by very large deformations, the VGM is found to predict failure more accurately then the SMCS because of its ability to capture triaxiality changes. A few situations are encountered where cleavage type fracture occurs, which is not captured by either the SMCS or VGM models. On the whole, the models are able to predict fracture with reasonable accuracy in conventional sharp crack specimens as well as situations without the presence of an initial sharp crack. The chapter concludes with some observed limitations of these models.
4.1 Different varieties of steel used in this study

In applying and verifying the models for this study, we investigated a large variety of steels to extend the applicability and demonstrate the generality of the models. Seven different batches of steel were used, four of them were manufactured in the United States, and three of them were manufactured in Japan. Table 4.1 (at the end of the chapter) lists the steels, and a notation for future reference. The steels are coded in a meaningful way, i.e., AW50 refers to an American steel of Grade 50 that is cut from a W section. Similarly, JP50HP refers to a Japanese steel of Grade 50 cut from a plate having high performance designation. The table also includes other information such as the orientation of the fracture specimens with respect to the rolling axis, as well as the thickness of the plate or the flange from which the specimens are obtained.

4.2 Basic Tests for Material Characterization

A variety of basic tests for material characterization are run on each of the steel varieties. These tests aim to gather and help correlate information about material microstructure and properties of strength, toughness and ductility. The tests in this category are –

1. Uniaxial Tensile Tests: These tests provide the basic stress strain data for calibration of FEM models, as well preliminary information about the steel strength and ductility.

2. Charpy V-Notch Tests: The Charpy V-Notch Energy value is a commonly used indicator of material toughness. It gives an overall measure of material toughness which has a loose correlation to fracture indices such as the $J_{IC}$, $\alpha$ and the $\eta$. The Charpy test also provides an indication of the transition temperature above which the mode of fracture is fibrous and driven by void growth and coalescence and below which the mode is more brittle, driven by processes such as intergranular cleavage.
3. Chemical Spectroscopy Tests: These tests determine the quantity by weight of different primary elements in the steel alloy.

4. Grain size tests: Grain size is important to establish the characteristic length scale, since inclusions are likely to be located at grain boundaries.

The Charpy, spectroscopy and grain-size measurement tests were outsourced to an off-campus commercial testing company. The other tests were done by the author.

4.2.1 Results of Uniaxial Tensile Tests

Two uniaxial are run for each of the steels tension tests according to ASTM standard E8. These tests provide basic engineering parameters such as the modulus of elasticity E, the yield stress $\sigma_y$, the ultimate stress $\sigma_u$, and the ductility to failure. The tests are run on standard ASTM E8 type specimen as shown in figure 4.1 (a) with an extensometer of 1 inch gage length. The specimens are slightly wasted at the center to encourage necking at that location. These are tested in 20 kip servo-hydraulic MTS load frame under displacement control. The data channels acquired from the test are time, actuator displacement, extensometer displacement, and force. After the testing, the necked diameter of the fractured specimen is measured for true strain and ductility calculations. Figure 4.1 (b) shows a photo of the uniaxial test setup.

The engineering stress and strain are calculated according to the following formulae –

$$\sigma_{eng} = \frac{F}{A_{initial}}$$

$$\varepsilon_{eng} = \frac{\Delta}{L_{initial}}$$

Where the $A_{initial}$ is the initial cross sectional area of the round bar (equal to 0.1963 sq. in.) $L_{initial}$ is the initial length of the gage of the specimen equal to the extensometer gage
length (equal to 1 inch). Δ is the displacement measured by the extensometer over the gage length and \( F \) is the applied force.

The true stress-strain properties are calculated from the engineering properties as shown in the following equations (4.3) and (4.4) –

\[
\sigma_{\text{true}} = \sigma_{\text{eng}} \left( 1 + \varepsilon_{\text{eng}} \right) \tag{4.3}
\]

\[
\varepsilon_{\text{ln}} = \ln \left( 1 + \varepsilon_{\text{eng}} \right) \tag{4.4}
\]

Figure 4.2 (a) shows a sample engineering stress-strain curve obtained from such a test of the AP50 steel, while 4.2 (b) shows a true stress-strain curve obtained from the same test. At some point during the test, the specimen starts to neck in an unstable fashion, i.e., the hardening of the material can no longer compensate for the reduction of area caused due to the additional strain. At this point, the extensometer reading becomes useless because the straining across the extensometer gage length is concentrated in the necking region and no longer uniform. The extensometer is removed at this point to protect it from damage. For most steels in this study, this happens at around 10-15% engineering strain levels. Since the strains for ductile crack exceed these levels by almost an order of magnitude, it is necessary to recover strains even after the extensometer has been removed. To calculate the true stress and strain at fracture, we measure the necked diameter and the breaking force of the specimen. The true stress and strain are calculated according to equations (4.5) and (4.6) which employ conservation of volume arguments for their derivation.

\[
\varepsilon_{\text{true}}^{\text{fracture}} = \ln \left( \frac{d_0}{d_f} \right)^2 \tag{4.5}
\]

\[
\sigma_{\text{true}}^{\text{fracture}} = \frac{F_{\text{fracture}}}{\pi d_f^2 / 4} \tag{4.6}
\]

Where \( d_0 \) is the initial diameter of the round bar, and \( d_f \) is the final or fracture diameter of the bar measured after the test is complete. The last point of the true stress-strain curve
shown in figure 4.2 (b) is obtained by such a method. It must be noted that at the point of fracture, the stress and strain distributions across the necked cross section are not uniform, so for calibration, it might sometimes be more useful to match the load displacement curve from FEM analysis to the actual specimen. In our study, both approaches gave almost identical material properties. Table 4.2 summarizes the key results and values obtained from the tests of all the steels.

Table 4.2 Data from the uniaxial stress-strain tests on all the steel varieties

<table>
<thead>
<tr>
<th>Steel</th>
<th>Yield Stress (ksi)</th>
<th>Ultimate Tensile Stress (ksi)</th>
<th>Ductility $d_0/d_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Test 1</td>
<td>Test 2</td>
<td>Mean</td>
</tr>
<tr>
<td>AW50</td>
<td>65.0</td>
<td>57.6</td>
<td>61.3</td>
</tr>
<tr>
<td>AP50</td>
<td>56.3</td>
<td>56.3</td>
<td>56.3</td>
</tr>
<tr>
<td>AP110</td>
<td>112.1</td>
<td>119.7</td>
<td>115.9</td>
</tr>
<tr>
<td>AP70HP</td>
<td>87.1</td>
<td>83.0</td>
<td>85.1</td>
</tr>
<tr>
<td>JP50</td>
<td>47.4</td>
<td>47.8</td>
<td>47.6</td>
</tr>
<tr>
<td>JP50HP</td>
<td>59.2</td>
<td>60.5</td>
<td>59.9</td>
</tr>
<tr>
<td>JW50</td>
<td>49.1</td>
<td>49.1</td>
<td>49.1</td>
</tr>
</tbody>
</table>

Referring to Table 4.2, the results are fairly repeatable for each variety of steel. The yield stresses of most steels are not exactly what the grade prescribes them to be, but often higher, which is likely due to the manufacturer’s effort to obtain multiple certification for the steels (satisfying the minimum ASTM requirements for multiple steel categories – because ASTM defines a grade of steel as the minimum yield strength for that steel). The high-performance steels, i.e. AP70HP and JP50HP (from USA and Japan), are seen to have a relatively higher ductility values (as compared to most of the other steels). Also, the AP50 and to have lower ductility values, and this correlates well with some of the other ductility and fracture measures that are described later. At this point, it might be useful to note that ductility measurements based on uniaxial tests may often show combined effects of material ductility as well as material hardening behavior, and may not be transferable to situations with different stress conditions. This issue will be revisited again later while dealing with the notched bar tests.
4.2.2 *Charpy V-Notch Tests*

Standard Charpy V-Notch tests (ASTM Standard E3) are run to determine the temperature transition curve and the upper shelf energy value for all the steels. A typical set of Charpy temperature transition curves is shown in Figure 4.3 (shown in this case for steel AW50). The Charpy energy is sensitive to the rate of loading, and typically the Charpy tests are carried out as impact tests, with a high rate of loading. To be meaningful when compared with standard fracture tests, which have near-static loading rates; the Charpy curve obtained from dynamic tests needs to be corrected. This is done by shifting the curve to the left on the temperature axis by a shift temperature given by (Barsom and Rolfe, 1987) –

$$T_{\text{shift}} = 215 - 1.5\sigma_y$$  \hspace{1cm} (4.7)

Where $\sigma_y$ is the yield stress of the steel (in ksi units and $T_{\text{shift}}$ is the temperature shift in degrees Fahrenheit). Figure 4.3 shows the curves before and after the temperature shift.

The graph can be divided into the lower shelf, the transition region and the upper shelf. The lower shelf corresponds to the “brittle” mechanisms of fracture, such as intergranular cleavage. As we increase the temperature, more slip systems are available for the material to yield and plastically deform, and there is a combination of the brittle mechanisms as well as ductile mechanisms such as void coalescence. This corresponds to the rising part of the Charpy curve known as the transition region. The fracture surface for test specimens in this region shows a combination of the shiny cleavage facets interspersed with the more fibrous ductile tearing areas. As the temperature is increased even further, the fracture mechanism transitions to the completely ductile void growth and coalescence type fibrous fracture mechanism. The saturation value of the Charpy curve is typically referred to as the upper shelf value. Since this study primarily deals with ductile mechanisms of fracture, the upper shelf value is of the most interest to us. The Charpy curve also indicates the temperature above which the material will experience ductile tearing, so we can ensure that our tests are all conducted above this temperature. There is
also the effect of constraint which affects the transition temperature, but most of our tests such as the notched bar tests typically have lower constraint situations than the Charpy tests, so the temperature indicated by the Charpy tests is a safe threshold above which to test. However, for some of the sharp-cracked ASTM fracture tests, we do observe instances of brittle fracture even above the transition temperature, because of the elevated constraint.

The Charpy test data is regressed according to equation (4.8), where \( \beta_1 \) to \( \beta_4 \) are curve-fitting parameters. The upper shelf value is recovered as \( \beta_1 + \beta_2 \), whereas the lower-shelf value is recovered as \( \beta_1 - \beta_2 \).

\[
CVN = \beta_1 + \beta_2 \tanh[\beta_3(T - \beta_4)]
\]  \hspace{1cm} (4.8)

Where the CVN is the energy expressed in ft.lbs. In Japan, the test data is often calibrated using an alternative relationship proposed by Nogata and Masaki in 1982 which is given by –

\[
\nu E(T) = \frac{\nu E_{Shell}}{\exp\{-a(T - \nu T_E)\} + 1}
\]  \hspace{1cm} (4.9)

The curve fit for equations (4.8) and (4.9) is almost identical, and the upper shelf toughness values presented here are found to be coincident using either expression. Table 4.3 summarizes the upper and lower shelf Charpy values which are a useful basis to compare trends with respect to other toughness measures.
Table 4.3 – Summary of Charpy Test data for the different steels

<table>
<thead>
<tr>
<th>Steel</th>
<th>Upper Shelf Energy Value (Foot Pounds)</th>
<th>Lower Shelf Energy Value (Foot Pounds)</th>
<th>Fitting Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\beta_1$</td>
</tr>
<tr>
<td>AW50</td>
<td>188</td>
<td>30</td>
<td>109</td>
</tr>
<tr>
<td>AP50</td>
<td>108</td>
<td>8</td>
<td>58</td>
</tr>
<tr>
<td>AP110</td>
<td>107</td>
<td>9</td>
<td>58</td>
</tr>
<tr>
<td>AP70HP</td>
<td>206</td>
<td>42</td>
<td>124</td>
</tr>
<tr>
<td>JP50</td>
<td>156</td>
<td>10</td>
<td>73</td>
</tr>
<tr>
<td>JP50HP</td>
<td>246</td>
<td>22</td>
<td>134</td>
</tr>
<tr>
<td>JW50</td>
<td>242</td>
<td>8</td>
<td>117</td>
</tr>
</tbody>
</table>

It is interesting to note that on an average, the Japanese steels seem to show higher values of toughness than their American counterparts, e.g. the High Performance Japanese steel JP50HP has about a 25% higher upper shelf value than the AP70HP. Another interesting observation is that for steels of identical grade (such as AW50 and AP50 – or JW50 and JP50), the Charpy values from the steel cut from the W-section are much higher (55-85%) higher than those from the steel cut from the rolled plate. A similar trend is observed in other fracture parameters. The higher strength steel AP110 has the lowest upper shelf value, which is consistent with the commonly held perception that stronger steels are in general less tough. Similar trends are observed in other fracture parameters.

Another interesting observation is that the correlation between the upper and lower shelf values is poor which is consistent with the reasoning that the upper shelf and lower shelf energies are governed by independent mechanisms.

4.2.3 Chemical Analysis

Spectrochemical analyses were performed to determine the chemical composition of each of the steels. For the American steels, quantities of the elements are limited by ASMT specifications. It is interesting to note the correlation between the occurrence of certain elements and trends in toughness of that particular steel. Table 4.4 (at the end of the chapter) lists the steels and tabulates their chemical contents for some of the important elements. The values in the parentheses indicate the maximum allowable percentage of the element allowed as per that standard.
Some important observations can be made looking at this data –

- The Japanese steels, JW50, JP50HP and JP50 have a lower Sulfur content in general as compared to the American Steels.
- The carbon content is very strongly negatively correlated with the toughness of the steel. The most ductile steels, such as AW50 and JP50HP have much less carbon as compared to some of the more brittle varieties, such as Steel AP50 or AP110.
- The American high performance AP70HP is that it contains very high quantities of Chromium (0.69%), and Molybdenum (0.07%). In the all the other steels, these are present in much smaller quantities. Chromium is typically in the range of 0.02-0.05% and Molybdenum is in the range of 0.007-0.008%.

A possible explanation for the Chromium might be the hardness that it imparts to the steel. Though hardness tests were not conducted for the steels, the AP70HP steel was found to be much harder to machine as compared to most of the other steels. Moreover, it is important to remember that the AP70HP (known as HPS 70W), was derived from the A709 steel and it is likely that the Chromium content is maintained from the parent variety of steel. The important feature of the AP70HP is the low carbon, which increases ductility and weldability. On the flipside, the low carbon content can hurt the yield strength of the material. Molybdenum has a positive effect on the yield strength as well as the ductility, and this might be reason for the unusually large content of Molybdenum in the high performance steel. Molybdenum also promotes finer grain structure, which in turn increases toughness.

4.2.4 Grain Size Determination

Grain sizes are measured according to ASTM E 112-96, which specifies a process of polishing and etching with a 2% Nital solution. Photomicrographs, such as the one shown
in figure 4.4 (shown here for AP50) are obtained for each of the steel samples. The microstructure for all the steels studied was found to be ferrite and pearlite.

Grain sizes are an important quantity for establishing the length scale of the micromechanical models, since the fracture-initiating carbide inclusions can sit at grain boundaries (which are one grain-size apart). There are some practical difficulties involved in using the grain size directly as the characteristic length due to some material sampling issues discussed in detail in the section dealing with the use of the micromechanical models. ASTM E112-96 outlines methods for determining grain size by counting the number of grains in a given area, which relates to the ASTM grain size number, which relates to the grain size. Table 4.5 tabulates the ASTM grain size numbers and the corresponding grain sizes for the different steels used in this study.

Table 4.5 Grain Size Data

<table>
<thead>
<tr>
<th>Steel</th>
<th>ASTM Grain Size Number</th>
<th>Average Grain Diameter in Inches (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AW50</td>
<td>8.5</td>
<td>7.441e-4 (0.0189)</td>
</tr>
<tr>
<td>AP50</td>
<td>8.5</td>
<td>7.441e-4 (0.0189)</td>
</tr>
<tr>
<td>AP110</td>
<td>8.5</td>
<td>7.441e-4 (0.0189)</td>
</tr>
<tr>
<td>AP70HP</td>
<td>11.5</td>
<td>2.638e-4 (0.0067)</td>
</tr>
<tr>
<td>JW50</td>
<td>8.5</td>
<td>7.441e-4 (0.0189)</td>
</tr>
<tr>
<td>JP50HP</td>
<td>11.3</td>
<td>2.874e-4 (0.0073)</td>
</tr>
<tr>
<td>JP50</td>
<td>8.5</td>
<td>7.441e-4 (0.0189)</td>
</tr>
</tbody>
</table>

The grain size numbers of both the high-performance steels (AP70HP, JP50HP) are higher than those for the other steels (meaning the grain sizes are smaller). Reducing grain size increases the surface area of the grains thereby creating better fit between the grains, in turn reducing locations for carbides to accumulate, increasing toughness. Smaller grain size also increases yield strength by grain boundary hardening mechanisms. Thus, grain size reduction has two beneficial effects of increasing both strength and toughness.
4.3 Tests for micromechanical model parameter calibration

The micromechanical models involve two parameters. The first parameter (toughness index) appears in the model equation, such as the $\alpha$ in the SMCS or the $\eta$ in the VGM. The other parameter (characteristic length) establishes the length scale over which the micromechanical failure criterion must be satisfied to trigger fracture.

The toughness index is obtained through testing and finite element analyses of circumferentially smooth-notched tensile specimens – such as the one shown in Figure 4.5. These tests referred to as SNTT – smooth-notched tensile tests have the same overall geometry as the smooth round bars with a circumferential notch machined into them to produce a triaxial stress condition. The triaxiality is varied by changing the notch size. Typically, the fracture condition is a function of the toughness index as well as the characteristic length. However, since the contours of the SMCS or VGM fields (refer Chapter 2) are very flat across the cross section of the notched round bars, ductile fracture initiation in these specimens appears to take place virtually simultaneously over most of the central portion of the bar cross-section area. This strong dependence of the failure on the $\alpha$ or the $\eta$ criterion as opposed to the characteristic length, $l^*$ makes the SNTT tests suitable for calibration of the toughness index.

Ductile fracture initiation in notched bars can be defined to coincide with sudden change in slope in the load versus deflection plot – see Figure 4.6. One might question if the ductile crack initiation actually occurs prior to the load drop, but the flatness of the contours of the crack initiation parameters convinces us that a large crack would initiate all at once, causing the load drop. Using finite element simulations, the distributions of effective plastic strain, von Mises (or effective) stress and hydrostatic stress at ductile fracture initiation (i.e., load drop) can be examined to back-calculate the toughness index as described in section 4.3.1

The characteristic length can be determined through the micro-structural measurements and observation of the fracture surface, as described in section 4.4. Once the complete
SMCS and VGM criteria are calibrated, various other specimens (such as the three-point bend and compact tension fracture specimens as well as the non-conventional blunt notched compact tension specimen) can be investigated to verify the parameters of the micromechanical models, i.e., the correctness of the combination of the calibrated $\alpha$ and $\eta$ parameters and the characteristic length.

4.3.1 Notched Round Bar Tests (Smooth Notched Tensile Tests – SNTT) for calibrating the $\eta$ and $\alpha$ parameters of the VGM and the SMCS

Figure 4.5 shows a typical SNTT test specimen. Two different notch radii are investigated: $r^* = 0.06$” and $r^* = 0.125$” for all steels. In addition, a notch radius of $r^* = 0.25$ is investigated for the steels AP50 and AP110. In each case, a minimum section diameter of 0.25 inches is maintained. These different notch severities provide variations of triaxiality at the notched section where the ductile fracture initiation occurs. Smaller notch radii provide larger triaxialities, and larger radii provide smaller triaxialities (a radius of infinity would reduce the test to a uniaxial stress-strain test). The grips, extensometer (with gage length 1 inch), and loading apparatus is the same as in the smooth tensile specimens. Multiple tests are conducted for each notch radius (all the results are summarized in Table 4.6). The elongation ratio is determined by the deformation over the extensometer gage length. Figure 4.6 shows the load vs. elongation curve a typical notched bar test indicating the point of ductile crack initiation (shown here for a $r^*=0.125$ test on the AW50 steel). The point indicated on the figure as fracture elongation, $\Delta_f$, corresponds to the initiation of ductile fracture. This displacement is used as the controlling displacement in the companion finite element analyses to back-calculate the fracture parameters $\alpha$ and $\eta$. Beyond this displacement, the falling load displacement graph indicates the tearing of the material in the circumferential shear lips after the ductile crack initiation in the middle of the bar. Figures 4.7(a) and (b) show the photographs of the cup-cone fracture surfaces of the specimens with $r^* = 0.125$ and $r^* = 0.06$ inches. The central penny-shaped flat surface (the location of the crack initiation) is rough with lots of dimples and is surrounded by shear lips at 45 degrees from the tensile
axis. This outer ring surrounding central dimpled fracture surface is relatively smoother and has a fibrous appearance.

4.3.2 Analysis Models and Results

Finite element simulations are used to analyze notched tensile specimens and obtain the critical fracture properties for the VGM and SMCS criteria. Elastic-plastic finite element analyses with incremental plasticity are conducted using ABAQUS/CAE 6.2.

The finite element solutions employ nonlinear, large-deformation plasticity models. The plasticity behavior is modeled using incremental theory with a von Mises yield surface, associated flow rule, and isotropic strain hardening. The nominal flow property used for each of the steels is a piece-wise linear fit to the measured true stress-log strain curve obtained from the smooth tensile tests (e.g. Figure 4.2). Though these plasticity models cannot be extended as such to the reversed cyclic loading, we prescribe them here because they are easier to calibrate as compared to some of the more sophisticated cyclic-plasticity models that may contain numerous parameters that are difficult to calibrate. In the chapter on cyclic plasticity, we will discuss other models that we use to model cyclic behavior, which aim to simulate the monotonic as well as the cyclic behavior. The model parameters for the monotonic and cyclic plasticity models for all steel varieties are provided in Appendix A.

Two-dimensional, axisymmetric finite element analyses are performed for the notched tensile specimens. As shown in Figure 4.8, the element size is refined to about 0.01 inch in the notch area, which is sufficient to capture the stress-strain gradients in that area based upon detailed convergence studies on various mesh densities by Chi et al., 2000. The analytical prediction of the load deformation curve is very close to the experimental observation for all the three notch geometries as can be seen in Figures 4.9 (a), (b) and (c). These are for steel AP50, but similar agreement is observed for all the other steels as well.
4.3.3 Determination of Critical Values of Toughness Indices $\alpha$ and $\eta$

The calibration process for the $\alpha$ parameter of the SMCS model and the $\eta$ parameter for the VGM is almost identical, but for some small modifications in the mathematical expressions used. We will describe the process for the SMCS parameter $\alpha$ in detail, and then briefly describe the key differences between the SMCS and the VGM.

The critical value of the SMCS model parameter $\alpha$ is determined from the testing and analysis of notched tensile specimens with varying notch severity. The tensile tests are conducted to identify the displacement corresponding to fracture initiation $\Delta_f$ (see Figure 4.6). Finite element analyses of each notched tensile geometry are performed to obtain the stresses and strains at the displacement $\Delta_f$ corresponding to fracture initiation. Substituting these critical stress and strain states at the cross section into the SMCS criterion, equation (4.10) to enforce a zero value at the section center determines the fracture parameter $\alpha$.

$$\varepsilon^\text{critical}_p = \alpha \exp(-1.5T)$$

(4.10)

The finite element predictions of the SMCS parameters, the triaxiality ($T = \sigma_m/\sigma_e$) and equivalent plastic strain ($\varepsilon_p$), at the load corresponding to the fracture initiation shown in Figure 4.10(b) indicate that the triaxiality at the center of the specimens (analyses shown here for Steel AW50) is much higher than at the free surface (i.e., constraint is higher at the center). However, as seen in Figure 4.10(a), the equivalent plastic strain at the free surface is much higher than at the center where the fracture initiates first. The critical effective plastic strain at the center of the specimen with $r^* = 0.125$ is much higher (by typically approximately 25%) than for $r^* = 0.06$, whereas the triaxiality of the large-notched specimen is approximately 30% lower than the value of the smaller radius notch. These results reveal two important points. First, the contradiction between the location of maximum equivalent plastic strain and the location of ductile fracture initiation (at the center) demonstrates that equivalent plastic strain alone is not a good ductile fracture index. Second, the result is qualitatively consistent with the failure plastic strain criterion.
proposed by Hancock and Mackenzie (1976) that critical effective plastic strain is
decreased as triaxiality increases. Moreover, the observed failure surface suggests that the
ductile crack initiates in the center and moving outwards until it changes to tearing – see
Figure 4.7.

