ACCOUNTING RULES FOR DEBT COVENANTS

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I certify that I have read this dissertation and that, in my opinion, it is fully adequate in scope and quality as a dissertation for the degree of Doctor of Philosophy.

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ABSTRACT

This thesis examines the properties of accounting rules designed to maximize the efficiency of accounting-based debt covenants in a setting with incomplete contracts and asset substitution. Accounting takes the role of a state-contingent decision facilitator by inducing covenant violations, and hence a transfer of decision rights from the borrowing firm to the lender, whenever the lender has less detrimental decision incentives than the firm. This representation of accounting is distinctive in two respects. First, accounting does not have to enlarge the contracting parties’ information sets but is nonetheless valuable because it provides verifiable information and hence improves contract efficiency. Second, accounting is an endogenously chosen information aggregation rule rather than an information signal with exogenous properties. The optimal accounting rule in this setting is a function of cash flows and fair value estimates and implements a debt contract with a unique optimal tradeoff between decision rights and interest rates. The optimal degree of conservatism of this accounting rule, measured as the tendency to yield low accounting measures and hence more frequent covenant violations, is higher for firms with high leverage, low profitability, and high liquidation values. Firm value is a convex function of financial leverage in this setting.
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1. **Introduction**

The objective of this thesis is the derivation and characterization of accounting rules that maximize the efficiency of debt contracts, defined as the extent to which the contract maximizes firm value. In practice, debt contracts frequently contain covenants that impose restrictions, duties, or sanctions on the borrower under specified contingencies. These contingencies are usually stated in terms of accounting variables such as book value of equity, leverage, or net income. For example, the firm may be required to keep its financial leverage ratio below a certain value or maintain a minimum level of working liquidity. The properties of the accounting rules underlying the computation of these accounting variables thus control when borrowers are or are not in compliance with the covenant’s requirements.

Covenant violations, also referred to as ‘technical default,’ have economic consequences that affect firm value by inducing changes to the borrowing firm’s operating or financing decisions. For example, the lender may receive the right to call the debt and thus potentially force the firm into liquidation by insisting on immediate payment of the borrowed amount, or, in exchange for waiving its right to do so, compel the firm to curtail dividends, undertake or forgo certain investments, or pay a higher rate of interest. Covenants can also specify remedies directly, e.g., an automatic requirement for additional collateral or an increase in interest rates upon violation. The design of the accounting rules underlying the covenant therefore has effects on real economic outcomes.

Both covenant violations and the absence of covenant violations can potentially lead to inefficient decision outcomes. In debt contracts, the asymmetric distribution of payoffs between borrower and lender induces the well-known asset substitution problem, in which the borrower has a preference for (possibly inefficiently) high risk while the lender has a preference for (possibly inefficiently) low risk. A covenant designed to maximize firm value induces decision outcomes that are least affected by these possibly detrimental incentives. For example, a covenant violation that permits the lender to call the debt should only occur in circumstances when the lender’s incentive to do so would maximize firm value. The accounting variable underlying the covenant acts as a mediator in the
agency conflict by summarizing verifiable information about these circumstances and thereby implementing a state-contingent resolution of decision problems.

Two preliminary remarks should be made. First, it is instructive to contrast this role of accounting in the debt contracting problem with the approach taken in existing research. Accounting in this thesis serves the role of a decision facilitator by aggregating information in a deliberate, strategic manner, but accounting does not by itself generate new information. In contrast, some prior theoretical research on accounting and debt contracts, e.g., Gigler, Kanodia, Sapra & Venugopalan (2009), Göx & Wagenhofer (2010), and Li (2013), has interpreted accounting primarily as an information signal rather than as an aggregation rule. Other models either consider one-dimensional contractible information, in which case the functional form of the decision rule is very simple (Sridhar & Magee, 1997), or prescribe specific functional forms of the accounting rule ex ante (Caskey & Hughes, 2012).

Second, the asset substitution problem underlying the latent contractual inefficiency in this setting is specific to the payoff structure of the debt contract. Permitting a deviation from this payoff structure would eliminate it. Hence, the model cannot explain the optimality of debt financing relative to other forms of capital and takes the contracting parties’ payoff functions as exogenously given. Prior research has identified a large number of settings in which debt is preferable over other forms of financing, e.g., information asymmetry in capital markets, the need for costly state verification, signaling to outside investors, or the existence of private benefits. While not modeled explicitly, these settings provide a motivation for assuming debt contracts to be desirable in practice. Section 2.2 reviews related prior research in more detail.

The model employed in this thesis investigates how the optimal contract terms and the underlying accounting rule are determined and what their properties are. The model addresses the following questions. i) How do the decision incentives of borrower and lender relate to the first-best decision rule that maximizes total firm value? How are these incentives affected by information about past and future outcomes? ii) What is the functional form of the accounting rule that implements the optimal debt covenant? Is it unique? iii) How does the optimal accounting rule vary with the borrower’s leverage, the
profitability of the underlying investment, the opportunity cost of decision alternatives, and the variance of payoffs? iv) How does project selection affect the optimal contract?