Substituting the plastic strain and triaxiality distributions into equation (4.10), the SMCS
value \( \text{SMCS} = \varepsilon_p - \varepsilon_p^{\text{critical}} \) is fit such that a value of zero is obtained at the center of the
specimens. The \( \alpha \) value from the different notch sizes is very similar suggesting that it
can be assumed to be a material property indicative of toughness. It is very interesting to
note that after combining the plastic strain and triaxiality distributions using the
calculated value of \( \alpha \), the final SMCS distributions look identical for the different notch
geometries, as can be seen from Figure 4.11 (a).

The calibration process for the \( \eta \) parameter of the VGM is similar to that of the SMCS,
however, some of the mathematical expressions used are different. As outlined in Chapter
2, the \( \eta \) for a given test can be calculated by evaluating the following expression at the
point of failure –

\[
\eta = \ln \left( \frac{R}{R_0} \right)_{\text{critical}} = \int_0^{\varepsilon_p} \exp(1.5T) d\varepsilon_p
\]

(4.11)

Where \( R, R_0, C \) and \( T \) are as defined in Chapter 2. As in the case of the SMCS calibration,
the calibration is based on the experimental displacement corresponding to fracture
initiation \( \Delta_f \). Multiple analysis increments enable the numerical integration of the stress
and strain histories to calculate the right hand side of equation (4.11). The parameter \( \eta \) is
a measure of the void growth, and as in the SMCS model, failure occurs at the center.

This process of combining the tests and analyses to calibrate the toughness indices is
repeated for all the steels, and results are tabulated in Tables 4.6 (a) through (g). The
coefficients of variation (COV) for the parameters \( \eta \) and \( \alpha \) are in the range of 3% to
The low variation, despite using data from different triaxiality situations, demonstrates the ability of the micromechanical models to capture fracture under varied stress conditions.

Figure 4.11(a) shows a representative plot the SMCS distributions of large and small-notched tensile specimens at the fracture initiation loads. From these graphs, it is evident that the SMCS distributions for these specimens have virtually identical shapes with a peak value of 0 at the center. This indicates that the SMCS criterion is able to appropriately account for the interaction of the triaxiality and plastic strain effects. Furthermore, both SMCS distributions have very low gradients at the central area, agreeing with the experimental observation that ductile fracture initiation occurs over the penny-shaped dimpled surface simultaneously at the sudden drop point along the load versus deformation curve. Figure 4.11(b) shows a similar plot for the VGM, where the quantity in equation (4.12) is plotted versus the distance from the center.

\[ \int_0^\infty \exp(1.5T) d\varepsilon_P - \eta \]  

(4.12)

Tables 4.6 (a-g) Summary of the SNTT Tests for SMCS $\alpha$ and $\eta$ calibration

<table>
<thead>
<tr>
<th>Notch Size</th>
<th>Test #</th>
<th>$\Delta_l$ (inches)</th>
<th>$\alpha$</th>
<th>$\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r^* = 0.125''$</td>
<td>1</td>
<td>0.0564</td>
<td>2.10</td>
<td>2.20</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.0738</td>
<td>2.80</td>
<td>2.90</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.0742</td>
<td>2.81</td>
<td>2.96</td>
</tr>
<tr>
<td></td>
<td>4*</td>
<td>0.0675</td>
<td>2.55</td>
<td>2.67</td>
</tr>
<tr>
<td></td>
<td>5*</td>
<td>0.0637</td>
<td>2.50</td>
<td>2.51</td>
</tr>
<tr>
<td>$r^* = 0.06''$</td>
<td>1</td>
<td>0.0444</td>
<td>2.55</td>
<td>2.91</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.0419</td>
<td>2.50</td>
<td>2.72</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.0469</td>
<td>2.75</td>
<td>3.11</td>
</tr>
<tr>
<td></td>
<td>4*</td>
<td>0.0456</td>
<td>2.70</td>
<td>3.00</td>
</tr>
<tr>
<td></td>
<td>5*</td>
<td>0.0456</td>
<td>2.70</td>
<td>3.00</td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td>2.59</td>
<td></td>
<td>2.80</td>
</tr>
<tr>
<td>COV</td>
<td></td>
<td>8%</td>
<td>10%</td>
<td></td>
</tr>
</tbody>
</table>

(b) Steel AP50

<table>
<thead>
<tr>
<th>Notch Size</th>
<th>Test #</th>
<th>$\Delta_f$ (inches)</th>
<th>$\alpha$</th>
<th>$\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r^* = 0.125''$</td>
<td>1</td>
<td>0.0412</td>
<td>1.44</td>
<td>1.47</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.0335</td>
<td>1.17</td>
<td>1.10</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.0372</td>
<td>1.32</td>
<td>1.29</td>
</tr>
<tr>
<td>$r^* = 0.06''$</td>
<td>1</td>
<td>0.0218</td>
<td>0.84</td>
<td>0.81</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.0238</td>
<td>1.16</td>
<td>1.01</td>
</tr>
<tr>
<td>$r^* = 0.25''$</td>
<td>1</td>
<td>0.0490</td>
<td>1.18</td>
<td>1.13</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.0502</td>
<td>1.19</td>
<td>1.14</td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td></td>
<td>1.18</td>
<td>1.13</td>
</tr>
<tr>
<td>COV</td>
<td></td>
<td></td>
<td>15%</td>
<td>18%</td>
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</table>

(c) Steel AP110

<table>
<thead>
<tr>
<th>Notch Size</th>
<th>Test #</th>
<th>$\Delta_f$ (inches)</th>
<th>$\alpha$</th>
<th>$\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r^* = 0.125''$</td>
<td>1</td>
<td>0.0283</td>
<td>1.64</td>
<td>1.87</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.0268</td>
<td>1.61</td>
<td>1.78</td>
</tr>
<tr>
<td>$r^* = 0.06''$</td>
<td>1</td>
<td>0.0160</td>
<td>1.76</td>
<td>1.88</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.0135</td>
<td>1.41</td>
<td>1.44</td>
</tr>
<tr>
<td>$r^* = 0.25''$</td>
<td>1</td>
<td>0.0530</td>
<td>1.41</td>
<td>1.22</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.0390</td>
<td>0.93</td>
<td>0.80</td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td></td>
<td>1.46</td>
<td>1.50</td>
</tr>
<tr>
<td>COV</td>
<td></td>
<td></td>
<td>20%</td>
<td>28%</td>
</tr>
</tbody>
</table>

(d) Steel AP70HP

<table>
<thead>
<tr>
<th>Notch Size</th>
<th>Test #</th>
<th>$\Delta_f$ (inches)</th>
<th>$\alpha$</th>
<th>$\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r^* = 0.125''$</td>
<td>1</td>
<td>0.0662</td>
<td>3.20</td>
<td>3.48</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.0551</td>
<td>2.69</td>
<td>2.93</td>
</tr>
<tr>
<td>$r^* = 0.06''$</td>
<td>1</td>
<td>0.0337</td>
<td>2.90</td>
<td>3.27</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.0326</td>
<td>2.81</td>
<td>3.08</td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td></td>
<td>2.90</td>
<td>3.19</td>
</tr>
<tr>
<td>COV</td>
<td></td>
<td></td>
<td>7%</td>
<td>7%</td>
</tr>
</tbody>
</table>
4.4 The Characteristic Length *l* calibration issue

The low gradients in the notched bar tests provide ease of calibration of the *α* and the *η* parameter (due to the local micromechanical fracture criterion being simultaneously satisfied over a large area all at once, and the convenience of using a coarser mesh). However, for this same reason, the notched bar tests do not provide data to validate the characteristic length of the material.
The concept of the characteristic length relates to the fact that every model (whether micromechanical or macro-scale) needs a length scale to over which the fracture criterion must be satisfied for failure to occur, e.g. the macroscopic stress-based yield model would not work if the length scale chosen was smaller than the average dislocation spacing. Similarly, for the micromechanical models to work, a length scale needs to be established to ensure sufficient sampling. If the critical stress-strain state of the SMCS or VGM model was reached in a localized area between two inclusions, then obviously the voids would not nucleate, grow or coalesce. Thus a minimum volume of material must be identified, over which the failure criterion must be satisfied to trigger ductile crack initiation. This distance (referred to as the characteristic length $l^*$) is often determined based on microstructural measurements, obtained from scanning electron micrographs. Tests with higher stress-strain gradients, such as sharp-cracked fracture tests, provide another means of evaluation (validating) the characteristic length.

Existing literature on this topic suggests that there many different approaches have been adopted for determination of the $l^*$ parameter, but there is little consistency among them. The approaches have typically been governed by convenience, or limited by the lack of tools for a detailed evaluation. In any case, there is no clear choice as regards the most accurate way of determining this parameter, or why one method would be superior to another. We aim to look at various approaches and ideas that have been proposed, compare them, look at the reasons behind them, and finally present a proposed approach which incorporates attractive features from other approaches. Previously proposed approaches for evaluation of characteristic length include the following.

1. **Based on Inclusion Spacing or Dimple Diameter**

Since the ductile crack initiation process is based on coalescence of adjacent voids, the characteristic length is likely related to the inclusion spacing (the distance between two potentially coalescing voids). Also, the dimple diameter (the diameter of the half-void left behind after coalescence) might be an indicator of the inclusion spacing. This approach
has been used by Ritchie et al. (1979), by Panontin (1995), and Chi (2000). Figure 4.12 is a Scanning Electron Microscope (SEM) scan showing the dimples in the AW50 steel. For illustration, a dimple is highlighted in the figure.

Though it is fairly evident that the dimple size is a significant microstructural feature, its precise relation to the characteristic length remains somewhat of an issue of debate. Ritchie et al found that for different steels the characteristic length could be as small as twice the dimple size or as large as 6 or 7 times the dimple size. The factor of 2 seemed to work well for Panontin as well, and a similar factor was used by Chi since it predicted fracture in the validation tests with good accuracy. These different numbers suggest that ductile crack initiation is not a function only of the void size, but also of the number of voids coalescing to form the macrocrack.

2. Grain Size

Panontin (1995) proposed that the characteristic length be related to the grain size as follows –

\[ l^* = d \sqrt{n} \]  

(4.13)

Where \( d \) is the grain diameter, and \( n \) is the Ramberg-Osgood Coefficient for hardening properties of the metal. The key advantage of this method is that well-established standards (ASTM 112) exist for measuring the grain size so test results can be consistently reproduced, as opposed to dimple diameter measurement which is somewhat more subjective. This method worked well for the steels investigated in the Panontin study, but the exact physical relationship of the grain size to the microstructural length scale affecting ductile crack initiation is not perfectly clear. The grain size controls the inclusion spacing to some extent (because carbides are typically located at grain boundaries) – implying that a smaller grain size would have a smaller characteristic length. At the same time, it is well known that finer grains have less intergranular volume for inclusions to form – an observation which seems to contradict the earlier assumption.

Hancock and Cowling (1980), point out that the relationship between grain-size and the
fracture process is clear only for cleavage type fracture, which is peak-stress based and requires that the stress state be severe enough over a certain number of grains. They refer back to Ritchie’s paper (1979), in which the distinction is made between upper shelf and lower shelf behavior, making the point that using the grain-size for predicting upper-shelf type ductile fracture might not be the best strategy.

3. Mechanical Testing and Finite Element Analysis Based (Back Calculated from FEM)

This method is conceptually very straightforward and likely is very accurate, but the key disadvantage is that it does not relate fracture behavior to the microstructural features, and hence is not as clean in its predictive nature as some of the other microstructure based approaches. The idea is to run a test (such as an ASTM fracture test), which has a sharp strain gradient at the sharp crack tip, and then predict fracture in this test with different $l^*$ values assuming one already knows $\alpha$ and $\eta$ from the notched bar tests. The $l^*$ that best predicts fracture the best is retained as the calculated $l^*$ value. Fig 4.13 shows a brief outline of the methodology, using the SMCS criterion ahead of the crack tip.

Though this method should work and has been demonstrated to be effective by many investigators in the past, e.g., Beremin (1981), McMeeking (1977) and Norris et al.(1978), there are some questions that can be raised about this –

a) Is the fracture type test and the associated detailed and expensive finite element analysis a cost effective way to estimate the $l^*$, as opposed to running the notched bar tests and making some microstructural observations?

b) Is it appropriate to tie the definition of macrocrack formation to the concept of crack initiation in the ASTM definition of $J_{IC}$? Though the ASTM method is tried and tested for predicting toughness, it still is approximate in some sense (e.g., using the blunting line intercept to predict the critical J-value). In other words, should a method like the ASTM 1820, designed primarily for single parameter
toughness measurements be used as the basis for defining micromechanical models that aim to capture the fundamental process of fracture?

4. Observation of castellated fracture surface

Hancock and Mackenzie (1977) noted that failure initiation must involve a minimum amount of material that is characteristic of the scale of the physical events leading up to fracture. They observed that the “physical event leading up to fracture” is often the linking of two or more holes formed from inclusion colonies. In other words, first some inclusions nucleate voids to form larger holes (inclusion colonies); shear bands are then formed between adjacent inclusion colonies, and fracture finally occurs when shear localizes in these bands causing the two halves of the material to pull apart, leaving behind a castellated fracture surface. Figure 4.14 illustrates this process.

This mechanism suggests that for fracture to occur the material sampled must be at least as large as the diameter of the inclusion colonies (in an analogy somewhat similar to the critical distance being comparable to grain size for cleavage fracture). This methodology is particularly useful since it makes no assumptions about a grain size multiplier or a dimple size multiplier, rather it relates the observed failure surface from the notched bar test to the characteristic length a logical way.

How to measure the size of the inclusion colonies is another issue. Hancock and Mackenzie cut longitudinal sections of notched bars pulled until just before the point of failure and measured the colony (hole) sizes directly. The longitudinal section looked somewhat like the one in Figure 4.15.

Another way of measuring this is to make scans of the fractured surface at a 45 degree angle to the surface, so that the topographic features of the castellated surface are visible. We tried such an approach using an SEM, and clearly, some features (plateaus and troughs) similar to those observed by Hancock and Mackenzie are observed. Figures 4.16 (a) through (g) show the observed fracture surface from such a viewing angle for the
different steels, while 4.16 (h) shows the scanning electron microscope setup for such measurements. The scanning electron microscope provides the electron beam in a direction parallel to the horizontal of the figures creating shadows where the plateaus and troughs exist, the lengths of the dark and bright regions can be measured as estimates of the inclusion colony size.

4.4.1 Proposed approach for characteristic length calibration

The approach we propose (which is an extension of the one proposed by Hancock and Mackenzie), is to have two bounds and a most likely value for the characteristic length $l^\ast$. The lower bound would be twice the average dimple diameter. The reason is simple and physical, since we could, in the simplest sense, define fracture as the coalescence of two adjacent voids. Moreover, twice the dimple size has been shown to work reasonably by previous investigators. This lower bound value could be used as a conservative estimate for the $l^\ast$.

The upper bound on $l^\ast$ would be the length of the largest plateau or trough observed in the angled fractograph. This would be the most liberal interpretation of the characteristic length, but would give an idea of the maximum likely toughness that a material would exhibit, and together, the lower bound dimple definition and upper bound plateau/trough should help explain some of the experimental scatter. The mean value of $l^\ast$ would be arrived at by taking an average over roughly ten measurements of the lengths of the plateaus and troughs such as those shown in Figure 4.16(b) through (i). This would be the most likely estimate of the $l^\ast$ value.

In Table 4.7 values of $l^\ast$ for the upper bound, mean and the lower bound are tabulated for each steel. We then use these values to predict three values for the point of failure in test specimens or real structures that have high stress/strain gradients, where the characteristic length significantly affects the prediction. In the case of the notched bar, all these three values would give a nearly coinciding estimate of the failure point, because of the flat gradient of the SMCS or the VGM criterion over a large area. For other geometries, with
high gradients (such as sharp cracked geometries), the \( l^* \) has a significant effect on the fracture prediction.

The characteristic length values for all the steels lie roughly in the same range, i.e. – 0.005-0.012 inches. There is no strong correlation between the characteristic length and the CVN values or the toughness indices for the various steels.

Table 4.7 Estimate of Characteristic Lengths of different steels

<table>
<thead>
<tr>
<th>Steel</th>
<th>Characteristic Length (Inches)</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lower Bound (2 X Dimple Dia)</td>
<td>Mean Value (Average plateau)</td>
<td>Upper Bound (Largest Plateau)</td>
<td></td>
</tr>
<tr>
<td>AW50</td>
<td>0.0035</td>
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<td>0.015</td>
<td></td>
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<tr>
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<td>0.017</td>
<td></td>
</tr>
<tr>
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<td>0.019</td>
<td></td>
</tr>
<tr>
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<td>0.012</td>
<td>0.016</td>
<td></td>
</tr>
<tr>
<td>JP50</td>
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<td>0.014</td>
<td></td>
</tr>
<tr>
<td>JP50HP</td>
<td>0.0022</td>
<td>0.005</td>
<td>0.009</td>
<td></td>
</tr>
<tr>
<td>JW50</td>
<td>0.0024</td>
<td>0.009</td>
<td>0.014</td>
<td></td>
</tr>
</tbody>
</table>

4.5 Tests for micromechanical model parameter verification

To summarize, the micromechanical model parameters \( \alpha, \eta \) are calibrated based on tests of notched round bars and the \( l^* \) is determined by microstructural measurements. To validate the combination of \( \alpha, \eta \) and \( l^* \) obtained from these independent tests, three different types of validation tests are conducted that offer sharper stress and strain gradients, where the ductile fracture is expected to involve \( \alpha \) and \( \eta \), as well as \( l^* \). Two of these tests are standard fracture mechanics tests, which seek to relate the micromechanical models to conventional fracture indices, whereas the third is a new test to evaluate fracture initiation at a blunt flaw tip without a stress singularity. Critical fracture toughness values or J-integrals (corresponding to the nonlinear energy release rate for fracture) are recovered from the standard fracture tests, whereas a critical deformation at material rupture is obtained from the blunt notch test. These critical indices are compared. We start by describing the three point bend fracture tests followed by the compact tension fracture tests and the blunt notched fracture tests.
4.5.1 *Three Point Bend Tests*

The single edge crack three-point bending specimens shown in Figure 4.17 are based on ASTM E399 standard. These are known as Single Edge Notched Bending (SENB) specimens. The specimens have a thickness of 1 inch, height is 2 inches and the span is 8 inches. At the section of the crack, the specimen is side-grooved to a net thickness of 0.8 inches to promote uniform constraint across the crack width and encourage straight-through crack growth. The side grooves help avoid tunneling of cracks (faster growth at the center of the specimen due to the higher constraint conditions present there). The crack is initially machined using an Electric Discharge Machine (EDM) with a 0.003 inch radius followed by fatigue pre-cracking to extend and sharpen the crack tip for a distance of 0.08 to 0.16 inches beyond the EDM-machined crack front. The overall notch geometry conforms to that specified in ASTM E399 for fracture toughness testing. The clip-on displacement gage is mounted to the notched edge to record the Crack Mouth Opening Displacement (CMOD). Two a/W ratios of 0.17 (one test) and 0.60 (two tests) are studied. A J-resistance curve is produced according to ASTM E1820, and a critical fracture toughness $J_{IC}$ is calculated. The rationale for two different crack depths is to generate different crack tip constraint conditions to test the micromechanical models. These three point bend tests are run only on the AW50 steel, because it was the first batch of steel tested, and it was later determined that compact tension tests provide us with similar data while using much smaller amounts of material.

4.5.2 *Compact Tension Specimens – Sharp Cracked (CT)*

As an alternative to the three point bend specimens, compact tension fracture specimens are also used for making fracture initiation predictions. These are referred to as Single Edge Notched Tension (SENT) specimens. They are machined in accordance with ASTM E399, with an a/W ratio of 0.50 for all steels except AW50, for which we use an a/W ratio of 0.6. This is a standard “IT” specimen, whose thickness and width are 1 and 2 inches respectively, as shown in Figure 4.18. A displacement gage is mounted to the notched edge to measure the displacement at the load line. The critical J, as well as the load line displacement corresponding to it is determined. As in the three point bending
specimen, the compact tension specimens are side grooved to a net section of 0.8 inches width to minimize crack tunneling. Load displacement curves for one such test (test details) are shown in Figure 4.19. The procedure involves loading and unloading the specimen at different points. The unloading slopes are measured and the current crack length is calculated. This is then used to plot the J-R or J-resistance curve which provides us with the critical J value. One such curve is shown in Figure 4.20. The curve plots the J-integral versus the crack extension. Other specimens show similar behavior, however they are not included here to preserve brevity and readability. For a detailed description of the test process, we refer to ASTM E399 and ASTM E1820 standards. The ASTM defines JIC based on a blunting line intercept concept, which uses material flow stress and empirical observations to infer a crack initiation point based on the test data. The important results from the tests are summarized in Tables 4.8 and 4.10. The commentary on this data is provided later. The test data generated by the author was consistent with standard ASTM requirements and the toughness values were valid as per the procedure.

4.5.3 Compact Tension Specimens – Blunt Notched (BN)

We developed a new type of specimen which has the same overall geometry (See Figure 4.21) as the CT specimen, but different type of notch. Instead of having a fatigue-precracked, “atomically sharp” crack in the specimen, a machined notch is introduced which has a notch radius of \( r^* = 0.03125 \) (\( \frac{1}{32} \)) inch. These tests are run for only four of the seven steels due to budgetary and time constraints. The purpose of this test is to mimic more realistic defect/flaw situations in structural connections, which may have strong stress concentrations, but not necessarily sharp cracks. The other advantage of this specimen is that it allows for cyclic loading as opposed to the sharp cracked situation where crack closure prevents cyclic loading. The load-line (or mouth opening) displacement at which the material is visually observed to rupture is noted for comparison with the analytical prediction. For all materials except AP70HP, the load line displacement is measured, for AP70HP the mouth opening displacement on the material surface is measured because some irreversible machining errors prevented the use of the displacement gage at the load line. In these cases, it had to be placed at the material
surface instead – see figure 4.22. Two different thicknesses 1 inch and 0.5 inch (0.5 inch used only for the AW50) with different constraint (i.e., triaxiality) conditions are tested to assess the capabilities of the models. These specimens are not side grooved because of two reasons – the crack propagation is not studied, and secondly because the crack has finite width (because of the blunt notch geometry), it is difficult to machine a side groove that will produce the desired results of uniform constraint. The finite element analysis is expected to capture effects of through thickness constraint variations.

4.5.4 Finite element models and fracture prediction procedures for the different test configurations

To verify the SMCS and the VGM criteria, a common methodology is adopted, which involves tracking the stress and strain fields over a volume of material ahead of the crack tip in the finite element model. As discussed earlier, the SMCS criterion is assumed to be satisfied at any point if the equivalent plastic strain at that point exceeds the critical equivalent plastic strain, i.e. –

\[ \varepsilon_p - \alpha \exp(-1.5T) > 0 \]  

(4.14)

Similarly, the VGM criterion is satisfied if the void growth calculated from integrating the stress and strain histories exceeds the critical value, i.e. –

\[ \int_0^{\varepsilon_{\text{critical}}} \exp(1.5T)d\varepsilon_p - \eta > 0 \]  

(4.15)

The basic procedure involves tracking the value of the expressions in equations (4.14) and (4.15) over a distance ahead of the crack tip, and recording the point in the loading history when their value exceeds zero over a given distance. This distance is compared to one of the bounds or the mean of the characteristic length for the steel under consideration. This point in the analysis is then related to a global measure of loading, which may be described in terms of the applied J-integral, or crack opening displacement.
The following sections describe in detail the application of the SMCS and the VGM criteria to the three point bend and compact tension fracture specimens, and the blunt notched compact tension specimens.

*Three point bend specimens*

To verify the micromechanical models, the two-dimensional plane strain finite element models of the deep (a/W = 0.60) and shallow (a/W = 0.17) crack three-point-bending specimens are analyzed to simulate the mid-plane fracture behaviors. Both models possess the same crack tip element mesh, which is shown in Figure 4.23 (a) (the model for a/W = 0.6 is shown). The size of the elements surrounding the crack tip is about 0.0004 inches. This size is selected so that there are at least 10 or 15 elements included within the lower bound estimate of characteristic length so as to eliminate the effect of mesh densities on the results. The crack tip is modeled with a finite (half) radius of 0.0003 inches in anticipation of substantial crack tip blunting. This is much smaller than the anticipated critical crack tip opening displacement (about 0.01 inches), which helps ensure that the finite radius does not affect the final results. Because of the finite plastic strain algorithms applied in these analyses, the blunted crack tip assumption aids in numerical convergence. The equivalent plastic strain, the effective stress and the hydrostatic stresses are monitored at a number of points (equal to a multiple - typically 5, of the characteristic length) ahead of the crack tip, and the J-integral is calculated for the crack tip at each stage of loading. The J-integral is calculated using many contour, typically 50 (with radii ranging from 0.0003 inches to about 1 inch), so that consistency in estimates between the different contours is obtained.