Addressing these questions yields several noteworthy insights. First, the optimal debt contract features a covenant that delegates decision rights to the contracting parties rather than prescribes actions directly. Second, the optimal debt contract is unique, which implies that the optimal tradeoff between contract terms, including interest rates and covenant restrictiveness, is not an arbitrary one. The accounting variable that underlies the optimal covenant is increasing in past cash flows and decreasing in the firm’s estimated liquidation value. Third, firm value is a convex function of the firm’s financial leverage and of its profitability. Fourth, accounting rules designed to optimize debt contracts are more conservative, in the sense that they induce more frequent covenant violations, when the borrowing firm has higher leverage, lower profitability, a high liquidation value, or discretion in selecting its investments. Debt financing becomes more costly if the firm has high operating risk.

The thesis is organized as follows. Section 2 provides a discussion of the research questions, the motivation for debt financing, and the roles of risk, information, incentives, and accounting in the debt contracting problem, in the context of extant research on these topics. Sections 3 through 5 develop the main results of the thesis, including the properties of the optimal debt contract and the accounting rule the covenant is based on. Sections 6 introduces the option for the firm to choose among projects with different risk and return parameters after the contract has been signed. Section 7 provides a summary and some concluding comments.

2. Conceptual Framework, Scope and Related Research

The purpose of this section is to describe the critical aspects of the research question examined in this thesis in the context of extant research. The first subsection defines and motivates the scope of the research question. The second subsection provides a discussion of the roles of risk, incentives, and information in the context of the debt contract problem. The subject of the third subsection is the role of accounting in debt covenants.
The fourth subsection discusses the implications of contractual incompleteness and renegotiation. The final subsection examines the role of accounting in debt contracts.

2.1. Scope of the Research Question

A financing contract must address two broad questions. First, what payoff does the investor receive in which states of the world? Second, what rights does the investor have in influencing decisions that affect the enterprise financed by the investment? A comprehensive security design problem would attempt to answer both questions simultaneously. In contrast, this thesis is confined to studying a standard debt contract, defined here as an agreement between a borrower and a lender under which the lender receives all payoffs generated by the borrower’s enterprise up to a fixed threshold value (the maturity value of the debt), unconditional on the state of the world and the decision history that produced the payoffs. The independence of the borrower’s payment obligation from the state of the world is critical. All results and conclusions drawn in the following analysis are primarily a consequence of this non-state-contingency of the debt payment. The reason for choosing this type of financing arrangement is taken as exogenous. Possible motivations for assuming debt financing are discussed in Section 2.2.

The objective of this thesis is to answer the second question: what decision rights does the investor, i.e., the lender, have, or, more broadly, how are decisions arising over the course of the contract period resolved? As Section 2.3 demonstrates, debt financing generates conflicting risk preferences not found in other forms of financing, which can lead to inefficient decisions. This inefficiency, referred to as the agency cost of debt, necessitates a contractually specified resolution. Consistent with the terminology used in practice, these contractual resolutions will be referred to as covenants. For the purpose of the analysis in this text, a covenant is broadly defined as a contract clause that specifies, possibly as a function of the state of the world, how decisions over the course of the contract period are to be resolved.

In summary, this thesis examines the design and the properties of covenants but assumes an underlying standard debt contract as the payoff function. The criterion for an optimally designed covenant is its degree of efficiency, i.e., the extent to which the total
value of the enterprise, including debt and equity claims, is maximized under the contract. In particular, the following research questions are studied.

i) How do the borrower’s and the lender’s incentives relate to efficiency, and how do those incentives depend on risk, information, and the state of the world?

ii) Does an optimal debt covenant delegate decision rights to the contracting parties or prescribe actions directly?

iii) What is the tradeoff between interest rates and control rights in an optimally designed debt contract? Is the optimal contract unique?

iv) What are the properties of the accounting rule underlying the optimal debt contract?

v) How are the optimal contract terms and the contractual efficiency affected by the borrowing firm’s financial leverage, profitability, liquidation value, and discretion in selecting investments?

2.2. Costs and Benefits of Debt Financing

An analysis of the inefficiency of debt financing makes the implicit assumption that either debt confers benefits that outweigh the inefficiency or that alternative financing options also incur an inefficiency of some form. The following discussion covers some settings, proposed in prior research, in which debt can dominate other forms of financing despite the presence of agency costs.

The foundational result of the theory of capital structure is the invariance principle of Modigliani & Miller (1958): in a frictionless world, firm value is the same irrespective of how the firm is financed. An initial theory why heterogeneity of debt-to-equity ratios is observed in practice has centered on the tradeoff between bankruptcy costs (e.g., Baxter, 1967) and the tax benefits associated with debt as a result of the heterogeneity in the taxation of bond interest, dividends, and corporate profits (e.g., DeAngelo & Masulis, 1980). Scott (1977) proposes a setting in which the issuing of secured (and hence prioritized) debt can increase firm value because the priority of debt payments implies that certain future expenses of the firm paid to outsiders, such as liabilities arising from lawsuits or certain types of taxes, are subordinated in bankruptcy, which increases the wealth retained by the firm’s capital providers. Yet, as Miller (1977) argues, doubt remains wheth-
er the magnitude of bankruptcy costs and tax rate differentials alone is sufficient to argue that capital structure affects firm value substantively.

Subsequent attempts to motivate the role of debt financing have focused on frictions between lenders, owners, managers, and other constituents. Jensen & Meckling (1976) argue that equity incurs an agency cost if managing (inside) equity owners can consume private benefits at the expense of outside equity owners and hence at the expense of total firm value. Conversely, debt incurs an agency cost in the form of asset substitution, i.e., equity owners may increase risk to an inefficient level in order to extract wealth from debtholders. Debt financing may then be an optimal choice if its agency cost is below that of equity. Leland (1998) combines tax benefits and agency costs in a single model and finds that a firm’s optimal leverage ratio may actually increase with the potential for asset substitution.

Aghion & Bolton (1992) study a security design problem in a setting with incomplete contracts and private benefits affected by discretionary decisions to be taken at interim dates. In their setting, common equity, preferred equity, or debt can all potentially be optimal financing instruments, depending on the correlation between the borrower’s and lender’s private benefits across states of the world and decisions. In particular, the debt financing alternative is optimal if the state-contingent allocation of decision rights yields more efficient outcomes than an unconditional allocation or joint ownership.

Information asymmetry between a firm’s current owners and potential outside investors underlies the pecking order of financing options proposed by Myers & Majluf (1984). Under the assumption that capital for new investments must be raised from outsiders, firms prefer issuing riskless debt over equity because information asymmetry produces an adverse selection problem in the equity capital market. In a similar setting, Noe (1988) obtains the same dominance of riskless debt over equity in a signaling equilibrium but notes that this result depends on the assumption that insiders observe the firm value without error. Another qualification is raised by Halov & Heider (2011), who observe that the pecking order does not necessarily extend to risky debt if the information asymmetry between insiders and outside investors is with respect to the firm’s business risk.

The resolution of information asymmetry between entrepreneur and outside capital provider via a costly verification of the state of the world is analyzed by Townsend
If the verification strategy is constrained to be deterministic, the standard debt contract is the optimal financing arrangement in this setting. Gale & Hellwig (1985) reach the same conclusion in a similar setting. A proviso is that stochastic verification strategies, along with financing tools other than debt, may dominate deterministic strategies. Harris & Raviv (1990) extend the setting to a multi-period learning process by which lenders infer (imperfectly) the borrowing firm’s value through observing whether the firm defaults or not and through investigating the firm following a default. Kareken & Stecher (2009) introduce the amount of debt raised as a choice variable to the costly state verification model and demonstrate that debt financing can then be optimal even with stochastic verification strategies.

Ross (1977) proposes a signaling role for debt. A firm’s manager whose compensation depends on the market value assigned to the firm in a capital market with uninformed investors may convey private knowledge of the firm’s true value by financing the firm with debt. Under the assumption that bankruptcy would be a costly event to the manager, managers of firms with higher true values issue more debt. The idea of using debt to establish credibility also underlies the moral hazard setting in Grossman & Hart (1982), where managers issue debt in order to commit to taking some non-contractible action that maximizes firm value rather than managerial compensation. The effectiveness of debt as a bonding device in this setting is again based on the assumption that bankruptcy of the firm would be personally costly to the manager.

Hart & Moore (1998) examine a setting in which an entrepreneur can divert his business’ cash flows, but not its invested assets, from outside investors. A debt contract, including the investor’s right to seize and sell the business’ assets if the entrepreneur fails to repay the investor at an interim date, is optimal in this setting if some regularity conditions on the business’ investment returns are satisfied.

2.3. Risk, Information and Incentives in Debt Contracts

The assumption of a standard debt contract is by itself not sufficient to make the designing of covenants to minimize inefficiency a well-defined problem. As the following discussion illustrates, a well-specified asset substitution problem only arises if firm and
lender face decision options that can be ranked with respect to their risk. The purpose of this subsection is to elaborate on the relationship between debt, risk, and decision incentives, based on the following formal illustration.

Let \( L \) denote the maturity value of a loan that a firm borrowed from a lender, and let \( c \) denote the ultimate cash inflow to the firm. The cash flow is unknown initially and has a prior distribution with cumulative probability \( F \) and support \([0, \bar{c}]\). Once \( c \) is realized, the lender receives cash inflows up to \( L \) and the firm’s owners receive any residual amount if \( c > L \) and zero otherwise. Initially, the parties’ expected payoffs are therefore

\[
u = E(\min(c, L)) = L - \int_0^L F(c) \, dc
\]

for the lender and

\[
x = E(\max(0, c - L)) = \bar{c} - L - \int_L^{\bar{c}} F(c) \, dc
\]

for the firm’s owners.