The point of crack initiation is predicted to be where the fracture parameter first satisfies the SMCS or the VGM criterion (according to equations (4.14) and (4.15)). The mean values of the $\alpha$ and $\eta$ values as reported in Table 4.6 are used in the evaluation of these expressions. The corresponding J-integral (as obtained from the finite element analysis model) is defined as the critical initiation value, denoted by $J_{IC}^{\text{Analysis}}$ for that particular
failure criterion. Figure 4.24 schematically illustrates this idea, where the $J_{IC}^{\text{Analysis}}$ would correspond to $J_2$ in the figure where the critical index is satisfied over $l^*$. 

The stress-strain histories are monitored at three points directly ahead of the crack corresponding to the lower bound, mean and upper bound on the characteristic length $l^*$, producing three $J_{IC}^{\text{Analysis}}$ values. The predicted $J_{IC}^{\text{Analysis}}$ values are compared with the experimental results in the first three rows of Table 4.8(a) for the SMCS and Table 4.10(a) for the VGM. Also see Figures 4.30 (a) and (b). It is apparent that the bound values predicted using the SMCS and VGM criteria bracket the experimental values in most of the cases for the SMCS, and the mean analytical value is close to the experimental values. To save material, three point bend tests were conducted only for AW50. Compact tension tests were conducted for this and all the other steels, and they are discussed in the next section.

**Compact Tension Specimens – Sharp Cracked**

Two dimensional plane-strain models for the sharp-crack compact tension geometry were analyzed. The crack tip mesh is similar to that of the three point bend specimens, as shown in Figure 4.23 (b). J-integral values are calculated for the upper bound, lower bound and mean values of the characteristic length, as are the loads. Tables 4.8 (a) through (g) and 4.10 (a) through (g) show a comparison of the predicted J-integral values to the experimental observations. Figures 4.30 (a) and (b) show the same results graphically. There are several observations that can be made from these tables –

- For the AW50, AP50 and JP50 steels, the experimental J-values are fairly consistent and agreeable with the analytical predictions. The VGM seems to overpredict the critical value of the J-integral, but this may be due to a combination of inherent uncertainty in the model parameters, especially the characteristic length, whose measurement is somewhat debatable.
- For the other steels, the predictions of the experimental values of the J-integral are again fairly consistent, but the analytical predictions based on the three different
length values return J values that are very different (about a 6X difference). This indicates that the J value, especially for sharp crack specimens with very high strain gradients is very sensitive to the length scale parameter. This is a disadvantage of the model, when applied to sharp-crack situations.

- For high ductility steels such as AP70HP or the JP50HP, the VGM seems to do a better job than the SMCS because of the large geometry changes involved. In these situations, a triaxiality change of about 70% is observed during loading, which contradicts the SMCS assumptions. The VGM accounts for this change and reduces the discrepancy.

In general, the lower bound on the characteristic length seems to predict fracture with as much accuracy as the mean. This might be due to a couple of reasons. The sharp crack tests, due to the extremely high stress situations, can experience local cleavage type failure that is not modeled by the SMCS and VGM models. Secondly, the characteristic length measurement, as described earlier, is somewhat subjective in nature, and this, coupled with the sensitivity of the fracture process in such high gradient situations to this length is partly responsible for the discrepancy between the mean estimate of failure and the experimental observations.

Traditional fracture mechanics combines the effects of characteristic length, toughness indices and micromechanisms typically in one parameter. The high-stress, high-gradient situations created by the sharp cracked case might be more reliably captured by traditional fracture mechanics than by the micromechanical models.

*Compact Tension Specimens – Blunt Notched*

Three-dimensional models were used for analyzing the smooth notched compact tension specimens. The blunt crack led to large deformations of the specimens before fracture – with extensive necking and bulging (see Figure 4.25) which is not accurately captured in a two-dimensional model. Moreover, for the AW50 steel, we used two thicknesses, i.e. 1 inch and 0.5 inch to explicitly investigate the effects of triaxiality and constraint. The
triaxiality differences between these situations would not be picked up by a two-dimensional model which is either plane-strain or plane-stress.

For the blunt notched specimens, the crack tip and mesh is modeled in a way similar to the sharp crack specimens, but the notch radius is 0.03125 inches. Figure 4.26 shows the picture of the 3-dimensional mesh, which has approximately 2500 elements and requires around 30 minutes to run on a Pentium 4, 1.6 GHz processor. The SMCS and VGM expressions are monitored ahead of the crack tip at many locations to find over what distance the failure criterion is satisfied. A failure displacement (at the load-line) is calculated in a way similar to the J-integral for the sharp crack specimens. Figure 4.27 shows this process in detail. Figure 4.28 shows such a calculation conducted for the SMCS criterion. The equivalent plastic strain, triaxiality and SMCS fields are monitored ahead of the crack tip. The SMCS is said to be satisfied if it is greater than zero (refer Chapter 2). Also shown in Figure 4.28 is the load displacement curve for the specimen (plotting load line displacement versus load). The different graphs for the gradients (numbered), correspond to the numbered points on the load displacement curve. Looking at the SMCS curves in Figure 4.28 (d), we see that the failure criterion – in this case the SMCS, is satisfied over the mean characteristic length ($l^* = 0.008$ inches, for AW50) somewhere between points 3 and 4 on the load-displacement curve. It is predicted that failure occurs between these two points, and the point of failure is marked on figure 4.28 (a). The mean failure displacement predicted by the SMCS model is also overlaid on the load-displacement curve. It is interesting to note that the specimen starts losing load capacity soon after this point. The drop is more gradual than the notched bars, because the crack initiation area is small as compared to the ligament length, which is still hardening and hence picking up load. Only when the reduction of area due to crack initiation overwhelms the increase in load carrying capacity due to the hardening, does the load begin to drop.

The results are tabulated in tables 4.9 and 4.11, and in Figures 4.30 (c) and (d). We can make a number of observations from these –
For most of the steels, the agreement between tests and analyses is much better than that observed for the sharp-crack specimens. The blunt notched specimens do not have stress/strain gradients as sharp as the sharp crack specimens, and are less sensitive to the characteristic length parameter. This is similar to the notched bars, which are insensitive to \( l^* \). This is supported by the fact that the lower and upper bound estimates of the predicted failure displacement are much closer to each other than the predicted J-integral values.

The thinner specimens for AW50 have lower constraint and consequently a much higher ductility than the thicker specimen. This would not have been picked up by traditional fracture mechanics or two-dimensional models (which are either plane-stress or plane strain), but is accurately captured by the micromechanical models.

For AP110, which is a high strength steel, both the SMCS and the VGM overpredict failure displacements. This might be due to the elevated stresses causing another mode of failure such as local cleavage or intergranular fracture not picked up by the micromechanical models geared towards void growth processes.

On the whole, the SMCS and VGM predictions for the blunt notched specimens are fairly repeatable and accurate with respect to the experimental observations. This demonstrates the ability of the micromechanical models to predict failures in conditions without sharp cracks, which traditional fracture mechanics cannot handle.

Data from the sharp and the blunt notched specimens suggests that the weak link the micromechanical models is the characteristic length determination. This creates problems for high-gradient situations such as sharp-crack geometries, but is less of an issue in blunt notched specimens or notched round bars.

### 4.6 Summary

Looking at tables 4.8 to 4.11, we observe that the micromechanical models are a useful means of predicting fracture in a variety of materials. The notched bar tests are much easier and cheaper to perform as compared to the costly ASTM fracture tests, but through
the micromechanical models, they predict ductile fracture effectively even in situations of large scale yielding where conventional fracture mechanics may not be as reliable.

The toughness indices $\alpha$ and $\eta$ are loosely correlated with the traditional $J_{IC}$ toughness measure, because the $J_{IC}$ integrates a number of effects into one index – that include the toughness parameter for the SMCS/VGM models, as well as characteristic length effects, which are not captured in the SMCS or VGM calibration.

The scatter in the toughness index ($\alpha$ and $\eta$) values from the notched bar tests combined with the scatter and subjectivity in the measurement of the characteristic length is sufficient to explain the uncertainty in the measured fracture toughness parameters such as the $J_{IC}$ or the load line displacement in the blunt notched compact tension specimens. It is again important to remember that the tables use a mean value of the toughness indices but in reality, it is likely that these will have a distribution of their own.

The toughness indices ($\alpha$ and $\eta$) are much better estimated than the characteristic length, and as a result, situations that are more sensitive to characteristic length are less reliably modeled by the SMCS and the VGM models. As an example, for the sharp-crack specimens, the micromechanical predictions of failure are not nearly as good for the blunt notch specimens, which have less sharp stress/strain gradients.

As we conclude this chapter, it is also interesting to see how the SMCS and VGM models compare to more conventional toughness measures. Figures 4.29 (a) and (b) plot the $\alpha$ and $\eta$ values for the different steels against the upper shelf Charpy V-Notch energy values for all the steels. There seems to be a very strong positive correlation between the CVN energy values and the micromechanical model parameters (as indicated by the high $R^2 \approx 0.9$ values for the regression line fit to the data – see figure). Encouraged by the strong correlation, the CVN values could be converted to $\alpha$ or $\eta$ values if time or budgetary constraints do not permit a complete micromechanical model calibration as described in this chapter. Equations (4.16) and (4.17) contain expressions for this conversion.
\[ \alpha_{CVN} = 0.021 \cdot CVN - 0.93 \]  \hspace{1cm} (4.16)

\[ \eta_{CVN} = 0.024 \cdot CVN - 1.30 \]  \hspace{1cm} (4.17)

The CVN values in the above equations are expressed in foot-pounds. Trends as clear as these were not observed for the other fracture indices, such as \(J_{IC}\), likely because the different constraint situation can change the mode of failure from ductile to more brittle, while the CVN upper shelf value is guaranteed to provide situations of ductile tearing. Moreover, the Charpy V-notch is not an infinitely sharp crack like the SENT or SENB specimens, and the sensitivity to the \(l^*\) can be assumed to be smaller for the CVN.

Figure 4.30 (a) provides a convenient histogram comparison of the experimental values and the predicted values due to the SMCS for all tests. Figure 4.30 (b) repeats this for the VGM. These figures, along with tables 4.8-4.11 form the basis of the observations that follow –

- The SMCS model is obviously more convenient to apply since it does not involve integrations of quantities, but in some cases, such as the \(J\)-integral tests for steels AP70HP, JP50HP and JW50, the SMCS predicts \(J_{IC}\) that are much larger than the experimentally observed values. See Figure 4.30 (a) and (b). It is interesting to note that these steels are some of the most ductile steels that we have tested, and as a result, large crack tip deformations, leading to significant blunting occurs in these materials prior to fracture. A direct consequence of this is that the triaxiality ahead of the crack tip drops substantially (by as much as 70%), as the plastic strain increases. The SMCS depends only on the instantaneous value of triaxiality, and because the triaxiality is low in the advanced stages of loading, the plastic strain has to reach very large values for the SMCS criterion to be triggered. As a result of this, the SMCS predicts unrealistically large values for the \(J_{IC}\), because it does not accurately account for the void growth that occurred in the earlier part of the loading when the triaxiality was very high. The VGM on the other hand is much more realistic in the sense that it tracks the void growth over the entire history of loading, and accounts for the fact that much of the void growth...
occurred in the early stages, especially for the fracture tests of the more ductile materials. As a result of this, the predictions of failure for these situations are better than those obtained from the SMCS model.

- For the AP110 steel, where the change in triaxiality in loading is not very high, either for the sharp cracked or the blunt notched tests, but the SMCS and VGM both seem to overestimate the measure of ductility (be it the $J_{IC}$ or the load-line displacement). See Tables 4.8 (c), 4.9 (c), 4.10 (c) and 4.11 (c). A possible reason for this could be that the high strength of this steel (about twice that of the other steels), elevates the stresses at the crack tip to be much higher than the cleavage stress for intragranular cleavage stress such as simulated in the RKR model – see Ritchie et al (1973). Due to this, it is entirely possible that the cleavage failure criterion is triggered before the SMCS or the VGM criterion is triggered (which have a requirement in terms of the plastic strain, unlike the cleavage criterion, which is typically stress-based). A similar situation was encountered in one of the notched round bars for AP50 ($r^* = 0.06''$), where the bar failed abruptly before even reaching yield – likely due to sampling a weak particle that triggered cleavage. This particular test was not included in any of the analyses.

Based on these observations, we reach a couple of important conclusions regarding the used of these models for fracture predictions.

The first conclusion is more specific to the models used – i.e., the SMCS versus the VGM. The SMCS is a pre-integrated form the VGM, and as a result, there is the possibility of losing important history related information. As a result, for situations where large triaxiality changes are present, the SMCS should not be the model of choice. However, the SMCS offers the advantage of easy implementation. The user should be aware of the relative advantages and disadvantages of the SMCS and the VGM models and make an informed decision as to the choice of one model over the other.

The other important issue to remember is that both the SMCS and the VGM assume that fracture will occur based on the void growth and coalescence method. However, this may
always not be the case. Small microcracks present within the material microstructure, or high stress concentrations due to a combination constraint and high-strength might sometimes lead to situations where intragranular cleavage or intergranular separation is the controlling mechanisms. As in the case of the CVN specimens, this transition from the ductile mechanism to the more brittle mechanisms may be caused due to a combination of high constraint, high strength and possibly low temperature. Moreover, the cleavage process is highly random in nature (as discussed in Chapter 2), and may also occur at surprisingly low constraint and force levels. Such occurrences were observed in at least two situations in the notched bar specimens tested by us.

In an ideal case, where a safety assessment of structural or mechanical components might be required, one would be well advised to apply the SMCS and the VGM models, as well as other models, such as the RKR model to ensure that cleavage fracture mode is accounted for.

Finally, it is observed that a weakness of the micromechanical models is the subjectivity in the measurement of the characteristic length. This uncertainty becomes very important in situations such as the sharp crack specimens that are very sensitive to the length scale. Thus, the micromechanical models are reliable for use in situations which do not have very sharp stress or strain gradients, but somewhat suspect in situations with sharp cracks. More research in the area of removing this uncertainty in the length scale issues is likely to solve the problem.

To summarize the findings from the part of our study reported in this chapter, we can say that the micromechanical models are reasonably accurate in predicting ductile crack initiation under monotonic loading, even under situations of large scale yielding as observed in the blunt notched specimens, and some of the more ductile steel varieties. The VGM model aims to be more exact, by capturing the stress and strain history effects, while the SMCS, though more convenient to use, depends only on the current stress and strain state and neglects history effects.
The other important issue to consider is that fracture may be caused due to a combination of micromechanisms such as void coalescence as well as cleavage. It is difficult to envision a single micromechanical model can model all of these mechanisms simultaneously. However, it is possible to envision a situation where different tests will be designed to specifically calibrate parameters of different models, each aimed at capturing a different mode of fracture. This is very much unlike test methods such as the Charpy V-Notch tests that give a combined view of toughness without understanding what mechanisms contribute to it. Such methods are difficult to generalize and will hopefully be replaced by the newer set of micromechanical models that work together to give extremely refined estimates of fracture initiation under different situations.

The research described by this chapter aims to promote and advance models for the void growth and coalescence method. Chapter 6 starts where this chapter ends – by applying these models to realistic situations that resemble structural components. How steels respond to the situation of ultra low cycle fatigue (as described in Chapter 2), is a completely different and more complex problem. It forms the focus of Chapter 3 and Chapter 5.
Table 4.1 Steels used as part of the study

<table>
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<tr>
<th>Designation</th>
<th>Notation</th>
<th>Description</th>
<th>Manufacturer</th>
<th>Orientation of Specimen*</th>
<th>Thickness of flange or plate (inches)</th>
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<tbody>
<tr>
<td>A572-Grade 50</td>
<td>AW50</td>
<td>Low carbon steel cut from flange of rolled W-section.</td>
<td>USA</td>
<td>L-T</td>
<td>1.3</td>
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<tr>
<td>A572-Grade 50</td>
<td>AP50</td>
<td>Low carbon steel cut from rolled plate shape.</td>
<td>USA</td>
<td>T-L</td>
<td>1.0</td>
</tr>
<tr>
<td>A514-Grade 110</td>
<td>AP110</td>
<td>High strength steel cut from rolled plate.</td>
<td>USA</td>
<td>T-L</td>
<td>1.0</td>
</tr>
<tr>
<td>HPS70W</td>
<td>AP70HP</td>
<td>High performance bridge steel cut from rolled plate.</td>
<td>USA</td>
<td>L-T</td>
<td>1.5</td>
</tr>
<tr>
<td>JIS-SN490B Grade 50</td>
<td>JP50</td>
<td>Low carbon steel cut from rolled plate</td>
<td>NSC – Japan</td>
<td>L-T</td>
<td>1.0</td>
</tr>
<tr>
<td>JIS-SM490YBTMC-5L Grade 50</td>
<td>JP50HP</td>
<td>High performance plate steel for bridge use</td>
<td>NSC – Japan</td>
<td>L-T</td>
<td>1.0</td>
</tr>
<tr>
<td>JIS-SN490B Grade 50</td>
<td>JW50</td>
<td>Low carbon steel cut from flange of W-section</td>
<td>NSC – Japan</td>
<td>L-T</td>
<td>1.1</td>
</tr>
</tbody>
</table>

* L-T means that the loading is in the Longitudinal direction, and the crack propagation is the Transverse direction. T-L means that the loading is in the Transverse direction, and the crack propagation is in the Longitudinal direction.
Table 4.4 Chemical Composition of the Test Steels by weight percentage

<table>
<thead>
<tr>
<th>Steel</th>
<th>C</th>
<th>S</th>
<th>Mn</th>
<th>P</th>
<th>Si</th>
<th>Cr</th>
<th>V</th>
<th>Ni</th>
<th>Mo</th>
<th>Cu</th>
<th>C&lt;sub&gt;eq&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>AW50</td>
<td>0.07</td>
<td>0.015</td>
<td>1.39</td>
<td>0.01</td>
<td>0.2</td>
<td>0.05</td>
<td>0.05</td>
<td>0.1</td>
<td>0.007</td>
<td>0.26</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>(&lt;0.23)</td>
<td>(&lt;0.05)</td>
<td>(&lt;1.35)</td>
<td>(&lt;0.04)</td>
<td>(&lt;0.4)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AP50</td>
<td>0.22</td>
<td>0.015</td>
<td>1.22</td>
<td>0.007</td>
<td>0.27</td>
<td>0.01</td>
<td>0.07</td>
<td>0.006</td>
<td>***</td>
<td>***</td>
<td>0.44</td>
</tr>
<tr>
<td></td>
<td>(&lt;0.23)</td>
<td>(&lt;0.05)</td>
<td>(&lt;1.35)</td>
<td>(&lt;0.04)</td>
<td>(&lt;0.4)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AP110</td>
<td>0.20</td>
<td>0.005</td>
<td>0.95</td>
<td>0.006</td>
<td>0.38</td>
<td>0.57</td>
<td>0.04</td>
<td>0.08</td>
<td>0.26</td>
<td>0.26</td>
<td>0.56</td>
</tr>
<tr>
<td></td>
<td>(0.15-</td>
<td>(&lt;0.035)</td>
<td>(0.80-</td>
<td>(&lt;0.035)</td>
<td>(0.20-</td>
<td>(0.50-</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.21)</td>
<td></td>
<td>1.10)</td>
<td></td>
<td>0.40)</td>
<td>0.80)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AP70</td>
<td>0.08</td>
<td>0.006</td>
<td>1.31</td>
<td>0.01</td>
<td>0.43</td>
<td>0.69</td>
<td>0.06</td>
<td>0.34</td>
<td>0.07</td>
<td>0.33</td>
<td>0.51</td>
</tr>
<tr>
<td>HP</td>
<td>(&lt;0.11)</td>
<td>(&lt;0.006)</td>
<td>(1.10-</td>
<td>(&lt;0.02)</td>
<td>(0.3-0.5)</td>
<td>(0.45-</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.35)</td>
<td></td>
<td></td>
<td>(0.70)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>JP50</td>
<td>0.16</td>
<td>0.003</td>
<td>1.47</td>
<td>0.013</td>
<td>0.45</td>
<td>0.02</td>
<td>***</td>
<td>0.01</td>
<td>***</td>
<td>0.01</td>
<td>0.41</td>
</tr>
<tr>
<td></td>
<td>(&lt;0.18)</td>
<td>(&lt;0.015)</td>
<td>(&lt;1.6)</td>
<td>(&lt;0.03)</td>
<td>(&lt;0.55)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>JP50</td>
<td>0.09</td>
<td>0.002</td>
<td>1.51</td>
<td>0.007</td>
<td>0.17</td>
<td>0.03</td>
<td>***</td>
<td>0.02</td>
<td>0.008</td>
<td>0.007</td>
<td>0.35</td>
</tr>
<tr>
<td>HP</td>
<td>(&lt;0.2)</td>
<td>(&lt;0.035)</td>
<td>(&lt;1.6)</td>
<td>(&lt;0.035)</td>
<td>(&lt;0.55)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>JW50</td>
<td>0.11</td>
<td>0.003</td>
<td>1.48</td>
<td>0.007</td>
<td>0.14</td>
<td>0.04</td>
<td>0.066</td>
<td>0.26</td>
<td>***</td>
<td>0.29</td>
<td>0.41</td>
</tr>
<tr>
<td></td>
<td>(&lt;0.18)</td>
<td>(&lt;0.015)</td>
<td>(&lt;1.6)</td>
<td>(&lt;0.03)</td>
<td>(&lt;0.55)</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The values in parentheses below each of the quantities denotes the values prescribed by the relevant ASTM or Japanese standard, e.g. the first row and column (for AW50) can be read as –

0.07 = Carbon content from spectroscopy tests

(<0.23) = Carbon content allowed as per ASTM standard A572 (applicable to AW50)

The Carbon Equivalent is calculated as per the following equation –

\[
C_{eq} = C + \frac{Mn}{6} + \frac{Cr + Mo + V}{5} + \frac{Ni + Cu}{15}
\]
Table 4.8 Comparison of experimental and analytical results for fracture specimens using the SMCS model

(a). Steel AW50

<table>
<thead>
<tr>
<th>Test</th>
<th>a/W</th>
<th>$J_{IC}^{Test}$ (ksi-in)</th>
<th>$J_{IC}^{Analysis}$ (ksi-in)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Lower Bound</td>
<td>Mean</td>
</tr>
<tr>
<td>TPB 1</td>
<td>0.6</td>
<td>1.525</td>
<td>0.943</td>
</tr>
<tr>
<td>TPB 2</td>
<td>0.6</td>
<td>2.007</td>
<td>0.793</td>
</tr>
<tr>
<td>TPB 3</td>
<td>0.17</td>
<td>2.408</td>
<td>0.722</td>
</tr>
<tr>
<td>CT1</td>
<td>0.6</td>
<td>1.606</td>
<td></td>
</tr>
<tr>
<td>CT2</td>
<td>0.6</td>
<td>1.560</td>
<td></td>
</tr>
<tr>
<td>CT3</td>
<td>0.6</td>
<td>2.408</td>
<td></td>
</tr>
</tbody>
</table>

(b). Steel AP50

<table>
<thead>
<tr>
<th>Test</th>
<th>a/W</th>
<th>$J_{IC}^{Test}$ (ksi-in)</th>
<th>$J_{IC}^{Analysis}$ (ksi-in)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Lower Bound</td>
<td>Mean</td>
</tr>
<tr>
<td>CT1</td>
<td>0.5</td>
<td>0.55</td>
<td>0.390</td>
</tr>
<tr>
<td>CT2</td>
<td>0.5</td>
<td>0.78</td>
<td></td>
</tr>
</tbody>
</table>

(c). Steel AP110

<table>
<thead>
<tr>
<th>Test</th>
<th>a/W</th>
<th>$J_{IC}^{Test}$ (ksi-in)</th>
<th>$J_{IC}^{Analysis}$ (ksi-in)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Lower Bound</td>
<td>Mean</td>
</tr>
<tr>
<td>CT1</td>
<td>0.5</td>
<td>1.051</td>
<td>1.106</td>
</tr>
<tr>
<td>CT2</td>
<td>0.5</td>
<td>0.750</td>
<td></td>
</tr>
</tbody>
</table>