If one were to define greater risk as applying a mean-preserving spread to \( F \), it is readily apparent that the firm would benefit from increased risk while the lender would incur a loss. This wealth transfer from lender to borrower as a function of increased risk is central to the asset substitution problem and has long been recognized as one of the primary sources of friction in debt financing (e.g., Jensen & Meckling, 1976). Accordingly, if there existed two cash flow distributions \( F_1 \) and \( F_2 \) to choose from, and if \( F_2 \) were obtained by applying a mean-preserving spread to \( F_1 \), the lender would prefer \( F_1 \) while the firm’s owners would prefer \( F_2 \). The empirical findings of Chava & Roberts (2008) are consistent with these risk preferences: after a covenant violation, firms’ capital expenditure, i.e., investment with uncertain future payoff, declines as lenders obtain the right to influence the firms’ business decisions. Similarly, Begley & Feltham (1999) observe that the existence and restrictiveness of debt covenants is negatively correlated with the degree to which CEOs’ compensation is aligned with the incentive of shareholders rather than lenders.

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1 Extant research, e.g., Gavish & Kalay (1983) and Décamps & Faure-Grimaud (2002), often only considers the incentives of the borrowing firm in a setting in which the lender has no decision rights.
The business decisions that firms make over the course of a loan contract period can be thought of as constituting choices between cash flow distributions. The most efficient decision is the one that produces the cash flow distribution with the highest expected value. As Chava & Roberts (2008) note, changes in firms’ investment behavior following a covenant violation may be either efficient, if the lender interdicts an unprofitable high-risk investment, or inefficient, if the lender interdicts a profitable high-risk investment. In this context, a risk-ordering by mean-preserving spreads is an impractical representation because then any decision would be equally profitable. Yet, relaxing the risk definition to, say, second-order stochastic dominance, would impose the unrealistic restriction that riskier decision options must have a lower expected payoff. In addition, the impact of higher risk on the equity payoff $x$ and hence on the preferences of the firm’s owners becomes ambiguous.

The preceding discussion demonstrates that the intuitive interpretation of the asset substitution problem that arises in debt finance, i.e., the preference for potentially inefficiently low risk on the part of the lender and the preference for potentially inefficiently high risk on the part of the firm, has no obvious or ‘correct’ formal representation. What is required is a notion of ‘higher risk’ that imposes no restriction on the expected value of the firm’s cash flow while permitting some ranking of the contracting parties’ risk preferences. The following definition meets this criterion.

**Definition 1** (Risk-ordering). A random outcome $c$ with cumulative probability $F$ has a higher risk than a random outcome $c$ with cumulative probability $G$ if $F(c) > G(c)$ when $c < c_0$ and $F(c) \leq G(c)$ when $c \geq c_0$ for some constant $c_0$.

The risk-ordering here takes the form of a single-crossing property of cumulative distribution functions. The definition is stricter than second-order stochastic dominance in that the latter permits multiple crossing points. But unlike second-order stochastic dominance, Definition 1 imposes no restrictions on the ranking of expected values, i.e., a high-risk outcome can have either a higher or a lower expected value than a low-risk outcome.

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2 Green & Talmor (1986) explore the implications of this particular type of risk-return tradeoff with respect to the firm’s incentives.
The definition implies an intuitive relationship between the risk preferences of the firm and the lender. Consider the differences
\[ \Delta u = u(G) - u(F) = \int_0^L (F(c) - G(c)) \, dc \]
in the lender’s payoff and
\[ \Delta x = x(G) - x(F) = \int_0^c (F(c) - G(c)) \, dc \]
in the firm’s payoff that arise if the payoff comes from a low-risk distribution \( G \) rather than a high-risk distribution \( F \). Applying the above definition yields
\[ \Delta x > 0 \Rightarrow \Delta u > 0 \]
and
\[ \Delta u < 0 \Rightarrow \Delta x < 0 \]
In other words, a decrease in risk that is beneficial to the firm is always beneficial to the lender as well, and a decrease in risk that is detrimental to the lender is always detrimental to the firm as well. Since the converse is not true, the risk-ordering captures the intuitive notion that the firm has a greater preference for high risk than the lender. For the remainder of this thesis, notions of ‘high risk’ and ‘low risk’ will be operationalized according to Definition 1.

It should be noted that the risk-ordering definition is always met if one of the two distributions is degenerate, i.e., the low-risk outcome is deterministic. For example, a common setup of the asset substitution problem is a choice between a fixed liquidation value and a random continuation value (Décamps & Faure-Grimaud, 2002; Gigler, Kanodia, Sapra & Venugopalan, 2009). Yet, it stands to reason that many decisions between risky alternatives in practice cannot be ranked according to the above definition. Hence, one should bear in mind that the results obtained in this thesis only apply to settings with risk-ordered decision options.

2.4. Covenants, Contractual Incompleteness and Renegotiation

The incentive misalignment and the resulting potential for asset substitution discussed in the previous section are sources of inefficiency specific to debt contracts. Debt con-
tracts therefore often include additional clauses, referred to as covenants, that specify restrictions, requirements, or contingencies designed to reduce the inefficiency. Covenants can be classified into three broad categories: affirmative, negative, and financial. Affirmative (or positive) covenants require the borrower to fulfill certain obligations, such as providing the lender with audited financial statements, maintaining certain assets as collateral, carrying insurance on its property, or notifying the lender if certain events occur. Restrictive (or negative) covenants prohibit the borrower from taking certain actions, such as issuing dividends beyond a certain amount, taking out additional loans, or engaging in major transactions, such as mergers, without the lender’s approval. Financial covenants require the borrower to maintain certain levels of specified financial numbers or ratios, such as book value of equity, net income, working capital, or financial leverage. If a covenant is violated (often referred to as technical default), the lender typically receives the right to call the debt, i.e., to force the borrower to repay the loan immediately. Covenants can also specify directly certain actions to be taken, e.g., require the borrower to supply additional equity capital, refrain from certain investments, make an immediate partial repayment of the debt, or reduce its leverage ratio.

Financial covenants differ from affirmative and restrictive covenants in a critical respect: while the compliance with affirmative and restrictive covenants is deterministic, i.e., the borrower has full control over whether to abide by the covenant provision or not, compliance with financial covenants is stochastic because financial numbers reflect economic events beyond the borrower’s control. Affirmative and restrictive covenants are therefore useful to resolve incentive problems already existing at the time of contracting, e.g., by restricting the amount of dividends the borrower may pay out. Financial covenants, on the other hand, can address problems that depend on future events. Specifically, the nature of the asset substitution problem studied in this thesis depends on the risk-return tradeoffs discussed in the previous section. These risk-return tradeoffs may not be known at the time the contract is signed but may be captured by the borrowing firm’s financial results at later dates.

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3 Asset substitution is the focus of this thesis but does not constitute the only source of inefficiency germane to debt financing. For example, borrowing firms may have an incentive to pay out large dividends to their shareholders or divert assets by other means and thereby leave their nominally prioritized creditors empty-handed (Hart & Moore, 1998). Unless prevented by covenants, this potential expropriation might make lenders reluctant to provide loans to firms ex ante and lead to underinvestment ex post.
Financial results are generally imperfect reflections of the state of the world, and hence the remedies specified in the event of a covenant violation, even if set optimally ex ante, are not necessarily optimal ex post. Debt contracts with financial covenants are therefore incomplete contracts. Contractual incompleteness implies that scope for profitable renegotiation of the contract may arise after the state of the world is realized, and indeed debt covenants are frequently violated and renegotiated in practice. Chava & Roberts (2008) document that 15 percent of all firm-quarter observations in a sample of current ratio and net worth covenants from 1994 to 2005 constitute covenant violations and that 37 percent of all firms in the sample violate some of their covenants during the loan period. Nikolaev (2012) provides evidence of a higher incidence of renegotiation for borrowers with low credit ratings, high leverage, low profitability, volatile businesses, and more growth options.

Covenant violations tend to be costly events for the borrowing firms even if the lender does not call the debt but agrees to a renegotiation. Beneish & Press (1993) document the consequences of technical violations of debt covenants for a sample of 91 firms during the period of 1983 to 1987 and find that subsequent renegotiation of the contract tends to result in higher interest rates, increases in collateral, the prohibition of certain investing or financing transactions, or the divesting of capital assets in order to meet payment obligations. While lenders may grant waivers to violating firms, Chen & Wei (1993) observe that waivers are generally only given to firms with low leverage and a low probability of bankruptcy. Dichev & Skinner (2002) find evidence that firms manage their financial measures in order to avoid covenant violations.

Firms and lenders are likely to anticipate the possibility of a renegotiation at later dates and design their covenants accordingly. In a model by Berlin & Mester (1992), covenants require firms to invest a minimum amount in low-risk activities. In this setting, optimal debt contracts with the option to renegotiate include more stringent covenants than non-renegotiated contracts because the restriction can be lifted in renegotiation ex post when investment in high-risk activities is optimal. The benefit of the option to renegotiate is higher when the lender is better informed because renegotiation becomes more efficient. Chemmanur & Fulghieri (1994) consider a multi-period setting in which multi-period lenders (banks) and single-period lenders (bondholders) can invest in acquiring
information about borrowers, which improves the efficiency of renegotiation. In equilibrium, banks invest more in information acquisition, charge higher rates of interest, and attract borrowers with higher business risk. A further question is whether a default or covenant violation is optimally resolved via renegotiation or formal bankruptcy. Berkovitch & Israel (1998) model both options and conclude that private negotiations are suitable when borrowers face incentives for underinvestment while a court-administered bankruptcy proceeding is optimal in case of incentives for overinvestment.