(d). Steel AP70HP

<table>
<thead>
<tr>
<th>Test</th>
<th>a/W</th>
<th>$J_{IC}^{Test}$ (ksi-in)</th>
<th>$J_{IC}^{Analysis}$ (ksi-in)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Lower Bound</td>
<td>Mean</td>
</tr>
<tr>
<td>CT1</td>
<td>0.5</td>
<td>1.180</td>
<td>1.825</td>
</tr>
<tr>
<td>CT2</td>
<td>0.5</td>
<td>0.892</td>
<td></td>
</tr>
<tr>
<td>CT3</td>
<td>0.5</td>
<td>2.403</td>
<td></td>
</tr>
</tbody>
</table>

(e). Steel JP50

<table>
<thead>
<tr>
<th>Test</th>
<th>a/W</th>
<th>$J_{IC}^{Test}$ (ksi-in)</th>
<th>$J_{IC}^{Analysis}$ (ksi-in)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Lower Bound</td>
<td>Mean</td>
</tr>
<tr>
<td>CT1</td>
<td>0.5</td>
<td>2.202</td>
<td>1.175</td>
</tr>
<tr>
<td>CT2</td>
<td>0.5</td>
<td>1.511</td>
<td></td>
</tr>
<tr>
<td>CT3</td>
<td>0.5</td>
<td>2.250</td>
<td></td>
</tr>
</tbody>
</table>
### (f). Steel JP50HP

<table>
<thead>
<tr>
<th>Test</th>
<th>a/W</th>
<th>$J_{IC}^{Test}$ (ksi-in)</th>
<th>$J_{IC}^{Analysis}$ (ksi-in)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Lower Bound</td>
</tr>
<tr>
<td>CT1</td>
<td>0.5</td>
<td>2.25</td>
<td></td>
</tr>
<tr>
<td>CT2</td>
<td>0.5</td>
<td>Invalid J-Curve</td>
<td>3.530</td>
</tr>
<tr>
<td>CT3</td>
<td>0.5</td>
<td>1.65</td>
<td></td>
</tr>
</tbody>
</table>

### (g). Steel JW50

<table>
<thead>
<tr>
<th>Test</th>
<th>a/W</th>
<th>$J_{IC}^{Test}$ (ksi-in)</th>
<th>$J_{IC}^{Analysis}$ (ksi-in)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Lower Bound</td>
</tr>
<tr>
<td>CT1</td>
<td>0.5</td>
<td>1.003</td>
<td></td>
</tr>
<tr>
<td>CT2</td>
<td>0.5</td>
<td>1.455</td>
<td>2.437</td>
</tr>
<tr>
<td>CT3</td>
<td>0.5</td>
<td>0.592</td>
<td></td>
</tr>
</tbody>
</table>
Table 4.9 Comparison of experimental and analytical results for blunt notched specimens using the SMCS model

(a). Steel AW50

<table>
<thead>
<tr>
<th>Test</th>
<th>a/W</th>
<th>$\Delta_{\text{Test failure}}$ (inches) Measured at load-line</th>
<th>$\Delta_{\text{Analysis failure}}$ (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Lower Bound</td>
<td>Mean</td>
</tr>
<tr>
<td>BN1</td>
<td>0.5</td>
<td>0.338</td>
<td></td>
</tr>
<tr>
<td>BN2</td>
<td>0.5</td>
<td>0.252</td>
<td></td>
</tr>
<tr>
<td>BN3</td>
<td>0.5</td>
<td>0.271</td>
<td></td>
</tr>
<tr>
<td>BN4</td>
<td>0.5</td>
<td>0.389</td>
<td></td>
</tr>
<tr>
<td>BN5</td>
<td>0.5</td>
<td>0.370</td>
<td></td>
</tr>
<tr>
<td>BN6</td>
<td>0.5</td>
<td>0.393</td>
<td></td>
</tr>
</tbody>
</table>

Note – BN1-BN3 have thickness 1 inch, while BN4-BN6 have thickness 0.5 inch.

(b). Steel AP50

<table>
<thead>
<tr>
<th>Test</th>
<th>a/W</th>
<th>$\Delta_{\text{Test failure}}$ (inches)</th>
<th>$\Delta_{\text{Analysis failure}}$ (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Lower Bound</td>
<td>Mean</td>
</tr>
<tr>
<td>BN1</td>
<td>0.5</td>
<td>0.077</td>
<td></td>
</tr>
<tr>
<td>BN2</td>
<td>0.5</td>
<td>0.086</td>
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</tr>
<tr>
<td>BN3</td>
<td>0.5</td>
<td>0.083</td>
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</tr>
</tbody>
</table>

(c). Steel AP110

<table>
<thead>
<tr>
<th>Test</th>
<th>a/W</th>
<th>$\Delta_{\text{Test failure}}$ (inches)</th>
<th>$\Delta_{\text{Analysis failure}}$ (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Lower Bound</td>
<td>Mean</td>
</tr>
<tr>
<td>BN1</td>
<td>0.5</td>
<td>0.069</td>
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</tr>
<tr>
<td>BN2</td>
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<td>0.094</td>
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</tr>
<tr>
<td>BN3</td>
<td>0.5</td>
<td>0.078</td>
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</tr>
<tr>
<td>BN4</td>
<td>0.5</td>
<td>0.061</td>
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</tr>
</tbody>
</table>

(d). Steel AP70HP

<table>
<thead>
<tr>
<th>Test</th>
<th>a/W</th>
<th>$\Delta_{\text{Test failure}}$ (measured at specimen surface)</th>
<th>$\Delta_{\text{Analysis failure}}$ (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Lower Bound</td>
<td>Mean</td>
</tr>
<tr>
<td>BN1</td>
<td>0.5</td>
<td>0.344</td>
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<tr>
<td>BN2</td>
<td>0.5</td>
<td>0.302</td>
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</table>
Table 4.10 Comparison of experimental and analytical results for fracture specimens using the VGM model

<table>
<thead>
<tr>
<th>Test</th>
<th>a/W</th>
<th>$J^\text{Test}_{IC}$ (ksi-in)</th>
<th>$J^\text{Analysis}_{IC}$ (ksi-in)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Lower Bound</td>
<td>Mean</td>
</tr>
<tr>
<td>(a). Steel AW50</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TPB 1</td>
<td>0.6</td>
<td>1.525</td>
<td>0.925</td>
</tr>
<tr>
<td>TPB 2</td>
<td>0.6</td>
<td>2.007</td>
<td>0.406</td>
</tr>
<tr>
<td>TPB 3</td>
<td>0.17</td>
<td>2.408</td>
<td>0.703</td>
</tr>
<tr>
<td>CT1</td>
<td>0.6</td>
<td>1.606</td>
<td>0.329</td>
</tr>
<tr>
<td>CT2</td>
<td>0.6</td>
<td>1.560</td>
<td>0.679</td>
</tr>
<tr>
<td>CT3</td>
<td>0.6</td>
<td>2.408</td>
<td>1.184</td>
</tr>
<tr>
<td>(b). Steel AP50</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CT1</td>
<td>0.5</td>
<td>0.55</td>
<td>0.329</td>
</tr>
<tr>
<td>CT2</td>
<td>0.5</td>
<td>0.78</td>
<td>1.184</td>
</tr>
<tr>
<td>(c). Steel AP110</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CT1</td>
<td>0.5</td>
<td>1.051</td>
<td>0.679</td>
</tr>
<tr>
<td>CT2</td>
<td>0.5</td>
<td>0.750</td>
<td>0.329</td>
</tr>
<tr>
<td>(d). Steel AP70HP</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CT1</td>
<td>0.5</td>
<td>1.180</td>
<td>1.184</td>
</tr>
<tr>
<td>CT2</td>
<td>0.5</td>
<td>0.892</td>
<td>0.329</td>
</tr>
<tr>
<td>CT3</td>
<td>0.5</td>
<td>2.403</td>
<td>1.184</td>
</tr>
<tr>
<td>(e). Steel JP50</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CT1</td>
<td>0.5</td>
<td>2.202</td>
<td>0.647</td>
</tr>
<tr>
<td>CT2</td>
<td>0.5</td>
<td>1.511</td>
<td>0.647</td>
</tr>
<tr>
<td>CT3</td>
<td>0.5</td>
<td>2.250</td>
<td>0.647</td>
</tr>
</tbody>
</table>
### (f). Steel JP50HP

<table>
<thead>
<tr>
<th>Test</th>
<th>a/W</th>
<th>$J_{IC}^{\text{Test}}$ (ksi-in)</th>
<th>$J_{IC}^{\text{Analysis}}$ (ksi-in)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Lower Bound</td>
<td>Mean</td>
</tr>
<tr>
<td>CT1</td>
<td>0.5</td>
<td>2.25</td>
<td></td>
</tr>
<tr>
<td>CT2</td>
<td>0.5</td>
<td>Invalid J-Curve</td>
<td>1.310</td>
</tr>
<tr>
<td>CT3</td>
<td>0.5</td>
<td>1.65</td>
<td></td>
</tr>
</tbody>
</table>

### (g). Steel JW50

<table>
<thead>
<tr>
<th>Test</th>
<th>a/W</th>
<th>$J_{IC}^{\text{Test}}$ (ksi-in)</th>
<th>$J_{IC}^{\text{Analysis}}$ (ksi-in)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Lower Bound</td>
<td>Mean</td>
</tr>
<tr>
<td>CT1</td>
<td>0.5</td>
<td>1.003</td>
<td></td>
</tr>
<tr>
<td>CT2</td>
<td>0.5</td>
<td>1.455</td>
<td>0.931</td>
</tr>
<tr>
<td>CT3</td>
<td>0.5</td>
<td>0.592</td>
<td></td>
</tr>
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</table>
Table 4.11 Comparison of experimental and analytical results for blunt notched specimens using the VGM model

(a). Steel AW50

<table>
<thead>
<tr>
<th>Test</th>
<th>a/W</th>
<th>$\Delta_{\text{failure}}^\text{Test}$ (inches)</th>
<th>$\Delta_{\text{failure}}^\text{Analysis}$ (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Measured at load-line</td>
<td>Lower Bound</td>
</tr>
<tr>
<td>BN1</td>
<td>0.5</td>
<td>0.338</td>
<td>0.249</td>
</tr>
<tr>
<td>BN2</td>
<td>0.5</td>
<td>0.252</td>
<td>0.274</td>
</tr>
<tr>
<td>BN3</td>
<td>0.5</td>
<td>0.271</td>
<td>0.249</td>
</tr>
<tr>
<td>BN4</td>
<td>0.5</td>
<td>0.389</td>
<td>0.315</td>
</tr>
<tr>
<td>BN5</td>
<td>0.5</td>
<td>0.370</td>
<td>0.301</td>
</tr>
<tr>
<td>BN6</td>
<td>0.5</td>
<td>0.393</td>
<td>0.348</td>
</tr>
</tbody>
</table>

Note – BN1-BN3 have thickness 1 inch, while BN4-BN6 have thickness 0.5 inch.

(b). Steel AP50

<table>
<thead>
<tr>
<th>Test</th>
<th>a/W</th>
<th>$\Delta_{\text{failure}}^\text{Test}$ (inches)</th>
<th>$\Delta_{\text{failure}}^\text{Analysis}$ (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Measured at load-line</td>
<td>Lower Bound</td>
</tr>
<tr>
<td>BN1</td>
<td>0.5</td>
<td>0.077</td>
<td>0.085</td>
</tr>
<tr>
<td>BN2</td>
<td>0.5</td>
<td>0.086</td>
<td>0.085</td>
</tr>
<tr>
<td>BN3</td>
<td>0.5</td>
<td>0.083</td>
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(c). Steel AP110

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<tr>
<th>Test</th>
<th>a/W</th>
<th>$\Delta_{\text{failure}}^\text{Test}$ (inches)</th>
<th>$\Delta_{\text{failure}}^\text{Analysis}$ (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Measured at load-line</td>
<td>Lower Bound</td>
</tr>
<tr>
<td>BN1</td>
<td>0.5</td>
<td>0.069</td>
<td>0.092</td>
</tr>
<tr>
<td>BN2</td>
<td>0.5</td>
<td>0.094</td>
<td>0.092</td>
</tr>
<tr>
<td>BN3</td>
<td>0.5</td>
<td>0.078</td>
<td>0.092</td>
</tr>
<tr>
<td>BN4</td>
<td>0.5</td>
<td>0.061</td>
<td>0.092</td>
</tr>
</tbody>
</table>

(d). Steel AP70HP

<table>
<thead>
<tr>
<th>Test</th>
<th>a/W</th>
<th>$\Delta_{\text{failure}}^\text{Test}$ (inches)</th>
<th>$\Delta_{\text{failure}}^\text{Analysis}$ (inches)</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(measured at specimen surface)</td>
<td>Lower Bound</td>
</tr>
<tr>
<td>BN1</td>
<td>0.5</td>
<td>0.3445</td>
<td>0.299</td>
</tr>
<tr>
<td>BN2</td>
<td>0.5</td>
<td>0.3022</td>
<td>0.299</td>
</tr>
</tbody>
</table>
Figure 4.1 (a) Test configuration for smooth round bars tensile test

Figure 4.1(b). Test setup for round tensile specimens
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Figure 4.2 (b) True Stress-Strain Curve for Steel AP50
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(b) Large Notch - $r^* = 0.125”$
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Figure 4.9 (b) Comparison of FEM and test data for force elongation curves for AP50
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Test Data

FEM Analysis

$r^* = 0.25''$
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Figure 4.11(b). Void growth distribution across notch cross section at failure showing the center to be the point of initiation.
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Voids
Nucleate

Inclusion Colonies or Holes formed

Fracture leaving behind a castellated surface

Figure 4.14. Proposed Mechanism of Fracture

Inclusion colonies

Figure 4.15. Longitudinal Section of Notched Bar just prior to fracture showing inclusion colonies
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Plateau indicative of inclusion

Figure 4.16 (b) AP50

Plateau
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0.03125” radius smooth machined notch

All dimensions in inches, figure not to scale
Figure 4.22. Displacement gage attachment at surface for AP70HP

All dimensions in inches, figure not to scale

0.03125” radius smooth machined notch

Displacement gage attached to surface instead of load line

MTS

W = 2

a = 1

2.5

2.4
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Figure 4.25. Bulging and Necking in blunt notched specimen creates complicated constraint situations.
Figure 4.26. Mesh for blunt notched compact tension specimen

Element size approx.
0.003 inches
Δ\text{failure} = Δ_2 \text{ because SMCS or VGM is satisfied over } l^* \text{ when } Δ = Δ_2

Figure 4.27. Method for analytically predicting the $J_{IC}$ for a material using the SMCS
Figure 4.28 (a). Load displacement curve for blunt notched compact tension specimen showing four points along the loading curve where the notch tip fields are shown in Figure 4.28 (b), (c) and (d) along with the predicted point of failure according to the SMCS criterion.
Figure 4.28 (b) Equivalent plastic strain ahead of the crack tip at the four loading points

Figure 4.28 (c). Triaxiality distribution ahead of the crack tip at four the four loading points
Figure 4.28 (d) SMCS distribution ahead of the crack tip at four the four loading points

Failure between loading points 3 and 4
Figure 4.29 (a) Strong positive correlation between Charpy V-Notch Energy and $\alpha$

\begin{align*}
\alpha &= 0.0211x - 0.9254 \\ 
R^2 &= 0.9041
\end{align*}

Figure 4.29 (b) Strong positive correlation between Charpy V-Notch Energy and $\eta$

\begin{align*}
\eta &= 0.0242x - 1.3059 \\ 
R^2 &= 0.9321
\end{align*}
Figure 4.30 (a) Comparison of test and SMCS predictions for sharp cracked fracture tests
Figure 4.30 (b) Comparison of test and VGM predictions for sharp cracked fracture tests
Figure 4.30 (c) Comparison of test and SMCS predictions for blunt notched fracture tests
Figure 4.30 (d) Comparison of test and SMCS predictions for blunt notched fracture tests
Chapter 5

Cyclic Tests and Analyses for Calibration and Validation of DSPS and CVGM models

Chapter 3 presented two models for predicting ductile crack initiation for ultra low cycle fatigue (ULCF). These are the Degraded Significant Plastic Strain (DSPS) model and the Cyclic Void Growth Model (CVGM). The DSPS model is an extension of the SMCS model, which relies on a new strain measure, known as the significant plastic strain to monitor the void growth process under cyclic loading. The CVGM model is an extension of the VGM model. The CVGM differs from the DSPS in that it involves the integration of the strain history to capture triaxiality changes, whereas the DSPS model treats the triaxiality as constant during loading. Both the models are similar in the sense that they rely on a damaged or degraded version of the critical void ratio based on the cumulative plastic strain.

Chapter 3 provides the mathematical background of each model and how the idealized stresses and strains relate to the underlying fracture. This chapter deals with the calibration of the cyclic models using notched round bars, where the flat stress and strain gradients minimize dependence on length scales. After calibrating the degradation parameter of the models ($\lambda_{DSPS}$ or $\lambda_{CVGM}$, as defined in Chapter 3), the combination of the calibrated parameters and the length scale is validated using blunt notched cyclic specimens with steeper stress-strain gradients. The chapter concludes by summarizing the performance of these models and recommending guidelines for their use, as well as discussing their limitations and some unexpected behavior that was observed in the tests.
5.1 Calibration of the DSPS and CVGM models

The DSPS and CVGM models are both calibrated using a set of notched bar tests, which are similar to the notched bar tests discussed in Chapter 4, except that here they are investigated subjected to cyclic loading. The notched bar geometry is shown in Figure 5.1. As with the monotonic tests, three notch sizes (r* = 0.06, r* = 0.125 and r* = 0.25 inches) are employed. The first two notch sizes are used for tests with all seven steels, whereas the large notch radius = 0.25 inches is used for AP50 and AP110.

We begin by discussing the calibration process for the DSPS model in detail, and then outline variations to the procedure for the CVGM. The CVGM model is different from the DSPS model in terms of some mathematical details, but much of the calibration methodology remains the same.

Table 5.1 shows a test matrix for the cyclic notched bar tests. A mix of the different notch sizes as well as loading histories is used for the test program. The aim of varying the notch sizes is to study the effect of stress state or triaxiality on ULCF, whereas the aim of the different load histories is to calibrate and validate the model for general loading histories.

Two types of loading histories are used. The first involves cycling the applied displacements between two predetermined levels until failure occurs, typically during the last tensile cycle. The second involves cycling the specimen for a finite number of cycles at lower amplitudes and then applying a tensile excursion until failure. The second kind of loading history is thought to mimic earthquake loadings, where a few smaller cycles damage the component prior to a larger cycle which causes failure. Figure 5.2 shows representative load displacement curves corresponding to each of the two different types of load histories.

The first type of history is termed Cycle to Failure (CTF), and the second is termed as Cycle and Pull to Failure (C-PTF). Figure 5.3 shows a schematic of the loading histories.
For some situations, we might combine different amplitude loadings, where the specimen is first cycled between two displacement levels, and then two other displacement levels. Table 5.1 records the failure point, which in case of the CTF history is defined by the failure cycle and for the C-PTF is defined by the failure displacement on the final pull.

As shown in Figure 5.2, the load displacement curves of these specimens show a sudden loss in slope at a certain point during the loading history. This always occurs on a tensile cycle, and as in the case of monotonic loading, this is assumed to be the point of ductile crack initiation under cyclic loading $\Delta_j^{cyclic}$. This assumption is reasonable because failure occurs almost simultaneously over the central part of cross section causing a sudden drop in load. The sharply falling load displacement curve beyond this point indicates tearing in the shear lips around the center where ductile crack initiation occurs first. The entire loading history leading up to $\Delta_j^{cyclic}$ is recorded for use in the finite element analyses.

5.1.1 Analysis Models and Results

Cyclic finite element simulations are used to analyze notched tensile specimens to calibrate the parameters of the DSPS and the CVGM models. The mesh for the FEM is identical to the monotonic analysis (see Figure 4.8) and the simulations are run on ABAQUS/CAE 6.2 on a Pentium 4 workstation.

Like the monotonic analyses, the finite element solutions employ nonlinear, large-deformation behavior, and cyclic plasticity. The cyclic plasticity model used for the simulations is based on work by Lemaitrê and Chaboche and uses a von Mises yield surface combining nonlinear isotropic and kinematic hardening. The isotropic hardening component is described by equation (5.1).

$$
\sigma^0 = \sigma_0^0 + Q_0 \left( 1 - e^{-h_0 \epsilon} \right)
$$

(5.1)
Where the current elastic range $\sigma^0$ is defined as a function of the initial elastic range $\sigma|_0$, the equivalent plastic strain $\varepsilon_p$, and two model parameters are $Q_\infty$ and $b$. The kinematic hardening component of the model is governed by equation (5.2).

$$\dot{\alpha} = C\dot{\varepsilon}_p \frac{1}{\sigma_0} (\tilde{\sigma} - \tilde{\alpha}) - \gamma \cdot \tilde{\alpha} \cdot \varepsilon_p \tag{5.2}$$

Where $\alpha$ denotes the back-stress and $C$ and $\gamma$ are material parameters. The dots on the quantities indicate incremental values, e.g. $\dot{\varepsilon}_p$ would indicate an increment in the plastic strain. This is the basic Ziegler law, to which a recall term $\gamma \cdot \tilde{\alpha} \cdot \varepsilon_p$ has been added to introduce the nonlinearity in response. Figure 5.4 shows the evolution of the yield surfaces for this model in principal stress space (refer ABAQUS Theory Manual Version 5.2). The center of the loading surface is contained within a cylinder of radius $\sqrt{2/3} \cdot C/\gamma$. This degenerates to the linear kinematic hardening model upon setting $\sigma^0 = \sigma|_0$ and $\gamma = 0$. For a more detailed discussion of these models, the reader is referred to Lemaitré and Chaboche (1990), and the ABAQUS Theory Manual, version 5.2.

The parameters for these models are calibrated based on cyclic test data from notched bar tests with different notch sizes. The $C$ and $\gamma$ can substituted in the ABAQUS model by an array of plastic stresses and strains. Figure 5.5 shows a plot of the cyclic test load displacement curve from a AP50 specimen superimposed on the FEM data. The agreement seen in this graph is representative of all the other steels. There are two important challenges to calibrating the material properties based on uniaxial tests. The first problem is that during reversed cyclic loading, the specimen is very susceptible to buckling during the compressive cycle. A shorter specimen with a 2-to-1 aspect ratio could not alleviate this problem because of alignment issues. The second issue was that necking in the specimens would destroy the uniaxiality of the stress state long before strains comparable to fracture situations were reached. This made the approach of using uniaxial specimens questionable, and we chose to use the notched bars as the calibration
tests for the cyclic plasticity model. As a result, the calibration of the models was a trial and error procedure based on achieving the best fit between the notched bar force-displacement data from the tests and the simulations. A detailed list of the parameter values used for the different steels is provided in Appendix A.

Once the constitutive or plasticity model has been calibrated, cyclic finite element analyses for the notched bars were run under the same protocol as followed in the tests. The calculated stresses and strains corresponding to the point when failure was observed in the tests, (i.e. at an applied deformation $\Delta^c$) were then interpreted through the cyclic fracture models to back-calculate the degradation parameters. Failure is always found to occur at the center of the specimen at the cross section, with very flat stress and strain gradients. As a consequence, the stress strain histories are monitored at only one location (the center of the critical cross section) over the entire load history.

For the DSPS model, the significant plastic strain (as defined in Chapter 3) is monitored leading up to failure. As discussed in Chapter 3, the significant plastic strain increases and decreases based on the sign of the triaxiality. This is different from the equivalent plastic strain, which always increases. The final significant plastic strain value (corresponding to $\Delta^c_f$) is recovered as the critical significant plastic strain $\varepsilon^*_{critical}$. This value is then compared to the mean critical equivalent plastic strain $\varepsilon_{p_critical}$ recovered from the monotonic analysis of a similar test. As discussed in Chapter 3 for the DSPS model, it is necessary to run a monotonic analysis for the exact same geometry (as the cyclic analysis) ahead of time so we can compare the critical significant plastic strain from the cyclic case to the critical equivalent plastic strain from the monotonic case. The ratio between the critical values of the significant and equivalent plastic strains from the cyclic and the monotonic tests represents the deterioration in the critical void ratio due to cyclic loading.

As discussed in chapter 3, the cumulative damage variable is defined by the cumulative equivalent plastic strain at the beginning of each tensile excursion. For the notched bars,
the tensile excursion defined in terms of the global loading coincides with the tensile excursion defined by the sign of the mean stress at the local level. Thus, equivalent plastic strain at the beginning of the failure cycle is the damage causing the failure. The plastic strain that increases during the final cycle itself is not counted, because it represents void growth and does not cause damage in the sense defined by the cyclic deterioration models.

The equivalent plastic strain at the beginning of the final or failure cycle is denoted as the damage variable \( D \), and the damage ratio (the degradation in the critical strain) is plotted against this quantity. This process is repeated for all the tests (including the different loading histories as well and notch sizes). Results of these analyses are summarized in scatter plots such as the one shown in Figure 5.7 for the AP50 steel. A least squares fit to the data returns the damage ratio as a function of the damage variable. An exponential function, of the type shown in equation (5.3) is chosen as the damage function.

\[
f_{\text{DSPS}}(D) = \exp(-\lambda_{\text{DSPS}} \cdot D) = \frac{\varepsilon_{\text{critical}}^*}{\varepsilon_p^{\text{critical}}} \quad (5.3)
\]

Figure 5.7 shows the exponential curve fit to the scatter-data. The y-intercept of the exponential curve-fit function is 1, which means that for \( D=0 \), the cyclic critical void ratio is equal to that in the monotonic case, and the DSPS model will degenerate to the SMCS model. At other values of \( D \neq 0 \), the ratio of \( \varepsilon_{\text{critical}}^* / \varepsilon_p^{\text{critical}} \) is reduced according to the data.

Such a procedure returns the \( \lambda_{\text{DSPS}} \) value for each of the steels. Table 5.2 lists the \( \lambda_{\text{DSPS}} \) values obtained for each of the steels from curve fitting to the scatter plots similar to those shown in Figure 5.7.

For the CVGM model, a similar strategy is employed, where the stress and strain histories are monitored at the middle node of the specimen until the point of failure for
Each test. The integrals on the right hand side of equation (5.4) is evaluated numerically at the point of failure –

$$
\eta_{\text{cyclic}} = \frac{\ln \left( \frac{R_{\text{critical}}}{R_0} \right)}{C} = \sum_{\text{tensile-cycles } \varepsilon_t} \int \exp \left( [1.5T] \right) d\varepsilon_t - \sum_{\text{compressive-cycles } \varepsilon_c} \int \exp \left( [1.5T] \right) d\varepsilon_c \quad (5.4)
$$

The left hand side of the equation is the cyclic demand which is to be compared to the deteriorated value of the of the VGM toughness index $\eta$. Thus, the damage ratio for each test can be estimated as the cyclic void growth capacity $\eta_{\text{cyclic}}$ calculated in equation (5.4) at the point of failure divided by the mean value of the monotonic toughness index $\eta$.

It is interesting to note that in the case of the CVGM, it is not necessary to run a monotonic analysis to obtain a damage ratio (as was required for the DSPS model), because the effects of triaxiality change on the void growth are already incorporated in the $\eta$ value. As in the case of the DSPS model, the damage ratio is plotted against the damage variable $D$, which is the equivalent plastic strain at the beginning of the final or failure tensile excursion. This damage variable is plotted against the damage ratio and repeating this procedure for the different tests results in a scatter plot similar to that obtained for the DSPS model. Figure 5.8 shows such a plot for the AP50 steel.

$$
f_{CVGM}(D) = \exp(-\lambda_{CVGM} \cdot D) = \frac{\eta_{\text{cyclic}}}{\eta} \quad (5.5)
$$

A nonlinear regression is performed to fit an exponential curve through the scatter plot providing the value of $\lambda_{CVGM}$. Table 5.2 lists the $\lambda_{CVGM}$ for each of the steels. There is a slight correlation between the $\lambda_{CVGM}$ and the $\lambda_{DSPS}$ values, which is expected because the SMCS and DSPS are different only in the sense of the approximation regarding the triaxiality during loading.
To summarize the contents of this section, cyclic finite element analyses of the notched round bars were used in conjunction with experimental data to calibrate the degradation parameter $\lambda$ of the ULCF models, i.e. the DSPS and the CVGM. This parameter represents the degradation in the critical void size for ULCF as compared to monotonic loading. The ULCF model requires three parameters to function. The first is the monotonic toughness index. This is the $\alpha$ for the SMCS, which is used by the DSPS, or the $\eta$ for the VGM, used by the CVGM. The calibration of this parameter is described in Chapter 4. In addition to this, there is the degradation parameter $\lambda_{DSPS}$ or $\lambda_{CVGM}$ whose calibration has been described in this section. Finally, the length scale parameter is required to apply the DSPS and CVGM models to situations with higher stress/strain gradients. This is discussed in the next section.

### 5.1.2 The characteristic length $l^*$ issue

As with the SMCS and the VGM models for monotonic loading, it is necessary to introduce a length scale (characteristic length) to apply the micromechanical models to conditions with steep stress and strain gradients. Recall that the notched bars are insensitive to the length scale because of the flat stress-strain gradients involved. This is why the notched bar tests are useful to calibrate the basic parameters for the fracture criterion. However, for this same reason, the notched bar results do not provide the basis for determining the characteristic length. Though the notched bars do not provide mechanical response data to determine the characteristic length, they provide samples of fracture surfaces from which microstructural measurements can be made to calculate this length. A detailed discussion of ways to relate the microstructure to the length scale is provided in Chapter 4.

An important issue to consider is whether cyclic loading affects characteristic length and if it is possible to quantify this effect. Studying and comparing micrographs from monotonic and cyclic loading shows no appreciable differences in terms of dimple spacing (though dimple diameters may be different due to different mechanisms of coalescence. Moreover, the characteristic length is related to material sampling of critical
particles, and it is difficult to argue in support of a physical connection between cyclic loading and the length scale.

Even for monotonic loading situations, the length scale measurement is one of the more imprecise and subjective calculations, because of the oddly shaped dimples, and surface plateau, as well as the non-homogeneity in the sizes of these features. As a result, the confidence level in the estimates of length scale is lower as compared to the other fracture parameters. As a result, we have not attempted an independent determination of the length scale from the cyclic tests, but will adopt the same estimates as used for monotonic loading. These lengths can be found in Table 4.7 in Chapter 4.

### 5.2 Validation Tests and Analyses of the DSPS and the CVGM Models

Having established all the three material dependent parameters for the DSPS ($\alpha, \lambda_{DSPS}, l^*$), and for the CVGM ($\eta, \lambda_{CVGM}, l^*$), we are now in a position to validate these. To test the combination of these parameters, especially with the length scale parameter included, blunt notched specimens identical to those employed for monotonic loading are used. Figure 5.9 shows a schematic of the blunt notched specimen with the notch radius of 0.03125 inches. The specimen is loaded at the holes, with the applied displacement being measured at the load-line, In the case of AP70HP steel, machining errors forced displacement instrumentation to be located at the edge of the specimen. Thus, as shown in Figure 5.10, the specimens for AP70HP were slightly different than the other steels. This requires a different interpretation of the measured displacements.

Unlike the notched bar specimens, the blunt notched specimens offer steeper stress-strain gradients, such that the effect of the length scale can be investigated. At the same time, they are more convenient than sharp-cracked ASTM type specimens, because they do not cause closure type problems in cyclic loading situations since the notch faces are separated by 0.0625 inches. This is sufficient to allow for reversed cyclic plastic loading without crack closure. Though the cyclic model calibration was performed for all the
steels, time and budgetary constraints allowed us to run validation studies for only the AW50, AP50, AP110 and AP70HP steels.

Like Table 5.1, Table 5.3 outlines the test matrix and describes the loading histories for each of the blunt notched specimen tests. Each of the tests was run under displacement control, with the displacement being monitored at the load-line, except for the AP70HP, where the displacement was monitored at the edge of the specimen. The main aim of the loading histories is to provide general and varied conditions for loading that are representative of an earthquake, and the histories used in this validation exercise appear to be sufficient for that purpose. The cyclic loading history for the specimens were generally chosen to be associated with the monotonic failure displacement of the same specimen (e.g. cycling between 33% of monotonic failure displacement and 0), however, such predetermined rules could not be used in each case for a couple of reasons. Firstly, in some cases, where the monotonic failure occurred after a relatively short period of cyclic loading, one-third of the monotonic loading displacement would cause very little or no plastic displacement, and consequently require a large number of cycles to failure. This is seen in steels such as AP50 and AP110.

Unlike the notched bar tests where crack initiation was closely followed by failure and a sudden load drop, the crack initiation in the blunt notched specimens was gradual and not associated with any apparent load drop. Therefore, the crack initiation was detected by visual inspection during a tensile cycle. The rupture occurs at the center of the notch as shown in Figure 5.11. It is possible that rupture initiates a short distance inside the material and then propagates outwards towards the surface where it is observed, but no suitable and economical techniques were available for the observation of such a sub-surface crack. Moreover our micromechanical models predict the rupture to occur at the surface. It is assumed that these two events (i.e. ductile crack initiation and visual observation), are sufficiently close, that the visually observed rupture can be used as the point of rupture initiation. The point in the loading history (i.e. the cycle number and the displacement to failure) is recorded as $\Delta_f^{\text{cyclic}}$ and is used for comparison with the
analytical predictions based on the DSPS and the CVGM models. The next describes the process of making these analytical predictions using the FEM simulations.

5.2.1 Analysis Models and Results

Figure 5.12 shows the finite element mesh used to analyze the blunt notched specimens. It is identical to the mesh used for monotonic loading in Chapter 4, except that the material properties associated with this have the combined isotropic/kinematic cyclic property associated with it to accurately capture effects due to cyclic loading. A typical analysis takes between two-four hours to run on a Pentium 4, 1.6GHz machine depending upon the number of cycles and the amplitude of the cycles. All analyses are run under displacement control. For load histories which involve cycling at given amplitude until failure is reached, a few cycles more than the experimentally observed initiation point are used in the analysis in case the failure prediction occurs at a cycle subsequent to what is observed in the test. As the analysis progresses, all the stress and strain quantities are monitored and recorded ahead of the notch tip over a distance equal to a few multiples of the characteristic length, and failure is predicted to occur when the fracture criterion is satisfied over the characteristic length. As in the case of monotonic loading, the predictions are made using the three bounds of characteristic length, resulting in three different predictions of failure. We will first describe this process for the DSPS model and then for the CVGM model.

DSPS Predictions:

For the DSPS model predictions, it is necessary to run a monotonic analysis prior to the cyclic tests to obtain the equivalent plastic strain at failure under monotonic loading. According to the monotonic SMCS criterion, the equivalent plastic strain is equal to the critical equivalent plastic strain when the SMCS reaches a value of zero. The critical condition is a function of triaxiality, which tends to be lowest at the free surface of the notch. Consequently, the critical plastic strain is higher than at a point slightly inside the
material. Figure 5.13 shows the envelope of critical monotonic plastic strain recovered from the monotonic analysis.

Once the critical monotonic plastic strain at each point is determined, the cyclically degraded values of the critical monotonic plastic strain at that point is calculated at the beginning of each tensile cycle for each sampling location using equation (5.6) –

\[ \varepsilon_{\text{critical}}^* = \exp(-\lambda_{\text{DSPS}} \varepsilon_p) \varepsilon_{\text{critical}} \]  

(5.6)

The tensile cycle for each sampling location begins at the same instant in the loading history as the global tensile cycle. This is a matter of coincidence, indicating that for these specimens there are no appreciable residual stress or non-proportional stress distributions under cyclic loading. However, this might be a specimen dependent property, and in a case where this is not observed, the procedure of applying the DSPS prediction would not change, as long as the damage variable and the other stress-strain variables are monitored independently for each sampling location. The important point is that the damage parameter at each sampling location should be based on the tension/compression excursions at that location, which may or may not correspond to excursions at other sampling locations or to the global loading protocol. For the tests run as part of this study, the tensile excursions happen to occur simultaneously for all locations of interest, which makes the counting easier.

Figure 5.14 shows the degraded critical significant plastic strain at the beginning of each tensile excursion for an AP110, Test # 2 which has 4 tensile excursions as can be seen from Table 5.3. Since the degradation is driven by the accumulated plastic strain, which is maximum at the free surface, the degradation is largest at the surface and reduces away from the notch tip. As a result, the degraded contour of the plastic strain capacity shows a change in shape, making it flatter than the undamaged envelope. Figure 5.15 plots the critical strain criterion given by equation (5.7) at four different loading instants during the failure excursion (the 4th tensile excursion in this case) over a distance ahead of the notch tip.
This quantity is the difference between the applied significant plastic strain (see chapter 3 for details) and the critical significant plastic strain, somewhat similar to the SMCS. Failure is assumed to have occurred at a point when the quantity in equation (5.7) is greater than 0. Figure 5.16 shows a load displacement curve of the specimen, indicating the points on the load history corresponding to the different instants for which the failure criterion was plotted in Figure 5.15. A prediction of ductile crack initiation is made when the expression in equation (5.7) exceeds zero over the characteristic length. In this case, Figure 5.15 (a) shows that for a mean characteristic length of 0.009 inches, failure occurs between the loading instants 2 and 3 on the final excursion. This corresponds to a displacement of 0.071 inches, which is indicated on graph along with the experimentally observed failure displacement of 0.089 inches. A similar process is employed for all the other tests, where predictions of failure are made for the lower bound, mean and upper bound of the characteristic lengths. Table 5.4 compares the analytical predictions to the experimentally observed points of failure. A detailed discussion regarding these tables is given in Section 5.3.

CVGM Predictions:

The failure prediction process using the CVGM model is similar to that for the DSPS model except for a couple of key differences. One important difference is that for the CVGM model does not require the user to run a monotonic analysis before running a cyclic analysis. Recall that for the DSPS model, one needs to know the tensile triaxiality distribution ahead of time in order to calculate the contour of critical plastic strain over the region of interest. Since the CVGM model integrates the plastic strain and triaxiality throughout the cyclic loading, the initial analysis is not necessary. Rather in this analysis, we use the VGM parameter $\eta$ which is independent of the triaxiality. While running the FEM simulation, the damage variable is calculated at each point ahead of the crack and used to degrade or diminish the critical value of the parameter $\eta$ according to the following equation –
\[ \eta_{\text{cyclic}} = \exp(-\lambda_{\text{CVGM}} \varepsilon_p) \eta_{\text{monotonic}} \]  

(5.8)

Figure 5.17 shows the degraded version of the parameter \( \eta \) at the beginning of each of the tensile excursions. These plots of degradation in the CVGM parameter are analogous to the degradation of the DSPS parameter. The key difference is that for the first excursion, i.e. the undamaged state, the \( \eta \) value is constant, in contrast to the \( \varepsilon_{\text{critical}}^* \) in Figure 5.14 which is a function of the triaxiality at a given point and changes with distance ahead of the crack tip. Again, it is observed that the damage is more severe at the crack tip than far away from it.

\[
\left( \sum_{\text{tensile-cycles}} \int \exp(1.5T) d\varepsilon_t - \sum_{\text{compressive-cycles}} \int \exp(1.5T) d\varepsilon_c \right) - \eta_{\text{cyclic}} \]  

(5.9)

Figure 5.18 plots the quantity in equation (5.9) ahead of the crack tip at different loading instants during the final excursion. Failure at a material point is assumed to occur when this quantity exceeds zero at a given point. As in the case of DSPS, ductile crack initiation is said to occur when the criterion is satisfied over the characteristic length, in this case assumed to be 0.009 inches. From Figure 5.18, we see that value of the expression in equation (5.9) exceeds zero between the loading instants 3 and 4 on the final excursion. This corresponds to a load-line displacement of 0.095 inches at crack initiation. This is shown on Figure 5.16 along with the DSPS prediction. Similar predictions of failure are made for all the steel types and the data is compared with experimental observations in Table 5.4.

5.3 Evaluation of the DSPS and the CVGM models for predicting fracture

The analytical predictions of crack initiation from the DSPS and the CVGM models are compared to the experimentally observed instants of crack initiation in Table 5.4. As in
case of the monotonic loading, these predictions are made for three values of the characteristic length.

An immediate observation is that the DSPS model predicts failure to occur sooner than the CVGM model in almost every case. This is especially pronounced in the case of the ductile steel, such as AW50, where the large deformations due and consequent changes in triaxialities invalidate the assumptions of the DSPS model.

For most of the other tests, the DSPS model does a reasonably good job of prediction failure. In almost every other case, it predicts the failure cycle accurately and the point of failure within that cycle with reasonable accuracy. But for the AW50 steel, most of the tests are included within the upper and lower bounds of the failure predictions by the DSPS model.

The DSPS model suffers from two major limitations, the first limitation is its obvious deficiency in characterizing the triaxiality change during the loading itself. The other issue, which is not so obvious, is that it uses the monotonic failure strain distribution as a starting point for calculating cyclic strain capacities. This has two potential problems. Firstly, the monotonic estimate of failure strain is again based on the instantaneous value of triaxiality at fracture, which will affect the accuracy of the prediction. The second issue is that monotonic failure often takes place at much larger displacements as compared to cyclic loading. As a result, for ductile materials, the stress triaxialities at failure in the monotonic loading might be entirely different than from that encountered during cyclic loading.

The CVGM model appears to be a much better tool for predicting failure under ULCF conditions. In almost every case (13 out of 14), the model predicts the failure cycle accurately, and in almost all the cases, the prediction of displacement at failure (based on the mean characteristic length estimate) is typically within 10-20% of the experimentally observed displacement. As expected, the difference in predictions due to the different
characteristic length choices are not large, because the stress and strain gradients are relatively shallow in the blunt-notched specimen.

The cyclic models can be expected to have more uncertainty than the monotonic models, because they carry all the uncertainty in the monotonic models along with uncertainty due (a) complexity of cyclic failure mechanisms and (b) the mathematical assumption of the exponential decay law. Another issue, which is to be discussed in section 5.4, concerns the fact that in some tests an entirely different mechanism for failure that was observed. This mechanism involves separation of material along grain boundaries. This leads to loss of force capacity on cyclic loading. This is difficult to observe in blunt notched specimens, because the crack area is small is small as compared to the ligament area.

To summarize, the DSPS model does a reasonable job of predicting ULCF in steel. However in ductile steels, it gives very conservative estimates of failure. The savings in not evaluating the numerical integral of void growth over the time history is somewhat offset by the fact that an explicit monotonic analysis is required before the cyclic analysis.

On the other hand, the CVGM model is the model of choice as far predicting ULCF is concerned, is recommended for use over the DSPS. It does not require a monotonic analysis to be run beforehand, and if the location of cracking is predetermined, the evaluation of the integral to calculate void growth is fairly straightforward and quick.

5.4 Cyclic softening and failure caused by intergranular separation

As mentioned earlier, some of the cyclic experiments exhibited fracture behavior is inconsistent that is with the void growth and coalescence mechanism assumed in the DSPS and the CVGM models. This section deals with other likely mechanisms of ULCF that have not been considered in the models presented.
Roughly 10% of the notched bar cyclic tests marked with an asterisk in Table 5.1 experienced a degrading type hysteresis response, where the strength degraded rapidly with each cycle until finally the specimen separated with complete loss in force capacity. A representative load-displacement curve corresponding to such behavior is shown in Figure 5.19. Figures 5.20 (a) and (b) show a scanning electron micrograph and a schematic of the longitudinal section of the failure surface. The micrograph and the schematic show material separation along clearly defined planes, which are indicative of grain boundaries. Our presumption is that the grain boundaries were weakened due to the cyclicity of the loading. Further, the load displacement curve in compression experiences stiffening or “pinching” behavior as shown in Figure 5.19. This is consistent with the assumption that cracks open up along grain boundaries during tensile cycles and close during compressive cycles to cause the pinching. Crack growth during the tensile cycles results in reduced force capacity at each cycle. When the cracks finally expand over the entire cross sectional area, force capacity is completely lost. In some cases, the specimen breaks suddenly before the force capacity is reduced to zero. This might be explained by the fact that the cracks opening up in the cross section trigger brittle, $K_{IC}$ type fracture within the notch area. This type of behavior is seen only in AP110, AP70HP and JP50HP steels.

It is interesting to note that for some specimens with identical loading histories and geometries, such as AP110 $r^* = 0.06$, Test #2 and Test #3 one specimen showed typical void growth and coalescence type behavior whereas the other showed the intergranular separation type mechanism. Moreover, most of these failures involving the grain boundary separation mechanism occurred in higher cycle tests (around 20 cycles) of smaller notch radiiuses which have higher triaxialities and larger principal stress components. The apparent randomness in this phenomenon is most likely affected by local material properties as well as a sensitivity to maximum principal stress components.

This type of behavior highlights four important points. Firstly, just as a single micromechanical model cannot be used for monotonic fracture, i.e. we may need to have a void growth type SMCS model and a cleavage type RKR model operating
simultaneously to predict ULCF failure. Thus, there is a need for a combination of models to capture the different possible failure mechanisms of failure. To address the need for multiple models, we first need to isolate and understand the different mechanisms independently of each other. Tests can be specifically devised to cause one type of failure versus the other so as to develop and calibrate models pertaining to particular mechanisms. Once this has been done, the statistical component of the model can be introduced based on more tests to enable operation of the two mechanisms or models in conjunction with each other. This research has focused, almost exclusively, on the void growth and coalescence mechanism and the proposed models are unique to that specific mechanism. Similar types of research are needed to model the other modes of failure identified in some of our tests.

The degradation type of fracture (Fig. 5.20) occurs in situations without sharp flaws, in some cases as early as ten cycles. This raises questions about the stable steel hysteresis loop that is heavily relied upon in order to absorb seismic energy. In other words, similar to concrete, it is likely that some degradation mechanisms are active even within the steel, and these need to be investigated in the future. The key difference is that these mechanisms are very localized as compared to concrete, and as such might not affect system performance appreciably. Since many of these failures occur at larger number of cycles (around 20), it is possible that this mechanism is a mechanism that bridges the ULCF mechanism and the high cycle fatigue mechanisms of slip and decohesion.

It is also worth noting that, none of the blunt notched specimens seemed to show this type of behavior. One reason could be the lower number of cycles in the blunt notched specimens, since they were conducted for verification rather than calibration purposes. Another explanation could be that grain boundary separation occurred in these, but because the separated area was small in comparison to the ligament cross section, its effects were not readily apparent. If the degrading fracture behavior did occur this could explain the tests where the DSPS or the CVGM models did not predict the point of failure with good accuracy.
5.5 Summary

This chapter begins by referring to chapter 3, where the DSPS and the CVGM models were introduced. The two important issues that need to be addressed are the calibration and the validation of these models. Apart from the monotonic parameters, each of the models has one additional parameter that needs to be calibrated to account for cyclic loading. This additional parameter ($\lambda_{\text{DSPS}}$ or $\lambda_{\text{CVGM}}$, as the case may be) is calibrated using cyclic tests of notched round bars with different notch sizes. A detailed procedure for such calibration is outlined, and the calibrated parameters for each of the steels are presented.

The characteristic length estimate from the monotonic loading segment of this study is assumed to apply in the cyclic tests, and is combined with the newly calibrated cyclic degradation parameter, along with other monotonic parameters to validate the DSPS and the CVGM models. The predictions incorporating this effect are validated using cyclic tests of blunt notched compact tension specimens that offer sharper stress and strain gradients as well the facility to apply reversed cyclic loading without crack closure.

The DSPS model performs averagely, and often under-predicts the life of the specimen. The CVGM model on the other hand is much more accurate, and in most cases predicts failure displacements that are within 10-15% of experimental observations. This model is the recommended model for failure prediction in structural components.