Agency conflicts and information asymmetry influence the optimal ex ante allocation of control rights. Gârleanu & Zwiebel (2008) analyze costly renegotiation in a setting in which the borrower is privately informed about its ability to undertake an inefficient wealth transfer at the expense of the uninformed lender after the contract is signed. To address the resulting adverse selection problem, the optimal debt contract allocates greater decision rights to the lender.

In addition to information acquisition costs, several factors can make renegotiation costly. As noted by Smith & Warner (1979), public bond issues tend to be dispersed among many investors and hence costly to renegotiate because of coordination or free-rider problems. Renegotiation may also fail to prevent an inefficient liquidation if continuation of the business would allow the borrower to expropriate the lender, as in the setting analyzed by Hart & Moore (1998). Further, a project selection problem, e.g., as analyzed by Caskey & Hughes (2012), involves an irreversible commitment of capital and hence cannot be resolved via renegotiation ex post.

2.5. Roles of Accounting in Debt Contracts

Accounting plays a critical role in debt contracts because the financial results and ratios used in financial covenants are based on accounting data.\(^4\) The effectiveness of financial covenants in reducing the negative effects of the incentive conflict between borrowers and lenders is therefore ultimately determined by the design of the underlying ac-

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\(^4\) Begley & Freedman (2004) examine a sample of debt contracts between 1975 and 2000 and observe that covenants involving dividend and borrowing restrictions are frequently based on accounting measures. However, the authors note a decline in the use of accounting numbers in covenants in later years of the sample period.
counting system. For example, Cohen, Katz & Sadka (2012) document that firms with US-GAAP-based debt covenants restricting leverage increased their debt levels when the reclassification of minority interest to equity under SFAS 160 increased their covenant slack.

The following discussion is organized around two broadly defined roles for accounting in debt contracts: as an information transmitter and as a decision facilitator. Most existing research can be classified by this dichotomy, although accounting often serves both functions in the same setting. The model analyzed in this thesis is focused on the decision facilitation role.

2.5.1. Information Transmission

Accounting information is observable and verifiable. Prior research has therefore proposed that accounting increases the set of contractible information on which covenants can be based and reduces the information asymmetry between the borrower and the lender. Accounting can thereby lessen the need for potentially costly renegotiation of the contract at later dates, improve the efficiency of the renegotiation outcome, and provide information to the party in control of business decisions and thus decrease the cost of decision errors.

In theoretical models of this information transmission role, accounting typically takes the form of an information signal, and the analysis tends to focus on its statistical properties. For example, Sridhar & Magee (1997) examine the impact of an increase in the information content of accounting information relative to other, non-contractible information and conclude that, under certain assumptions, a more informative accounting variable shifts the optimal allocation of decision rights in favor of the borrowing firm and results in a lower rate of interest.

The accounting property that has received the most attention in prior research is conservatism. Both the operational definition of accounting conservatism and the research results show considerable heterogeneity. One interpretation views conservatism as an asymmetry in the precision of an accounting signal whereby high accounting values are more informative than low accounting values. Gigler, Kanodia, Sapra & Venugopalan
(2009) apply this definition in a debt contract setting with complete information and analyze its effect on the decision error in a choice between a fixed liquidation value and a random continuation value. The authors find that conservative accounting achieves the least efficient outcome and that the opposite property, i.e., an accounting signal with precise low realizations, would instead be efficient.\(^5\)

Göx & Wagenhofer (2010) adopt a similar framework of asymmetric precision but consider a setting in which a lender learns about the value of a potential borrower’s assets by observing an accounting signal before signing the contract. The lender only provides debt funding to the borrowing firm if the asset value inferred from the accounting signal is sufficiently high. Conservative accounting, by the definition from Gigler et al. (2009), reduces the probability of obtaining high inferred asset values and hence can prevent the funding of profitable investments.\(^6\) Similar representations of accounting conservatism can also be found in Chen & Deng (2010), where the borrower commits to a degree of accounting conservatism to signal its business risk to uninformed lenders, and in Callen, Chen, Dou & Xin (2010), where the borrower signals, through conservatism, its commitment not to expropriate the lender ex post.

Burkhardt & Strausz (2009) define conservatism via the lower-of-cost-or-market principle and contrast this accounting treatment with carrying an asset at its historical cost at all times. Conservatism then implies more informative accounting. The borrower in their model can engage in asset substitution only after acquiring liquidity by selling its exiting investment, but the potential buyers’ inability to observe the investment’s market value may lead to an adverse selection problem. More informative accounting can resolve the adverse selection problem and hence indirectly enable more firms to engage in asset substitution. The authors therefore conclude that conservative accounting can be detrimental for highly levered firms, for whom the cost of asset substitution is high.