The chapter ends with a review of degrading hysteresis type behavior observed in roughly 10% of the notched bar cyclic tests. Visual observation of fracture surfaces and scanning electron micrographs show that the mode of failure in these situations is very different from the void growth and coalescence type mechanism, and apparently involving intergranular separation. This alerts us to the existence of another mechanism that may cause low cycle fatigue, and hopefully motivates further research in that direction.
Table 5.1 Test matrix of notched bar cyclic tests

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</tr>
<tr>
<td></td>
<td></td>
<td>2*</td>
<td>CTF</td>
<td>(0→0.03)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
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<td>(0→0.02)</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>4</td>
<td>CTF</td>
<td>(0→0.02)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.06</td>
<td>1*</td>
<td>CTF</td>
<td>(0→0.02)</td>
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<tr>
<td></td>
<td></td>
<td>2*</td>
<td>CTF</td>
<td>(0→0.02)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>CTF</td>
<td>(0→0.04)</td>
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<td>4</td>
<td>CTF</td>
<td>(0→0.04)</td>
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<tr>
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<tr>
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<td>2</td>
<td>CTF</td>
<td>(0→0.06)</td>
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<tr>
<td></td>
<td></td>
<td>3</td>
<td>CTF</td>
<td>(0→0.03)</td>
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<td></td>
<td>4</td>
<td>CTF</td>
<td>(0→0.03)</td>
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</tr>
<tr>
<td></td>
<td>0.06</td>
<td>1</td>
<td>CTF</td>
<td>(0→0.04)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>CTF</td>
<td>(0→0.04)</td>
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<tr>
<td></td>
<td></td>
<td>3</td>
<td>CTF</td>
<td>(0→0.02)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>CTF</td>
<td>(0→0.02)</td>
<td></td>
</tr>
</tbody>
</table>
Table 5.2 The $\lambda_{DSPS}$ and the $\lambda_{CVGM}$ values for the different steels

<table>
<thead>
<tr>
<th>Steel</th>
<th>$\lambda_{DSPS}$</th>
<th>$\lambda_{CVGM}$</th>
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<tbody>
<tr>
<td>AW50</td>
<td>0.38</td>
<td>0.11</td>
</tr>
<tr>
<td>AP50</td>
<td>0.49</td>
<td>0.32</td>
</tr>
<tr>
<td>AP110</td>
<td>0.48</td>
<td>0.37</td>
</tr>
<tr>
<td>AP70HP</td>
<td>0.43</td>
<td>0.31</td>
</tr>
<tr>
<td>JP50</td>
<td>0.85</td>
<td>0.71</td>
</tr>
<tr>
<td>JP50HP</td>
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<td>0.20</td>
</tr>
<tr>
<td>JW50</td>
<td>0.41</td>
<td>0.31</td>
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Table 5.3 Test Matrix for blunt notched cyclic tests

<table>
<thead>
<tr>
<th>Steel Type</th>
<th>Test #</th>
<th>Loading</th>
<th>Type</th>
<th>Series</th>
<th>Failure</th>
<th>Cycle</th>
<th>Failure Displacement</th>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>AW50</td>
<td>1</td>
<td>C-PTF</td>
<td>2(0↔0.184)</td>
<td>3</td>
<td>0.151</td>
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</tr>
<tr>
<td></td>
<td>2</td>
<td>C-PTF</td>
<td>1(0↔0.188)</td>
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<td>0.184</td>
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<td></td>
<td>3</td>
<td>C-PTF</td>
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<td>0.198</td>
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<tr>
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<td>4</td>
<td>C-PTF</td>
<td>3(0↔0.1)</td>
<td>4</td>
<td>0.222</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AP50</td>
<td>1</td>
<td>C-PTF</td>
<td>3(0↔0.05)</td>
<td>4</td>
<td>0.088</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>C-PTF</td>
<td>3(0↔0.05)</td>
<td>4</td>
<td>0.093</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>CTF</td>
<td>(0↔0.07)</td>
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<td>0.060</td>
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</tr>
<tr>
<td></td>
<td>4</td>
<td>CTF</td>
<td>(0↔0.07)</td>
<td>3</td>
<td>0.069</td>
<td></td>
<td></td>
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<tr>
<td>AP110</td>
<td>1</td>
<td>C-PTF</td>
<td>1(0↔0.056)</td>
<td>2</td>
<td>0.081</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>C-PTF</td>
<td>3(0↔0.056)</td>
<td>4</td>
<td>0.089</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AP70HP</td>
<td>1</td>
<td>CTF</td>
<td>(0↔0.25)</td>
<td>2</td>
<td>0.171</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>CTF</td>
<td>(0↔0.25)</td>
<td>2</td>
<td>0.173</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>C-PTF</td>
<td>3(0.025↔0.126)</td>
<td>4</td>
<td>0.221</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>C-PTF</td>
<td>3(0.025↔0.126)</td>
<td>4</td>
<td>0.146</td>
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Table 5.4 Comparison of analytical predictions to experimental observations for blunt notched compact tension tests

<table>
<thead>
<tr>
<th>Steel Type</th>
<th>Test #</th>
<th>$\Delta_f^{cyclic}$ (test)*</th>
<th>$\Delta_f^{cyclic}$ (DSPS)*</th>
<th>$\Delta_f^{cyclic}$ (CVGM)*</th>
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<td></td>
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<td>Lower Bound</td>
<td>Mean</td>
<td>Upper Bound</td>
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<td>1; 0.180</td>
<td>2; 0.073</td>
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<tr>
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<td>2</td>
<td>3; 0.184</td>
<td>1; 0.168</td>
<td>2; 0.002</td>
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<tr>
<td></td>
<td>3</td>
<td>4; 0.198</td>
<td>3; 0.071</td>
<td>3; 0.093</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>4; 0.222</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AP50</td>
<td>1</td>
<td>4; 0.088</td>
<td>4; 0.005</td>
<td>4; 0.065</td>
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<tr>
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<td>4; 0.093</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>3; 0.060</td>
<td>2; 0.050</td>
<td>3; 0.025</td>
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<tr>
<td></td>
<td>4</td>
<td>3; 0.069</td>
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<tr>
<td>AP110</td>
<td>1</td>
<td>2; 0.081</td>
<td>2; 0.072</td>
<td>2; 0.080</td>
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<td>2</td>
<td>4; 0.089</td>
<td>4; 0.006</td>
<td>4; 0.071</td>
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<tr>
<td>AP70HP</td>
<td>1*</td>
<td>2; 0.171</td>
<td>2; 0.037</td>
<td>2; 0.116</td>
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<td>2*</td>
<td>2; 0.173</td>
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<td>3*</td>
<td>4; 0.221</td>
<td>3; 0.06</td>
<td>4; 0.136</td>
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<td></td>
<td>4*</td>
<td>4; 0.146</td>
<td></td>
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</tbody>
</table>

*The numbers in the columns indicate the tensile excursion and the displacement on that excursion at which failure occurred or was predicted to occur, e.g.,

4; 0.0134

would indicate that failure occurred on the fourth tensile cycle at a displacement of 0.0134 inches

**For AP70HP, the displacement numbers reported here correspond to the edge displacement and not the load line displacement, please refer figure 5.10.
Figure 5.1. Cyclically Loaded Notched round bar geometry with circumferentially machined notch, shown here for $r^*=0.125$ inches
Figure 5.2 (a) Load history involving cycling between predetermined levels of displacement (shown here for JW50, $r^* = 0.125$, load history = H18)

Figure 5.2 (b) Load history involving cycling between predetermined levels and then pulling to failure (shown here for AP50, $r^* = 0.125$, load history = H4)
Figure 5.3 (a) $(0 \leftrightarrow 0.035)$ CTF

Cycle to failure

Time

Displacement

Figure 5.3 (b) $(0 \leftrightarrow 0.025)$ C-PTF

Pull to failure

Time

Displacement

Figure 5.4 Evolution of different yield surfaces in the $\pi$-plane for the Lemaitré-Chaboche model (ABAQUS Manual, Version 5.2)
Figure 5.5 Comparison of load displacement curves from test (AP50, \( r^* = 0.125 \)) and analysis from ABAQUS

Figure 5.6 Representative plot of significant plastic strain \( \varepsilon^* \) versus the analysis time step
\[
\frac{\varepsilon_{\text{critical}}^*}{\varepsilon_p} = \exp(-0.49 \cdot D)
\]

Figure 5.7 Scatter plot and curve fit for calibrating $\lambda_{DSPS}$ for AP50 steel

Figure 5.8 Scatter plot and curve fit for calibrating $\lambda_{CVGM}$ for AP50 steel
Figure 5.9 Smooth Notched Compact Tension Specimen geometry and Experimental Setup

Figure 5.10 Displacement gage attachment at surface for AP70HP
Figure 5.11 Fracture originating in the center of the specimen
Figure 5.12 Mesh for blunt notched cyclic specimen is identical to the mesh for the monotonic specimen.
Figure 5.13 Undamaged plastic strain capacity for AP110 blunt notches specimen

Figure 5.14 Degraded significant plastic strain capacity envelope at the beginning of each tensile cycle in AP110, D7
Figure 5.16 Load displacement graph of an AP110 Test showing the 4 points corresponding to Figure 5.15.

Figure 5.15 Plot of $\varepsilon_- - \varepsilon_{\text{critical}}$ ahead of crack tip versus distance at four different instants during the 4th tensile excursion of an AP110 test.

Figure 5.16 Load displacement graph of an AP110 Test showing the 4 points corresponding to Figure 5.15.
Figure 5.17 Damaged void ratio capacity or $\eta_{\text{cyclic}}$ envelope at the beginning of each tensile cycle for AP110

Figure 5.18 Plot of difference between the degraded void ratio capacity and the applied void ratio demand at four different instant during the 4th tensile excursion for AP110
Figure 5.19 Degradation type hysteresis loop observed in JP50HP
Figure 5.20 (a) Cross sectional view of fracture surface of AP110 specimen showing degradation type behavior, with separation of material along sharp boundaries

Figure 5.20 (b) Schematic diagram of the fracture surface for specimens showing degradation type behavior
Chapter 6

Application of Micromechanical Simulation Techniques to Structural Details

Through detailed tests, finite element analyses and microstructural studies, the VGM and SMCS model have been found to be a suitable means of predicting ductile crack initiation for a variety of steels under varying constraint conditions. The Cyclic Void Growth Model (CVGM) and the Degraded Significant plastic strain (DSPS) are the cyclic loading counterparts of the VGM and the SMCS models. The tests described in the previous chapters include standard tension tests, notched bar tests, sharp cracked and blunt notched fracture specimens, which are directed primarily towards calibrating and verifying the micromechanical model parameters. To demonstrate these models as predictive tools for simulating crack initiation in practical building design details, tests are conducted of structural details with lower stress gradients and triaxialities, representative of buildings. Earlier studies (Chi et al, 2000) have made headway in this direction by analyzing fracture in pull-plate type specimens previously tested by other researchers (Dexter and Melendrez, 1999) and post-Northridge steel moment connections (Stojadinovic et al, 1998). As the tests were not designed with the aim of using the SMCS or VGM models, detailed calibration studies using notched bar type tests and SEM micrographs were not available for the metals that were used in the component tests. As a result, findings from Chi et al’s study are more qualitative in terms of the location and nature of the fracture, rather than quantitative and predictive.

This chapter describes a series of tests to simulate fracture using the SMCS and VGM, and ULCF using the DSPS and CVGM models. These tests are run on two steels – the AP50 and the AP70HP. These steels were chosen because represent a wide range of
behavior in terms of ductility. Three basic types of specimens are designed. Two of these have bolt holes and are meant to mimic members exhibiting net section failure at bolted connections. Monotonic tests are run for these specimens. The third has a dog-bone shape which is meant to mimic the flange of a beam with the post-Northridge reduced beam section (RBS) type detail. Monotonic as well as reversed cyclic (ULCF type) tests are run on this type of specimen. The subsequent sections describe each of these tests and results in detail. Estimates of ductility from the SMCS and VGM models are compared to those from simpler critical strain measures, such as the longitudinal strain, but micromechanical models are observed to be distinctly more accurate than conventional measures.

6.1 Specimens to simulate net section failure of members of bolted connections

Net section failure is an important mechanism for failure of joints with tension members. Figure 6.1 (a) shows such a member, and equation (6.1) summarizes the check that would be used to calculate the nominal load capacity of the member,

\[ P_n = \sigma_u A_e \]  \hspace{1cm} (6.1)

where \( \sigma_u \) is the ultimate strength of the steel used and \( A_e \) is the effective area of the section - equal to the total cross sectional area minus the area of the holes, see Figure 6.1 (b). This equation assumes that the entire section will attain ultimate tensile strength before failure. However, as the member is loaded, there typically is a stress gradient leading to a stress concentration near the hole. When the stresses near the hole reach, the material starts plastically deforming and the stress concentration at the hole is changed into a strain concentration with redistribution of stresses. Gradually, more and more of the material plastifies until the entire cross section has reached its tensile strength. For this to happen, the material near the hole must have sufficient capacity for accommodating large plastic strains. Thus, the net section failure formula assumes sufficient material ductility to achieve full plastification. Moreover, the formula is force-
based and does not provide information about the ductility of the joint, so it has limited use in understanding performance in earthquake type situations.

Another question about the bolted connection behavior relates to the manner of application of load. In an actual bolted connection, the load is introduced by the bolts bearing on the surface of the holes. However, the net section assumption, which is typically applied in design makes no distinction between this situation and one where a plate with holes would be pulled at opposite ends (see Figure 6.2). This raises the question whether the stress-strain combination that drives failure is affected by the manner of application of the load.

6.1.1 Pull plate specimens with bolt-holes

The pull plate specimens shown in figure 6.3 (a) were designed to study fracture under conditions representative of the bolted end of a tension member. The instrumentation and test setup is shown in figure 6.3 (b). Referred to as the bolt-hole (BH) tests, the specimen is cut out of a 2” X 1” X 6” plate, and as can be seen from the figure, the central 3 inches of the plate are milled down to a thickness of 0.375 inch. The load is applied to the specimen at the ends by means of pins which are connected to the actuator by means of a custom fixture.

Two LVDTs (Linear Voltage Displacement Transducers) are attached to the sides of the specimen to monitor the elongation. These LVDTs are attached to the specimen using specially constructed fixtures that clamp on to the ends of the specimen. The two LVDTs enable us to capture any bending or unsymmetrical tension in the specimens. The average of the readings of the LVDTs is used for comparison to the finite element analysis. Four such tests are run (two each for AP50 and AP70HP).

Figures 6.4 (a) and (b) show the load displacement curves for these tests of the AP50 and AP70HP steels respectively. As the test begins, the load rises to reach a maximum, at which point the steel ligaments between the holes start to neck, and the load starts
dropping. This is similar to the notched round bar tensile tests. After this point, the plastic strain increases until a critical value of plastic strain is encountered, and the material fractures. The fracture typically initiates at the outside edge of the hole (see Fig. 6.5) and consequently, the ligament fractures first. This causes a significant load drop, after which further straining causes the middle ligament to fail. For purposes of this study, fracture of the edge ligament is considered the point of failure and the corresponding displacement $\Delta_{\text{failure}}$ is considered the failure displacement. Fig. 6.6 shows a picture of a failed BH specimen.

The AP50 specimens reach a maximum load of 34 kips versus a value of 32 kips calculated by the simple net section equation (6.1) (using an ultimate tensile strength of 85.3 ksi as tabulated in Chapter 4). This difference can be most likely attributed to the fact that the yield and ultimate stress of the steel are parameters obtained from uniaxial tests, and we would expect the stress triaxiality introduced by the holes to increase the force capacities somewhat.

The AP70HP steel reaches a maximum load of 41 kips – see figure 6.4 (b) as compared to a calculated value of 38 kips. Again, a logic similar to the one extended for AP50 may be applied here. Figure 6.6 shows a photograph of a failed BH specimen.

Both materials show failure displacements in the range of 0.15 to 0.20 inches. The measured displacements and the fracture analysis predictions are tabulated in Table 6.1. Finite element analyses for these tests will be presented and results for the companion tests are presented in the next section.

6.1.2 Pull plate specimens with load applied through pins at bolt-hole locations

As discussed earlier, there is a significant difference in the manner of load application between the situations shown in figures 6.1 and 6.2. In one case, the bolts bear on the inner surface of the holes, and this is equilibrated by the tensile force in the net area of the ligaments. In the other case, the load path passes directly through the net area. Typical
design formulae make no distinction between the two situations. However, it is of interest to investigate if indeed the two situations are similar especially regarding the stress-strain situations corresponding to fracture.

To compare with the BH tests, we ran companion tests, termed bolt bearing (BB) tests, that represent the loading shown in Fig. 6.1, where, instead of pulling on the ends of a single plate, the load is applied by means of pins passing through the bolt holes. The geometry and loading apparatus of the specimen is as shown in Fig. 6.7(a) and (b). Dowel pins of 0.5 inch diameter pass through the bolt holes and transfer the load between the specimen and the test fixture.

As in the BH tests, two LVDTs are attached to the specimen using a specially designed fixture that allows us to measure the displacement between the ends of the specimen. The holes are reamed to within 0.001 inch, so the fit between the holes and the dowel pins is without significant play or relative movement. The average of the LVDT readings is treated as the controlling displacement. A total of four BB tests are conducted – two for each variety of steel.

Similar to the BH tests, fracture initiation in the BB tests occurs in the edge ligament, and the failure pattern is almost identical to that in BH. The load displacement curves look very similar (in nature and in magnitude) to that in BH, as shown in figures 6.8 (a) and (b) for the AP50 and the AP70HP steels. The AP50 specimens reach a maximum strength of approximately 35 kips, which is slightly higher than the 32 kips calculated using equation (6.1), and the elongation to failure is in the range 0.15-0.20 inches which is comparable with the elongation seen for the BH tests. The AP70HP specimens reach a maximum load of around 41 kips (as compared to a calculated strength of 38 kips), and a maximum elongation of around 0.15-0.20 inches.

Looking at the load-displacement graphs Figs 6.4 and 6.8 for the BH and the BB tests – the maximum load, displacement and ductility are almost identical. However, one can question if this is a mere coincidence, or is the physics of the fracture and necking
process unaffected by the difference in configuration. If this were to be a coincidence, experimental data could not be transferred accurately to other configurations, so it is important to compare the evolution of the stress and strain fields that drive the fracture process under the two situations. For this purpose, we ran detailed finite element analyses of the experimental test specimens which are described in the next section.

6.1.3 Finite element models of BH and BB tests to predict failure

Three dimensional finite element models are constructed for both the geometries, i.e., the BH and the BB to capture complicated behavior such as out of plane necking. The models are built in ABAQUS/CAE 6.2 and use large deformation theory and isotropic incremental plasticity that are calibrated based upon material tests described in Chapter 2, and upon procedures described in Chapter 4.

Finite Element Simulation of the BH Configuration

The BH configuration has three planes of symmetry, so it is sufficient to construct a FEM model of one-eighth of the complete specimen. Figure 6.9 (a) shows the FEM model and figure 6.9 (b) shows the deformed model with the equivalent plastic strain contours. The boundary conditions are fixed according to the requirements of symmetry, and the model is loaded in displacement control at the face specified. The displacement at the locations where the LVDT is attached to the test specimen is compared to the measured test data. The finite element mesh has just under 1000 hexahedral elements, and the size of smallest elements (in the regions of fracture is of the order of 0.03 inches – roughly a tenth of the ligament width). This element size is seemingly large, as compared to the characteristic length for the materials, but after employing much finer mesh densities, it was found that the stress and strain gradients in this type of specimen are very flat (even more so than in the notched round bars), allowing us to coarsen the mesh substantially. This mesh greatly speeds up analysis time, especially for simulation of cyclic tests. The results from the coarser mesh have been compared with those from the finer mesh (elements of the order of 0.002 inches), and have been found to be virtually identical. The
force displacement curve predicted by the finite element analysis (see Fig. 6.11) illustrates the good agreement between experimental and analytical predictions of response. Moreover, due to the flat stress-strain contours, the specimens are insensitive to the length scale effect, and as a result the uncertainty in the measurement of the characteristic length does not adversely affect the estimated ductility of these specimens.

To examine fracture initiation in the test specimens, we monitor the stress and strain contours over the entire region of the critical section of the specimen. The critical SMCS and the VGM criteria are both found first occur at the location of fracture initiation observed in the tests (Fig. 6.5), and the stress strain profiles are found to be flat enough in the vicinity such that for a very small increment in global displacement, the SMCS and the VGM is suddenly satisfied over a very large area, many times the dimension of the characteristic length $l^*$. As a result of this, monitoring the stress and strain variables is necessary only at one location (rim of the hole near the outer edge of the specimen). The displacement is plotted against the SMCS value. In 6.10 (a), and the displacement corresponding to the point where the SMCS reaches a value of zero is calculated as the analytical prediction of the failure displacement $\Delta_{\text{failure}}^\text{Analysis}$. A similar process is carried out for the VGM, and the void growth quantity expressed by the equation (6.2) is plotted against the displacement in Figure 6.10 (b). When this quantity exceeds zero, failure is assumed to occur.

$$VGM = \int_0^{\varepsilon_p^{\text{critical}}} \exp(-1.5 \frac{\sigma_m}{\sigma_e}) d\varepsilon_p - \eta$$  (6.2)

Referring to figures 6.10 (a) and (b), the steel with the higher $\alpha$ and $\eta$ value AP70HP ($\alpha = 2.9$, $\eta = 3.2$) has a higher slightly higher $\Delta_{\text{failure}}^\text{Analysis}$ as compared to the less ductile AP50 ($\alpha = 1.2$, $\eta = 1.1$). These analytical predictions are compared to the measured failure displacements in Table 6.1 for both the steels. Figure 6.11 shows the analytical load-displacement curve overlaid on the experimental load displacement curve for AP50 (for the purposes of illustration), and it is striking to see the agreement not only between the points of failure, but also between the load displacement curve from the tests and analyses.
Finite Element Simulation of the BB Configuration

The BB configuration has only two axes of symmetry versus the three axes the BH configuration, so a larger model needs to be built for this case. A one-quarter model with about 3500 hexahedral and tetrahedral elements is used to analyze the BB configuration. The analysis requires about 30 minutes to run on a Pentium 4, 1.6GHz computer processor. Figures 6.12 (a) and (b) show the FEM model and the deformed FEM model with the equivalent plastic strain contours. As in the BH models, the stress/strain contours are very flat, and as such the elements are not required to be refined more than 0.03 inches.

As in the BH analyses, it is sufficient to monitor the stress and strain at only one point, and Figures 6.13 (a) and (b) are plotted for the BB specimens exactly as Figures 6.10 (a) and (b) are plotted for the BH, showing the predictions of failure based on SMCS and VGM models for both the steels. When the appropriate failure criterion is satisfied at the critical location, the corresponding elongation is determined as $\Delta_{\text{Analysis}}$. The results, along with the experimental observations can be found in Table 6.1. A detailed commentary on the trends in agreement between the experiments and analyses is presented in a subsequent section.

Two other issues will be discussed before continuing to the next section, i.e. the dog-bone type connection. One important issue is the comparison between the BH and the BB connections. It is important to determine if the similarities in the fracture displacements as well as the maximum load reached is a coincidence, or is it actually driven by the physics of the problem. The other issue deals with a more common practice for determining the ductility of such connection, i.e., using the longitudinal strain measure as an estimator for fracture. Using the eight tests described above, the effectiveness of this approach will be discussed and contrasted with the micromechanical model approach.

Figure 6.14 compares the graphs of SMCS versus the elongation for both the configurations. It is interesting to note that for both the steels, the graphs of SMCS versus
Specimen elongation are virtually coincident. This indicates that the local stress-strain situation at the critical ligament location is independent of the manner of load introduction, therefore substantiating the use of BH type tests or analyses to understand BB type behavior. It is also interesting to note that while the net section formula can predict the force capacity quite well, it does not provide an indication of the ductility capacity.

Table 6.1 Comparison of Test data and experimental predictions for BB and BH tests

<table>
<thead>
<tr>
<th>Steel</th>
<th>Test</th>
<th>$\Delta_{\text{Test failure}}$ (inches)</th>
<th>$\Delta_{\text{Analysis failure SMCS}}$ (inches)</th>
<th>$\Delta_{\text{Analysis failure VGM}}$ (inches)</th>
<th>$\varepsilon_{\text{failure}}^1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AP50</td>
<td>BH1</td>
<td>0.140</td>
<td>0.138</td>
<td>0.129</td>
<td>0.73</td>
</tr>
<tr>
<td>AP50</td>
<td>BH2</td>
<td>0.136</td>
<td></td>
<td></td>
<td>0.66</td>
</tr>
<tr>
<td>AP50</td>
<td>BB1</td>
<td>0.153</td>
<td>0.139</td>
<td>0.138</td>
<td>0.79</td>
</tr>
<tr>
<td>AP50</td>
<td>BB2</td>
<td>0.145</td>
<td></td>
<td></td>
<td>0.75</td>
</tr>
<tr>
<td>AP70HP</td>
<td>BH1</td>
<td>0.144</td>
<td>0.184</td>
<td>0.205</td>
<td>1.03</td>
</tr>
<tr>
<td>AP70HP</td>
<td>BH2</td>
<td>0.154</td>
<td></td>
<td></td>
<td>1.15</td>
</tr>
<tr>
<td>AP70HP</td>
<td>BB1</td>
<td>0.158</td>
<td>0.184</td>
<td>0.227</td>
<td>1.18</td>
</tr>
<tr>
<td>AP70HP</td>
<td>BB2</td>
<td>0.164</td>
<td></td>
<td></td>
<td>1.22</td>
</tr>
</tbody>
</table>

Table 6.1 compares the experimental observations of failure to the analytical predictions using the SMCS and the VGM model. An average error of about 10% is obtained for the models, which indicates good overall agreement between the experimental and analytical values. This illustrates the capability of the micromechanical models to capture fracture processes in realistic situations in the absence of sharp cracks and flaws.

However, a closer inspection of the data reveals many interesting facts. First, there is a somewhat consistent bias in the sense that the predicted values are higher (as compared to the measured values) for the AP70HP, as opposed to the AP50. There are two possible explanations for this bias. First, the SMCS model is based on the instantaneous values of
the stress and strain fields, whereas the void growth and coalescence process is in fact based on the evolutions of stress and strain histories. The larger geometry changes for the more ductile AP70HP can invalidate the assumptions of the SMCS model. However, this argument can be countered by observing the VGM predictions, which show even a stronger bias as compared to the SMCS. Thus, there can be other reasons that cause this bias. One possibility is the occurrence of cleavage fracture in the specimens (as evidenced by pockets of cleavage in the fractured ligaments). The AP70HP has a much higher ultimate tensile strength (100 ksi), as opposed to AP50 (85 ksi), which can elevate the overall level of stress in the material leading to cleavage. Obviously, the SMCS or the VGM will not accurately capture such behavior because of the different mechanism that it is directed towards. Moreover, uncertainty in the measured parameters themselves, coupled with consistent, unverifiable biases in the computer models versus the constructed experiments can be another reason for this difference.

Questions may be raised as to why simple strain measures, such as the longitudinal strain cannot be used as a fracture criterion for situations such as this. The last column of Table 6.1 lists the plastic fracture strains in the longitudinal direction at the point of fracture (obtained from the experiment). We see that for each steel, they are somewhat consistent. However, it must also be realized that because the tests presented here are almost identical, the stress situations at the location of fracture are similar as well. Since the critical plastic strain is assumed to be a function of triaxiality, the similarity in the plastic strain values seen in Table 6.1 is not evidence of the usability of the criterion, because the triaxialities in all the tests are virtually identical. These values are retained here for comparison with other situations where the triaxialities might be appreciably different. In any case, the critical plastic strain for such an application might be obtained from the uniaxial tensile tests of the smooth round bars, and these critical logarithmic strains are obtained as 0.81 for the AP50 and 1.35 for the AP70HP. The values for the AP70HP connection (1.03 to 1.22) are somewhat similar to those for the tension bar (1.35). For the AP50 steel, the tension bar fails at a value of 0.81, whereas the connection tests show values between (0.66-0.79). The differences between the micromechanical models and the gross measures of strain are not so pronounced in the BH and BB tests, likely because
the stress conditions in the BH and BB tests are similar to those in the tension bars. The differences become more significant in the RBS tests that are described in the subsequent sections.

6.2 Specimens to simulate failure of Reduced Beam Section (RBS) type connections

During the Northridge earthquake, welded connections in steel moment frames were found to be a weak link in what were otherwise thought to be very ductile structural systems. After Northridge, numerous studies were conducted with recommendations to improve connection performance during seismic events. One of the key problems was the complicated and often severe stress situation in the weld connection area coupled with heterogeneity of materials and possible loss of ductility in the heat affected zone (HAZ). As shown in Fig. 6.15, the RBS or dog-bone type connection detail was developed to concentrate the plastic hinge a certain distance away from the connection within the beam itself.

Though such connections have the potential to prevent sudden and brittle failures such as those in welded connections, there is always the possibility of ductile fracture under large plastic strains. These fracture strains often control ductility of the connection and often of the system. However, the current state of analysis and practice in civil engineering does not provide tools other than experimentation to determine the ductility of such connections, especially under reversed cyclic loads. It was thought that using the RBS connection as a test case for the new micromechanical models would provide not only a demonstration of the abilities of the models, but also a compelling case for their application to practical design.

The RBS tests were conducted on AP50 and AP70HP steels. Two monotonic and two cyclic tests were conducted on each steel type, bringing the total number of tests to eight. The basic geometry of the specimen is shown in figure 6.16 (a) and (b). The overall geometry of the specimen is similar to that of the BH specimen, but instead of bold holes, the central part of the plate is narrowed down to a width of 1 inch for the AP50 and 0.9
inch for the AP70HP (due to machining errors) by means of two circular cuts on either side. Due to some machining issues, the radius of the cuts could not be maintained for both the steel types, and as a result, the radius of the cut for AP50 is 0.75 inch – Figure 6.16 (a), and that for AP70HP is 0.5 inch – figure 6.16 (b). This difference is accounted for in the FEM analyses.

As shown in Fig. 6.17, the instrumentation and the fixtures used are identical to those used in the BH tests. Two LVDTs are used, and the average displacement is used as the controlling displacement. Figure 6.18 shows a photo of the instrumented RBS test (shown here for AP70HP) in the 50 kip MTS test frame.

6.2.1 Monotonic Tests of RBS specimens

Two monotonic RBS tests were conducted for each steel type. Figures 6.19 (a) and (b) show the load displacement curves obtained for AP50 and AP70HP steel respectively. Similar to the case of the BH and BB tests, the load displacement curve rises to a maximum value, and then the cross section necks down until fracture finally occurs.

The maximum force capacity reached by the AP50 steel was approximately 35 kips, which is slightly higher than 32 kips predicted by equation(6.1). The specimens from AP70HP reach capacities of around 37 and 34 kips (the difference is likely due to machining errors), which are approximately equal to the 34 kips predicted by the net section equation.

The fracture propagates quickly suggesting that the ductile initiation takes place over a large area and then all of a sudden the material fails by a mixture of tearing and ductile mechanisms, this is also evident from the fracture surfaces. The fracture surfaces have the appearance of ductile tearing dimples, smooth shear lips, as well as shiny cleavage facets. As a result, it is not clear which of these mechanisms governed failure. The fracture elongations are in the range of 0.20-0.25 for both the steel types. The results for these tests, along with the finite element predictions of failure are presented in the next section.
6.2.2 Finite Element Simulation of the RBS Tests

Similar to the BH model, the RBS model has three planes of symmetry so we construct a one-eighth FEM model as shown in Figure 6.20 (a) and (b) (for AP50). The mesh has about 1500 hexahedral elements with the minimum element size of the order of 0.05 inches. The analysis requires about 20 minutes running time. The element size is large because the SMCS contours are very flat in the region surrounding the fracture location as in the BH and the BB configurations.

Both the SMCS and the VGM criteria are first found to be satisfied at the center of the specimen – see figure 6.20 (b) and as in the BH and BB specimens, it is satisfied over a very large area (as compared to the \( l^* \)) around the center. Figures 6.21 (a) and (b) show plots of the specimen elongation versus the SMCS and VGM for both steels. It is important to keep in mind that the geometries of the both the steel specimens are slightly different, and so the curves might not exactly conform to intuition because the material ductility of AP70HP is offset by the more severe stresses due to the smaller radius of the cut. This causes a much stronger strain concentration with severely localized strains in the AP70HP, i.e. the smaller global deformations actually correspond to large local strains. This reduces the ductility of the AP70HP specimens to the extent that it is in the range of the AP50. Table 6.2 compares the values of the predicted values of failure elongation to the experimentally observed values, as well as the values of the longitudinal plastic strain at failure.

<table>
<thead>
<tr>
<th>Steel</th>
<th>Test</th>
<th>( \Delta_{\text{Test \ failure}} ) (inches)</th>
<th>( \Delta_{\text{Analysis \ failure}} ) SMCS (inches)</th>
<th>( \Delta_{\text{Analysis \ failure}} ) VGM</th>
<th>( \varepsilon_{\text{failure}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>AP50</td>
<td>RBS1</td>
<td>0.247</td>
<td>0.259</td>
<td>0.256</td>
<td>0.45</td>
</tr>
<tr>
<td>AP50</td>
<td>RBS2</td>
<td>0.248</td>
<td></td>
<td></td>
<td>0.45</td>
</tr>
<tr>
<td>AP70HP</td>
<td>RBS1</td>
<td>0.220</td>
<td>0.238</td>
<td>0.267</td>
<td>0.71</td>
</tr>
<tr>
<td>AP70HP</td>
<td>RBS2</td>
<td>0.206</td>
<td></td>
<td></td>
<td>0.63</td>
</tr>
</tbody>
</table>
Looking at Table 6.2, we see that for most cases, experimentally observed elongation at fracture is reasonably close to the analytically predicted values. The SMCS and the VGM predictions for both the steels are very similar. The high triaxiality ratios and correspondingly smaller plastic strains in the RBS specimens ensure that fracture occurs at relatively small levels of deformation. As a result, there is no significant difference between the SMCS and the VGM predictions.

For the AP70HP steel specimens, both the models slightly over-predict failure displacement. Apart from uncertainty in the material parameters, this might be attributed to the fact that fracture surfaces for both these tests showed a combination of cleavage and ductile fracture, so it is very likely that cleavage fracture initiated in these before ductile tearing, due to elevated stresses. As a result, application of the SMCS and the VGM models is not a very clean issue under such circumstances. However, despite these discrepancies, the SMCS and VGM still predict failure within about 20-30% of the measured. Even in the case failure actually was triggered by cleavage, the micromechanical models can work through some degree of correlation with the actual stress and strain state, even if the mechanism is not accurately captured. This, and the observations from Table 6.2 along with those from table 6.1 encourage the use of the SMCS and VGM models to predict failure in “flaw-free” structural details similar to the BH, BB and the RBS. The ductility calculated by the micromechanical models can be related to rotational ductility of the joints and ultimately to the ductility of the system.

6.3 Cyclic Tests of RBS specimens

Four reversed cyclic tests are conducted on the RBS specimens (two for each steel – AP50 and AP70HP) to examine the capabilities of the two cyclic micromechanical models (DSPS and CVGM). The simple cyclic load history shown in Fig. 6.22 was employed for this purpose. The load history involves two reversed cycles between displacements of zero and a tensile elongation $\Delta_0$, and then a final tensile pull until failure. The displacement $\Delta_0$ is controlled so as not to exceed the necking displacement of the monotonic tests, to avoid unstable necking or localization in the cyclic test which
may lead to unpredictable forms of bucking that might not be reliably modeled. The displacement is determined to be $\Delta_0 = 0.1$ inch for the AP50 and $\Delta_0 = 0.06$ inches for the AP70HP. The fixtures and instrumentation are identical to those for the monotonic tests, as shown in figure 6.17. The controlling displacement is the average LVDT displacement. Load displacement curves for both the steels (one test shown for each steel type) are shown in figure 6.23 (a) and (b). The finite element predictions of the load displacement behavior are overlaid on the experimental plots to illustrate the modeling accuracy. The failure displacement for the final excursion or pull is recorded and compared with the finite element analysis predictions in each case. See Table 6.3.

The AP50 specimens have a failure displacement on the final cycle in the range of 0.18-0.19 inch, which is about 70% of the monotonic failure displacement. The AP70HP, on the other hand shows a much smaller amount of degradation in terms of displacement with respect to the monotonic tests, i.e., the monotonic failure displacement is in the region 0.20-0.22 inches, and the cyclic failure displacement is in the same region as well. This could be attributed to a couple of phenomena. The damage function has variability in its estimate of damage, and in this case, the damage was small, and as a result, the variability overwhelmed the damage trend. Another explanation could be the variability in the machining of the specimens, which were found to have a larger tolerance (0.05 inches) due to some issues with the milling machines used. These results will be presented in detail and compared to the FEM predictions after a brief introduction to the finite element model.

6.3.1 Finite Element Simulation of the Cyclic RBS Tests

The finite element model is identical to the one-eighth model used for the monotonic RBS tests. A more complicated, full-model was analyzed as part of the planning phase of the tests to simulate inelastic buckling. The major difference between monotonic and cyclic loading is the material model used. The cyclic model used is a combined isotropic-kinematic model by Lemaitré and Chaboche (1990), as referenced in Chapter 5. Since the
stress and strain contours are extremely flat in relation to the characteristic dimension, the stress-strain histories are recorded only at one point at the center of the specimen.

The methodology for predicting failure is similar to that used in Chapter 5 for the blunt notched cyclic fracture specimens, except that the critical condition is monitored only at one point. At the beginning of each “tensile” cycle as defined in Chapter 3 and 5 for the DSPS model, predictions are made for the significant plastic strain capacity for the next tensile excursion. This is a degraded version of the equivalent monotonic plastic strain capacity:

\[ \varepsilon_{\text{critical}}^* = \varepsilon_{p}^{\text{critical}} \cdot \exp(-\lambda_{\text{DSPS}}) \varepsilon_{p} \]  

(6.3)

Where \( \varepsilon_{p}^{\text{critical}} \) is the equivalent plastic strain capacity under monotonic loading, \( \varepsilon_{p} \) is the cumulative equivalent plastic strain at the beginning of the tensile cycle, and \( \lambda \) is the degradation parameter for the particular steel. The point where this critical value of the significant plastic strain is exceeded by the actual significant plastic strain is the point of failure initiation, thus defining the corresponding displacement as the failure displacement. Table 6.3 records the values of the failure displacements for each of the four tests conducted, and summarizes the values predicted by the DSPS model.

A similar procedure is followed for the CVGM model, where the void growth is tracked as an integral shown in equation (6.4) and failure is predicted to occur when the right hand side of the equation exceeds the left hand side, which is the critical value of the CVGM function. A more detailed description of this process is available in Chapter 5.

\[ \exp(-\lambda_{\text{CVGM}}) \eta_{\text{monotonic}} = \sum_{\varepsilon_{t}} \exp([1.5T]) \int_{\varepsilon_{t}} d\varepsilon_{t} - \sum_{\varepsilon_{c}} \exp([1.5T]) \int_{\varepsilon_{c}} d\varepsilon_{c} \]  

(6.4)

The displacement corresponding to this event is recorded as the failure displacement, and the DSPS and the CVGM predictions of failure are compared with experimental values in Table 6.3.
Table 6.3 Comparison of Test data and experimental predictions for cyclic RBS tests

<table>
<thead>
<tr>
<th>Steel</th>
<th>Test</th>
<th>∆\text{failure}^{\text{Test}} (inches)</th>
<th>∆\text{failure}^{\text{Analysis}}_{\text{DSPS}} (inches)</th>
<th>∆\text{failure}^{\text{Analysis}}_{\text{CVGM}} (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AP50</td>
<td>RBS1</td>
<td>0.189</td>
<td>0.179</td>
<td>0.192</td>
</tr>
<tr>
<td>AP50</td>
<td>RBS2</td>
<td>0.188</td>
<td>0.178</td>
<td>0.182</td>
</tr>
<tr>
<td>AP70HP</td>
<td>RBS1</td>
<td>0.218</td>
<td>0.178</td>
<td>0.182</td>
</tr>
<tr>
<td>AP70HP</td>
<td>RBS2</td>
<td>0.206</td>
<td>0.178</td>
<td>0.182</td>
</tr>
</tbody>
</table>

Looking at table 6.3, we observe that the measured values of failure displacement are within reasonable limits of the predicted values, especially for the AP50 tests. The agreement is not as good for the AP70HP, but the predictions are within 15% of experimental observations. The intriguing aspect of the AP70HP is that the cyclic tests show negligible degradation as compared to the monotonic tests (whether measured in terms of elongation capacity or strain capacity), and this could be related to the variability in the model (which is assumed to be exponential decay, but might not capture the exact physics of the degradation process), or it could be related to variability in machine tolerances on the AP70HP specimens. Another reason for the variability might be the discrepancy between the real stresses and the calculated stresses which are a function of the cyclic plasticity material model used. Moreover, in the AP70HP, the elevated stresses (due to higher ultimate strength) are likely to trigger other mechanisms besides void growth and coalescence causing larger scatter.

Overall, the DSPS and the CVGM model appear to have a very good correlation with experimental data and Table 6.3 makes a strong case for the use of these micromechanical models for failure prediction due to ULCF in structures under seismic loads.
6.4 Summary

This chapter discusses the use of the SMCS, VGM, DSPS and the CVGM models to predict failure in test specimens similar to structural details under monotonic and reversed cyclic (ULCF type) loading. For monotonic loading, three different types of tests are conducted; they are the BH, BB and the RBS.

The BH and the BB tests aim to simulate failures at bolted connections, and are distinguished by the manner of application of load. The BB specimens are more realistic, i.e. the load is applied by means of pins (intended to simulate bolts) bearing on the insides of the bolt holes. The BH, on the other hand introduces load in the specimen by pulling on the ends of the specimen.

A total of eight tests are run for the BH and BB specimens (four for each steel type – AP50 and AP70HP). Failure initiates in the outer ligament (as shown in figure 6.5), and then proceeds to the inner ligament. This happens significantly after the uniaxial tensile strength has been reached in all three ligaments, which justifies the use of the net section to calculate the force capacity. It is observed that for the structural connection type configurations, the SMCS as well as the VGM are excellent tools to predict ductile crack initiation and they can predict failure with an error less than 10% in terms of the failure displacement. However, a bias is observed in the predictions, in the sense that the predicted values for the AP70HP steel are consistently somewhat higher than the measured values. This is attributed to mixed-mechanism fracture, as well as large changes in triaxiality, and machining tolerances (several machining issues were encountered for the AP70HP).

The stress-strain profiles and histories for the two configurations are compared for both the steels, and it is found that the evolution of stress and strain at the critical location and its relation to global elongation, is independent of the configuration (BH or BB),
therefore it is justifiable to transfer test results from one configuration to determine strength and ductility of another.

Often the longitudinal strain is used as a parameter to predict fracture in flaw-free situations. The critical value of this strain can be inferred from uniaxial tests. Tables 6.1 and 6.2 tabulate the values of longitudinal strain in the test specimens at the point of failure. It is seen that the values of the critical value of plastic strain are inconsistent within each steel type, suggesting that the critical value of plastic strain is a function not only of the material, but also of the stress state, which may be influenced by hardening and necking behavior. The values predicted by the longitudinal strain are within 10-20% for the BB and BH configurations (which have relatively uniaxial stress states), as opposed to the RBS specimens that are more highly constrained, and have disagreements of the order of 100%. The micromechanical models explicitly take the stress state into account to provide more accurate predictions of failure. As discussed earlier in Chapter 2, uniaxial fracture strains often conceal information about hardening and triaxiality situations and might not follow trends similar to the $\alpha$ or the $\eta$ parameters and as a result they are less reliable in predicting fracture in steels.

For the cyclic loading, we apply similar load histories to both the steels, and see that in either case, the predictions of failure are with 10-15%. It is interesting to see that in the case of AP70HP, there is negligible degradation in the elongation capacity with respect to the monotonic situation. This, along with the increased variability can be attributed to a number of factors. Firstly, the cyclic models (DSPS and CVGM) have an additional parameter as compared to the monotonic models (SMCS and VGM), and makes assumptions in addition to the SMCS model, that introduces variability. Secondly, the cyclic models are dependent on the cyclic plasticity model which is much more complicated to calibrate as compared to the monotonic model, thus there is actually the possibility of introducing a discrepancy between the determined stresses and the actual stresses in the material. Finally, the effect of geometric imperfections and misalignments cannot be ruled out.
To summarize, the micromechanical models are found to be extremely useful tools to predict fracture under monotonic and cyclic loads in structural components. Many structural component situations are flaw free and have flat stress-strain gradients, which obviates the need for a detailed calibration of the length scale parameter for these models. Consequently, all the calibration for these models can be done using relatively inexpensive notched bar tests, as opposed to the costlier J-integral type tests. Despite being cheaper to calibrate, these models go beyond the currently used J and $\Delta J$ models for ductile fracture and ULCF in the sense that they actually capture the micromechanics that drives the phenomenon. Moreover, they are applicable to situations such as the RBS and bolted connections that are flaw-free and ill-suited for the J or $\Delta J$ models. This chapter demonstrates the abilities of these models to predict failure under various situations of relevance to structural and earthquake engineering, and we hope that their use will become widespread in the future.
Figure 6.1. Bolted end of tension member showing transfer of forces through bolt bearing into the cross section

\[ P_n = \sigma_u A_e \]

Shaded regions represent the effective area

Transfer of force through bolt bearing

(a) Overview of member and loading

(b) Transfer of forces through the cross section
Figure 6.2. Member pulled at both ends, load path travels directly through the member.

Figure 6.3 (a). Overall geometry of the BH specimens.
Figure 6.3 (b). Instrumentation and test setup for BH specimens
Figure 6.4 (a) Load-displacement curves for BH tests on AP50

Figure 6.4 (b) Load-displacement curves for BH tests on AP70HP
Figure 6.5. Fracture initiation at edge ligament near the hole

Figure 6.6 Fractured BH specimen from AP50
Figure 6.7(a)  Basic geometry of the BB specimens
Figure 6.7 (b) Test setup for BB specimens
Figure 6.8 (a) Load-displacement curves for BB tests on AP50

Figure 6.8 (b) Load-displacement curves for BB tests on AP70HP
Figure 6.9 (a) One-eighth Finite Element Model for BH specimens

Displacement monitored at location of LVDT attachment in experimental specimen

Face loaded in displacement control

Minimum element dimension 0.03 inches
Figure 6.9 (b) Deformed FEM Mesh for BH specimen showing equivalent plastic strain contours
Figure 6.10 (a) SMCS versus specimen elongation for both steels (BH), showing the predicted value of failure displacement $\Delta_{\text{Analysis}}$.

Figure 6.10 (b) VGM versus specimen elongation for both steels (BH), showing the predicted value of failure displacement $\Delta_{\text{Analysis}}$. 

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Figure 6.11 Prediction of failure in BH tests using SMCS criterion (AP50)
Displacement monitored over the reduced width of the specimen

Face loaded in displacement control

Bottom half of the hole constrained in the longitudinal direction

Minimum element size 0.03 inches

Figure 6.12 (a) One-quarter Finite Element Model for BH specimens
Figure 6.12 (b) Deformed FEM Mesh for BH specimen showing equivalent plastic strain contours
Figure 6.13 (a) SMCS versus specimen elongation for both steels (BB), showing the predicted value of failure displacement $\Delta_{\text{Analysis}}^{\text{failure}}$.

Figure 6.13 (b) VGM versus specimen elongation for both steels (BB), showing the predicted value of failure displacement $\Delta_{\text{Analysis}}^{\text{failure}}$. 

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Figure 6.14 SMCS versus elongation for both steels and specimen configuration showing similarity between BH and BB

Figure 6.15 Reduced Beam Section (RBS) to concentrate plastic hinge at a distance from welded connection
Figure 6.16 (a) Basic geometry of RBS test for AP50

Figure 6.16 (b) Basic geometry of RBS test for AP70HP
Figure 6.17 Instrumentation and test setup for BH specimens
Figure 6.18 Photo of RBS specimen for AP70HP steel
Figure 6.19 (a) Load-displacement curve for RBS Test on AP50

Tests 1 & 2 have nearly coinciding load displacement curves.

Figure 6.19 (b) Load-displacement curve for RBS Test on AP70HP

Test 1

Test 2

$\Delta_{\text{Test failure}}$
Figure 6.20 (a) One-eighth finite element model for RBS tests
Figure 6.20 (b) Equivalent plastic strain contours for the RBS finite element model
Figure 6.21 (a) Specimen elongation for both steels (RBS), showing the predicted value of failure displacement $\Delta_{\text{Analysis failure}}$.