Beyer (2012) also considers a lower-of-cost-or-market notion of conservatism that must be applied asset-by-asset and contrasts it with a fair value accounting system in

\(^5\) A necessary condition for this result is that the prior expected continuation value exceeds the liquidation value.

\(^6\) Yet, in contrast to Gigler et al. (2009) and in an illustration of the heterogeneity in the definition of accounting conservatism, the authors note that conservatism could alternatively be defined as high precision of the low accounting signal because this interpretation would be consistent with the stronger reaction to unfavorable accounting reports, relative to favorable ones, commonly observed in stock markets.
which all assets are carried at their market values. Only the aggregate asset balance is contractible. Applied to a debt contract setting, either accounting system can produce efficient outcomes, depending on the borrowing firm’s capital needs, the variance of its asset values, its assets’ liquidation values, and the severity of the incentive conflict between firm and lender.

The informational role of accounting in debt contracts has also provided a framework for interpreting empirical findings. Bharat, Sunder & Sunder (2006) measure accounting quality by deviations in firm-level operating accruals from expected values implied by industry-wide data and observe that firms with higher deviations pay higher rates of interest and are more likely to borrow from banks rather than issue publicly traded debt. The authors posit that banks process information more efficiently than investors in bond markets and can therefore enter into more efficient debt contracts when accounting quality is low. Nikolaev (2010) documents that firms with more debt covenants apply more conservative accounting, measured by the timeliness of loss recognition, and argues that the resulting increase in the probability of a control transfer to the lender mitigates the agency conflict between firm and lender.

2.5.2. Decision Resolution and Control Rights

In an alternative (or additional) role, accounting can serve as a decision facilitator. If financial covenants are based on accounting numbers, the likelihood of a violation and thus the triggering of sanctions or the transfer of decision rights depend on the properties of the accounting rules. The task of accounting in this context is to induce the most efficient decision given the underlying state of the world, either directly or via a delegation of decision rights to one of the contracting parties. Accounting thereby serves as a mediator in an incentive conflict. Importantly, this role does not require accounting to convey new information to either the borrower or the lender. The following is a brief formal illustration of this idea.

Let $\mathbb{A} = \{a_1, a_2, \ldots, a_N\}$ denote the set of all courses of action available for a given decision, which includes both all possible direct actions as well as options to delegate the decision to another party. The decision-maker chooses the action $a_i \in \mathbb{A}$ that maximizes
his payoff. In the presence of incentive misalignment, this may not be the same action that maximizes total firm value. Further, let $A \subseteq \mathbb{A}$, denote the subset of actions that are contractible ex ante, i.e. any element of $A$ must be describable at the date of contracting. The distinction between $\mathbb{A}$ and $A$ can take various forms in reality. For example, it may be possible to specify contractually to what level inventories are to be increased in the event of certain increase in sales volume. In contrast, it is likely impossible to specify ex ante how a product line should be redesigned in the event of a change in consumer taste. Importantly, $A$ always contains the option to delegate the decision to one of the contracting parties. In the case of the debt contracting problem, these are the firm and the lender. Without loss of generality, the choices to delegate the decision to either one can be assigned to the first two elements $a_1$ and $a_2$, respectively. If no direct action is contractible so that $A = \{a_1, a_2\}$, $A$ is said to be degenerate.

Let $\Theta$ denote the set of contractible information. A covenant describes a partition $P = \{P_1, P_2, ..., P_n\}$ of $\Theta$ such that $a_i$ is implemented whenever the realized state $\theta \in \Theta$ is in $P_i$, for all $i = 1, 2, ..., n$. A contract will be called a delegation contract if the only non-empty sets in $P$ are $P_1$ and $P_2$, i.e., the decision rule is a state-contingent allocation of control rights. Naturally, the decision rule always corresponds to a delegation contract if $A$ is degenerate. If $P$ contains more non-empty subsets than $P_1$ and $P_2$, the decision rule may prescribe consequences directly for some $\theta$. The corresponding contract will be referred to as a direct decision contract. An example of a direct decision contract from practice is the use of performance pricing. Asquith, Beatty and Weber (2005) document that debt contracts frequently specify automatic reductions in interest rates if the financial performance of the borrowing firm improves.