Figure 6.21 (b) VGM versus specimen elongation for both steels (RBS), showing the predicted value of failure displacement $\Delta_{\text{Analysis failure}}$. 
Figure 6.22 Loading History for Cyclic RBS Tests

Figure 6.23 (a) Experimental and Analytical Load-displacement curves for Cyclic RBS test for AP50 Steel
Figure 6.23 (b) Experimental and Analytical Load-displacement curves for Cyclic RBS test for AP70HP Steel
Chapter 7

Conclusions, Recommendations and Future Work

This study was motivated by the limitations of traditional fracture mechanics to predict earthquake-induced fracture in steel-framed structures, which are characterized by large-scale cyclic yielding. Investigations prior to this study, conducted through the SAC joint venture e.g. (Chi et al, 1997) established that traditional fracture mechanics is a suitable tool to simulate brittle fractures such as those that occurred during Northridge. However, the fracture resistance of post-Northridge type connections with smaller flaws and tougher materials, which are susceptible to ductile fractures, is not reliably predicted by traditional fracture mechanics. Previous studies, largely by specialists in materials science and computational fracture mechanics, explored micromechanical models as alternatives to traditional fracture mechanics for ductile fracture initiation. The promise shown by these approaches coupled with the lack of data on mild steels used in civil engineering structures led to an exploratory effort by Chi et al (2000) to apply these models to structural engineering situations. Some important issues remained unsolved at the end of that investigation, which are addressed herein. Firstly, prior studies of the micromechanical models emphasized monotonic loading, and as such provide only limited insight into ultra-low cycle fatigue (ULCF) or Stage III fatigue mechanisms. Being important to earthquakes, this type of behavior needed to be investigated further. Another issue relates to the fact that most of the monotonic studies on micromechanical models had been conducted on materials used for pressure vessels or, shipbuilding. Thus, there was a desire to investigate their application their applicability to common structural steels. Finally, the micromechanical models had never been specifically applied to situations or details resembling structural connections in welded, steel-framed buildings. This study is intended to bridge these gaps by verifying the existing micromechanical
models for monotonic loading, developing models for ULCF, and applying them to structural engineering situations.

This chapter summarizes the key findings of this study and makes recommendations for future work. The chapter is divided into two sections. Section 7.1 summarizes the work done during this investigation, including key findings and recommendations for use of the models. Section 7.2 discusses some of the questions raised by this study, and future efforts/studies required to deal with these.

7.1 Summary and Conclusions

Seven varieties of steels were investigated as part of this study. The steels were all low carbon structural steels of types commonly used in building and bridge construction. The steels included four varieties obtained from steel fabricators in the United States and three varieties produced by Nippon Steel Corporation of Japan. The steels ranged from 50 ksi (340 MPa) to 110 ksi (760 MPa) yield strength. Two high performance bridge steels – HPS70W used in the US, and SM490YBTMC-5L used in Japan – were also investigated. Five types of tests, each with its own objective, were conducted on the steels. These include monotonic and cyclic notched round bar tests, blunt notched compact tension specimens, ASTM fracture mechanics tests and component tests representing conditions in bolted connections. RBS connection type tests and bolted connection type tests. More than two-hundred tests were conducted as part of this study. Detailed complementary finite element analyses, employing large deformations and cyclic multiaxial plasticity models, were used for simulating the stress and stress conditions for the test specimens.

Key tasks that have been conducted in this study include (1) calibration of monotonic model parameters for the seven steel types including correlation to microstructural features and modification to existing approaches to improve calibration (2) verification and evaluation of existing micromechanical models for structural steels, and comparison to traditional fracture mechanics approaches (3) postulating of mechanisms and ULCF models for ultra low cycle fatigue based on tests, micrographs and physical intuition (4)
calibration and verification of the proposed ULCF models for structural steels (5) application of both the monotonic and cyclic micromechanical models to experiments resembling structural components to illustrate the use of such models under realistic situations.

7.1.1 Calibration of the monotonic models and modification of existing approaches

The monotonic models were calibrated using circumferentially notched round bars with two or three notch sizes for the different steels. The material parameters $\alpha$ (for the SMCS model) and $\eta$ (for the VGM model) were calibrated for all the steels. A slight correlation between low-carbon content and fracture toughness was observed, e.g. the more ductile steels – (AW50, AP7HP and JP50HP), have lower carbon contents (<0.09%) than the other steels. Another interesting observation was that smaller grain sizes correlated to higher toughness, e.g. the grain size (0.007mm) in the high-performance, high-ductility steels as opposed to a grain size of (0.019mm) in all the other steels. Thus, it is reasonable to expect that small grain size and low carbon content enhances ductility. The material toughness parameters $\alpha$ and $\eta$ were statistically robust (typically less than 10% COV), and are tabulated for use.

The micromechanical models are both based on two material-dependent properties. One of these is the toughness parameter $\alpha$ (for the SMCS model) and $\eta$ (for the VGM). The second parameter defines the length scale (or characteristic length) which defines adequate sampling of the microstructural features, such as inclusions required for fracture. An accurate determination of the characteristic length is a more complicated issue than determination of the toughness parameters. Existing approaches for determining the length scale appear to be suitable only for their particular study, and cannot be generalized conveniently. A unified approach is presented in this study with borrowed aspects of the previously proposed approaches. The proposed method is based on three bounds of the characteristic length, calculated using a combination of dimple spacing and measurements of the castellated fracture surface. Sensitivity of the fracture process to length scale is an important issue, and as a result the different bounds on
length scale can result in substantially different predictions of fracture in situations with sharp stress or strain gradients.

7.1.2 Task 1 – Verification and evaluation of existing monotonic models for structural steels

The micromechanical models are found to provide a suitable alternative to traditional fracture mechanics methods to predict fracture in structural steel materials in the presence of large scale yielding, because (1) they are based on stress and strain states and not limited by assumptions of small scale yielding (2) they can handle three-dimensional constraint situations accurately (3) they do not require an infinitely sharp initial flaw for definition. The verification tests are compact tension and three-point bend tests with blunt and sharp notches, which to offer variable stress and strain gradients. Some general conclusions that can be drawn from this task are –

1. The monotonic micromechanical models, i.e. the SMCS and the VGM models, predict fracture of most specimens tested within reasonable bounds of uncertainty (determined by the bounds on characteristic length), under different stress conditions (triaxialities).
2. The toughness indices calibrated for the monotonic models show strong correlation with the upper-shelf Charpy energy value. A linear model to relate the SMCS/VGM models to the CVN values is proposed, which has an $R^2$ value of approximately 0.9.
3. The SMCS model is more convenient to apply as compared to the VGM model, because it does not require numerical integration of the stress and strain histories. However, in situations with variable triaxiality during loading, the SMCS cannot capture the changing stress state as well as the VGM leading to often inflated values of the predicted ductility. The problem is more pronounced in high-ductility steels where large deformations precede fracture. As a result the VGM model is recommended as the model of choice, though for less ductile steels, or high constraint situations with smaller deformations, the SMCS is fairly accurate.
4. The accuracy of the micromechanical models in predicting fracture for steep stress/strain gradients is highly dependent on the characteristic length which is based on the relevant microstructural feature. Measurement of the characteristic length is a subjective issue, and as a result there can be considerable scatter in ductility predictions of situations such as sharp cracked situations with sharp stress gradients which are very sensitive to length scale effects. For this reason, it is difficult to correlate accurately micromechanical toughness indices to conventional fracture parameters such as the CTOD or J.

5. Micromechanisms other than void growth and coalescence can contribute to fracture. Some of the tests conducted showed cleavage fracture, which cannot be captured by the micromechanical models, which are geared towards the void growth and coalescence mechanisms. This is a general issue with the micromechanical models because they are mechanism specific, unlike other traditional fracture mechanics indices which do not differentiate between mechanisms and typically are a combined index. As a result, while the micromechanical models can be very precise, they may not be accurate when many mechanisms interact to cause failure.

7.1.3 Mechanisms and models for ULCF

Reviewing the literature available on fatigue, it is observed that few advances have been made in the direction of predicting Stage III fatigue or ULCF under very large strain ranges. Based on tests, finite element analysis, micrographs and intuitive reasoning, a new mechanism and simulation model are proposed for ULCF.

The mechanism proposes that voids grow and shrink due to excursions of plastic strain depending upon the state of triaxiality. The voids grow under positive mean stresses, and shrink under negative or compressive mean stresses. Ductile crack initiation is assumed to occur when the void ratio reaches a critical value, which is a degraded fraction of the critical value for fracture under monotonic loading. This degradation in the critical void size is assumed to be caused by a number of contributing mechanisms –
1. Void shape change causing sharpening of void curvatures and consequently raising stresses in the inter-void ligament, thereby causing cleavage fracture between the voids.

2. Secondary void nucleation due to cumulative strains between the voids.

3. Larger strains between voids causing lower localized tangent modulus, leading to instabilities being reached faster, than under monotonic loading conditions.

Two new models are developed in this study to capture this micromechanism. The first one (called the Degraded Significant Plastic Strain model – DSPS) is based on a new strain measure developed in this study called the significant plastic strain. The significant plastic strain overcomes the inability of the equivalent plastic strain to capture void growth and coalescence by partitioning the equivalent plastic strain based on the sign of triaxiality. The second model (called the Cyclic Void Growth Model – CVGM) is a more rigorous model based on integrating the stress-strain histories over the entire loading protocol. Both the models use the cumulative plastic strain at the beginning of each tensile cycle (or excursion) as the damage variable to model the damage, and an exponential function to model the damage phenomenon. Equations (3.19) and (3.28) – referenced here again from Chapter 3 describe the degradation:

\[
\varepsilon_{\text{critical}}^* = \exp(-\lambda_{\text{DSPS}} \varepsilon_p) \cdot \varepsilon_{\text{critical}}
\]  \hspace{1cm} (3.19)

\[
\eta_{\text{cyclic}} = \exp(-\lambda_{\text{CVGM}} \varepsilon_p) \cdot \eta_{\text{monotonic}}
\]  \hspace{1cm} (3.28)

7.1.4 Calibration and Verification of the ULCF Models

Equations (3.19) and (3.28) show that in addition to the toughness indices, and the characteristic length (which is assumed to be identical to the one used for monotonic loading), the ULCF models have one additional material parameter (\(\lambda\)) that needs to be calibrated. The calibration tests are conducted using the circumferentially notched round bars. A simple testing methodology is developed for the calibration process which involves cycling the material to prescribed elongation levels which are a fraction of the monotonic failure displacement. The detailed calibration process is described in Chapter
5 and it results in the $\lambda$ parameter for the appropriate model. The observations that can be made from this data are –

1. The void growth and coalescence mechanism captured by either the significant plastic strain (DSPS) or the explicit integration method (CVGM) appears to be a major contributor to ULCF.

2. The equivalent plastic strain at the beginning of each of the tensile excursions is a combined damage index including effects of different damage parameters.

3. The data fit is somewhat better for the CVGM than for the DSPS models. This can be attributed to the triaxiality-tracking attributes of the CVGM over the DSPS.

4. The degradation model fit is not good for the AP110 steel (with a low $R^2$ value of 0.3). A possible reason for this is that different micromechanisms (intergranular separation) are contributing to failure in this material. Thus alternative models are required to capture different mechanisms.

5. Excepting one or two outliers, there appears to be a correlation between the monotonic toughness index and the rate of degradation, i.e. tougher materials do not show degradation as rapid as the materials with less toughness in the monotonic situation.

The verification of the models is based on blunt notched compact tension specimens that offer sharper stress-strain gradients as compared to the round bars, and also provide the ability to apply reversed cyclic loading, which is somewhat of an issue in sharp cracked geometries. As detailed in Chapter 5, the predictions are made based on the DSPS and the CVGM models, and compared to the analyses. A number of conclusions can be drawn from the test data –

1. The CVGM model predicts fracture much more accurately than the DSPS in most cases. This can be attributed to two main reasons –
   a. The DSPS relies on the significant plastic strain, which assumes that the triaxiality does not change over the course of cyclic loading, as a result, the demand void ratio or size might be different than the actual void size.
b. The DSPS model requires the user to run a preliminary monotonic analysis ahead of time. The critical plastic strain is thus based on the monotonic triaxiality at failure. As a result, if the monotonic triaxiality at failure is different from the triaxiality during cyclic loading (even if it does not change too much during the cycling), appreciable errors will be introduced in the predictions of failure. It is important to note that calibrations for both models are somewhat dependent on the cyclic calibration protocol used, but more assumptions regarding the stress-state are involved in the DSPS model.

2. For situations with constant triaxiality or lower ductility, the significant plastic strain appears to be a suitable measure to capture failure due to ULCF, while for more ductile materials the CVGM is a better model choice.

3. In almost every case, the CVGM predicts the failure cycle accurately, and the failure displacement is predicted to be within 10-20% of the actually observed displacement.

4. The CVGM is the model of choice for ULCF because of two reasons (1) it captures triaxiality changes accurately and (2) it does not require the pre-run of the monotonic analysis. However, for large structures, where the location of fracture is not predetermined, one can visualize the large computational effort associated with generating and storing all the integrals associated with the CVGM. The DSPS, or some future improved version of it, is recommended such cases.

7.1.5 Application of the micromechanical models to structural details

Chapter 6, which applies the micromechanical models to the specimens resembling structural components, offers insight into the applicability of the models, and some behavioral issues. Different specimens, resembling those with bolt holes, and a reduced beam sections (or a dog-bone) are tested monotonically as well as cyclically to evaluate the use of the micromechanical models. The key findings are –
1. The micromechanical models predict the failure displacements within a 10-15% error margin for monotonic as well as cyclic loading situations.

2. The micromechanical models predict the location of fracture accurately for each case. In case of the bolt hole tests, crack initiation was predicted to occur at in the outermost ligament at the edge of the hole, as outlined in Chapter 6. For the RBS type specimen, the crack initiation was predicted to occur at the center of the reduced section.

3. An interesting issue explored is if there is significant difference in behavior in two different types of bolted connections, which are distinguished by the manner of load transfer – one with bolt holes in plate being loaded at the ends, and another where the load is introduced by bolt bearing (a more realistic case). It is observed that in both the cases, the micromechanical models predict failure at the exact same location and the same ductility. This is encouraging, since current provisions make no distinctions between the two situations.

4. Structural components without flaws typically have shallow stress-strain gradients compared to the microstructural dimensions. Due to this, one does not need to refine the finite element mesh down to the microstructural level even for the micromechanical models. As a result, the micromechanical models can be applied without significant computational effort to many structures.

5. Simple and inexpensive calibration tests such as circumferentially notched bar tests can substitute the more expensive fracture mechanics tests for predicting failure in structures without sharp cracks under monotonic and cyclic loadings.

7.2 Recommendations for future work

This study has created many exciting opportunities for the simulation of complex fracture processes in structures, and it has raised many questions, which could be addressed in future studies. Some of these issues are very specific, and some are larger issues dealing with the position of micromechanical models in larger hierarchy of performance based engineering. The specific issues are –
1. For the monotonic as well as the cyclic models, the characteristic length parameter measurement is still somewhat of a subjective issue. A more thorough understanding of how the microstructural features trigger fracture is required, to enable more reliable methods to predict characteristic length.

2. For the cyclic models, the intergranular separation mechanism has been observed, but it has not yet been studied in detail. This is an important mechanism, especially since it contradicts the common perception about the stable hysteresis loop of steel.

3. For cases governed by the void growth and coalescence mechanism a number of improvements can be suggested for the DSPS and the CVGM models:

   a. The void growth and shrinkage rates are defined by the same equation with respect to triaxiality. An improvement, based on more calibration data would be to assign different weights to the growth and shrinkage processes.

   b. Making more detailed micromechanical observations and determining if (i) the cycling has any effect on the characteristic length – in our study we used the characteristic length dimension based on the monotonic tests, however it is likely that different micromechanisms contributing to cyclic loading might be sensitive to other characteristic length scales, and (ii) there are other mechanisms that can cause damage, e.g. the intergranular separation mechanism needs more attention, and complementary model development.

   c. A small pilot study (not reported in this dissertation) was carried out where an array of voids was modeled using FEM in a continuum to study the effects of load histories. Time constraints prevented a more detailed study. It would be very useful to conduct further such investigations studying the effects of different parameters on cyclic void growth. The qualitative inferences from such a study can be used to build more approximate and quantitative models.
Apart from the specific issues outlined above, the study raises some larger questions regarding the use of micromechanical models, and fracture and fatigue simulation in structures in general. These are –

1. If indeed it is feasible to apply these micromechanical models at a larger scale, e.g. in large-scale beam column connections under cyclic loading. Large scale modeling of specimens often requires transition from smaller to larger elements to capture local (microstructure-level) as well as global stress effects. As a result, large-scale modeling, especially with cyclic loading, can be computationally very demanding. Advances in computational technology suggest that it would be worthwhile to explore the feasibility of applying such models at the larger scale. The various options available may include parallel or cluster computing, advanced crack propagation algorithms or substructuring methods.

2. Ductile crack initiation is often just the beginning of failure, and failure can be driven by its transition to brittle propagation. This process is very statistical, based on random sampling of weak particles. It is important to study under what conditions this will occur in structures. An important challenge in this direction is the re-meshing, crack propagation algorithms. Newer meshless methods are a possible tool for exploring these issues.

3. All the tests in this study, and most of the tests conducted in earthquake engineering are quasi-static. However, earthquake loading rates can be substantially different from the quasi-static case. Moreover, the large inelastic deformations during cyclic earthquake loading can increase temperatures in the metals enhancing the toughness. These are exciting new research directions.

Finally, an important observation is that much of the existing research in fracture mechanics has been based on non-mechanism specific considerations (as described at length in Chapter 2). Fundamental models, which capture specific mechanisms appear to be very encouraging, especially with today’s computing technologies. However, these models are difficult to calibrate because experiments sample reality, which might consist of the interactions of various mechanisms. An interesting concept is that of the “master-curve” that aims to predict material fracture properties based exclusively on the material microstructure and constituent chemistry (McCabe et al, 1998). The toughness can be
modeled based on microstructure, and the scatter in properties can be attributed to the random distribution of fracture causing particles or features. In the future, one can envision tests that target specific mechanisms, such as cleavage, void growth, or intergranular separation, and calibrate micromechanical models for each of these. Further studies can model the interactions between these mechanisms, and finally comprehensive models can be built which realistically model the material to predict performance with confidence for different loading conditions, loading rates and temperatures. With the help of detailed FEM, hopefully the calibration tests will be simple and inexpensive, but provide the strong basis for a reliable model based simulation environment.
Bibliography


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Appendix A

Material model parameters for monotonic and cyclic constitutive models for steel

A1 Monotonic model parameters
As mentioned in Chapter 4, for the monotonic simulations, a von-Mises yield surface with isotropic hardening was used. The hardening was described in terms of a stress-strain envelope in ABAQUS under the *PLASTIC option. The material parameters for the different steel varieties are given below (directly copied from an ABAQUS *.inp file).

1. AW50

```
*ELASTIC, TYPE=ISO
  29000.,    0.3
**
*PLASTIC
  5.0000000e+01,   0
  5.0000000e+01,   8.2758621e-03
  6.0500000e+01,   2.2913793e-02
  6.9410000e+01,   4.7606552e-02
  7.6100000e+01,   7.3758621e-02
  7.9500000e+01,   9.7258621e-02
  8.2190000e+01,   1.2216586e-01
  8.4790000e+01,   1.4708724e-01
  8.6300000e+01,   1.7202414e-01
  8.8280000e+01,   1.9695586e-01
  9.5120000e+01,   2.9672000e-01
  1.0120000e+02,   3.9651034e-01
  1.0800000e+02,   4.9627586e-01
  1.1402000e+02,   5.9606828e-01
  1.2012000e+02,   6.9585793e-01
  1.2683000e+02,   7.9562655e-01
  1.3250000e+02,   8.9543103e-01
  1.3780000e+02,   9.9524828e-01
  1.4390000e+02,   1.0950379e+00
  1.5000000e+02,   1.1948276e+00
  5.0000000e+02,   8.5627586e+00
```
2. AP50

*Elastic
29000., 0.3
*Plastic
50., 0.
65., 0.02
71., 0.036
84., 0.053
90., 0.078
94., 0.103
154., 0.813

3. AP110

*Elastic
29000., 0.3
*Plastic
11.0000000e+01, 0
11.4000000e+01, 3.8360000e-03
11.9000000e+01, 5.4560000e-03
12.0000000e+01, 1.2818000e-02
12.2000000e+01, 3.9281000e-02
18.0000000e+01, 8.0000000e-01
28.0000000e+01, 2.0

4. AP70HP

*Elastic
29000., 0.3
*Plastic
80.0, 0
81.5, 0.0031
92, 0.026
96.7, 0.0337
104., 0.056
108., 0.0891
110., 0.103
191., 1.386

**
5. JP50

*Elastic
29000., 0.3
*Plastic
  50.000, 0
  56.872, 0.0244
  69.360, 0.0483
  76.950, 0.0735
  83.780, 0.1109
  90.475, 0.1738
  145.200, 1.2690
  200.200, 2.269
**

6. JP50HP

*Elastic
29000., 0.3
*Plastic
  60.000, 0
  64.600, 0.02556
  68.000, 0.0329
  72.300, 0.04434
  76.000, 0.05653
  78.400, 0.06657
  81.400, 0.0891
  84.400, 0.1156
  86.200, 0.1401
  190.200, 1.67
**

7. JW50

*Plastic
  50.000, 0
  53.33, 0.01428
  58.564, 0.0242
  65.4738, 0.04101
  70.970, 0.0597
  75.660, 0.09011
  140.989, 1.468
**
A2 Cyclic model parameters

As explained in Chapter 5, the Lemaitre-Chaboche model was used for simulating the cyclic plasticity in the steels. This includes isotropic as well as kinematic hardening parameters. The kinematic hardening is prescribed in terms of an envelope (table of values), and the isotropic hardening is described by the parameters $Q_\infty$ and $b$. This is included in ABAQUS by using the *CYCLIC HARDENING option. The values are given below – the $Q_\infty$ and $b$ are indicated for AW50.

1. AW50

*Elastic
29000., 0.3
*Plastic,hardening=combined
  50., 0.
  50., 0.004
  50., 0.00827586
  60.5, 0.0229138
  69.41, 0.0476066
  76.1, 0.0723759
  79.5, 0.0972586
  82.19, 0.122166
  84.47, 0.147087
  86.3, 0.172024
  88.28, 0.196956
  95.12, 0.29672
  101.2, 0.39651
  108., 0.496276
  114.02, 0.596068
  120.12, 0.695858
  126.83, 0.795627
  132.5, 0.895431
  137.8, 0.995248
  143.9, 1.09504
  150., 1.19483
  500., 8.56276
*Cyclic Hardening,parameters
  50., 17., 5.

\[ Q_\infty \quad b \]
2. **AP50**

*Elastic
29000., 0.3
*Plastic, hardening=combined
  50., 0.0
  85.4, 0.0356
  88.4, 0.08079
  91.4, 0.106
  93.1, 0.1364
  93.6, 0.1416
  120.0, 1.33
*Cyclic Hardening, parameters
  50., 17., 5.

3. **AP110**

*Plastic, hardening=combined
  110., 0.0
  115.4, 0.0356
  118.4, 0.08079
  121.4, 0.106
  133.1, 0.1364
  136.6, 0.1416
  137.0, 1.33
  139.0, 2.33
*Cyclic Hardening, parameters
  110., -5., 5.

4. **AP70HP**

*Elastic
29000., 0.3
*Plastic, hardening=combined
  80.0, 0.0000
  95.0, 0.0350
  96.0, 0.0356
  102.4, 0.054
  104.4, 0.06
  107.1, 0.115
  110.6, 0.138
  115.6, 0.2
  125.0, 1.386
*Cyclic Hardening, parameters
  80., 22., 5.
5. **JP50**

*Elastic  
29000., 0.3  
*Plastic, hardening=combined  
  45., 0.0  
  79.4, 0.0356  
  80.4, 0.08079  
  82.4, 0.106  
  85.1, 0.1364  
  86.6, 0.1416  
  89.0, 1.33  
  132.0, 2.33  
  134.0, 3.33  
*Cyclic Hardening, parameters  
  45., 25., 5.

6. **JP50HP**

*Elastic  
29000., 0.3  
*Plastic, hardening=combined  
  50., 0.0  
  65.4, 0.02  
  71.4, 0.023  
  75.4, 0.04  
  80.1, 0.069  
  88.9, 0.237  
  97.0, 0.306  
  100.0, 0.659  
  118.0, 2.33  
  128.0, 3.00  
  140.0, 3.33  
  140.0, 4.00  
  140.0, 4.50  
*Cyclic Hardening, parameters  
  50., 35., 5.

7. **JW50**

*Elastic  
29000., 0.3  
*Plastic, hardening=combined  
  45., 0.0  
  68.4, 0.02  
  74.4, 0.043  
  78.4, 0.066  
  83.1, 0.106  
  92.9, 0.37  
  100.0, 0.60
104.0, 1.33
114.0, 2.33
132.0, 3.33
*Cyclic Hardening, parameters
  45., 35., 5.