An accounting rule $BV: \Theta \rightarrow \mathbb{R}$ is said to implement the mapping from $\Theta$ to $A$ if $BV(\theta_i) < BV(\theta_j)$ for any $\theta_i \in P_i$ and $\theta_j \in P_j$ and $i < j$. In accounting terminology, $BV$ can be thought of as a set of accounting principles, and its output is an accounting variable computed using the contractible information $\Theta$. For example, $\Theta$ may include cash

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7 One might think of this approach in analogy to a multinomial logistic regression, where choosing the mapping from $\Theta$ to decision outcomes corresponds to assigning values to the categories of the dependent regression variable. The specific functional form of $BV$ inevitably depends on how the elements of $A$ are sorted, but $BV$ must always be a reflection of the information and the incentives that determine the optimal partition $P$, and thus the insights to be gained from constructing this function are the same regardless of how the $a_i$ are sorted. In particular, the sorting question is irrelevant in a delegation contract.
flow data, interest rates, market prices and other observable information, and $BV$ specifies how $\Theta$ is translated into journal entries. Further, if $BV$ implements a delegation contract such that low values of $BV$ implement an allocation of control rights to the lender, $\sup\{BV(\theta): \theta \in \Theta\}$ corresponds to the technical default threshold commonly observed in debt covenants in practice, e.g., a particular level of the firm’s book value of equity. The output of $BV$ could be the firm’s net income, its book value of equity, or any other accounting variable typically used in covenant. While the labeling of $BV$ is not of importance, $BV$ will, for the sake of concreteness, be referred to as the firm’s book value of equity hereafter.

In sum, accounting serves as an aggregation rule in this model and condenses a potentially multi-dimensional information signal to a real-valued accounting measure. This representation captures an essential aspect of the problem of designing an accounting system. Condensing $\Theta$ entails a loss of information, i.e., one cannot invert the accounting process to recover $\theta$ from an observed value $BV$. However, this loss of information does not reduce the efficiency of the contract because $a_t$ is, by construction, the optimal action or decision for all $\theta$ that could have induced the observed $BV$. In other words, accounting summarizes information such that all decision-relevant information content is reflected in the summary accounting measure and only decision-irrelevant information content is lost in the aggregation process. The notion of accounting as a strategic, purpose-specific information aggregator takes up the ideas of Butterworth (1972).

One may raise the question why designing an aggregation rule is a useful exercise. While accounting variables appear to be used quite frequently in practice as indicators of the state of the world, it stands to reason that one could alternatively specify contractually the outcomes for any given element of $\Theta$ rather than compress all information through an accounting rule $BV$. The first reason is the cost of contracting on a fully detailed description of $\Theta$. Generally, the state space $\Theta$ is highly complex in reality, and describing the consequences for all possible states is likely to be very costly, if not impossible. The second reason is that the functional form of the optimal accounting rule will reflect any

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8 In particular, if any dimension of $\Theta$ is measured on a continuum, it would be impossible to create an exhaustive list of all states.
regularities in the effects of information and incentives on the efficiency of the contract and can thereby highlight some general principles of efficient debt contract design.

Prior research on the decision facilitation role of accounting has mostly focused on delegation contracts. Sridhar & Magee (1997) consider an accounting-based debt covenant after whose violation the lender may interdict a risky and potentially opportunistic investment choice by the borrowing firm. The covenant is optimally set to balance the lender’s losses and the firm’s gains from this opportunistic investment. Caskey & Hughes (2012) analyze an asset substitution problem in which the firm makes a non-contractible choice between an inefficient high-risk investment and an efficient low-risk investment after the contract is signed. An accounting-based covenant that transfers control rights predominantly to the lender deters the firm from choosing the inefficient project because the lender may enforce an early liquidation of the inefficient investment after a covenant violation and thus deprive the firm of the benefit of increasing risk.

The dependence of the design and effectiveness of covenants on the properties of the underlying accounting rules has also been the subject of several empirical studies. Beatty, Ramesh & Weber (2002) find a substantial reduction in the rate of interest charged to borrowing firms if voluntary and mandatory changes in accounting standards are excluded from the computation of the accounting variables underlying the firms’ debt covenants. Beatty, Weber & Yu (2008) investigate to what extent accounting conservatism and covenant restrictiveness are used as substitutes and find that accounting conservatism and conservative covenant modifications are both frequently found in debt contracts and are often applied jointly, which suggests that the two are not completely substitutable in practice.

Ahmed, Billings, Morton & Stanford-Harris (2002) present evidence that conservative accounting practices correlate positively with financial leverage, dividend payout ratios, and the volatility in the borrowing firms’ return on assets, and negatively with firms’ credit ratings. The authors suggest that accounting conservatism mitigates the agency conflict in debt contracts and reduces the borrower’s cost of debt. Similarly, Lee (2009) documents a greater prevalence of conservative accounting among firms with high levels of debt. Zhang (2008) focuses on the relationship between accounting conservatism, covenant violations, and interest rates at the inception of the debt contract. Borrow-
ing firms with more conservative accounting practices are more likely to violate their debt covenants after negative stock price shocks but pay lower interest rates to their lenders, which the author interprets as evidence that accounting conservatism reduces the cost of debt.

2.5.3. Focus of this Thesis

This thesis is concerned with decision facilitation rather than information transmission. The two roles are not mutually exclusive, but focusing on decision facilitation is promising for several reasons. First, the state-contingent resolution of decision problems is plainly descriptive of the use of covenants in practice. For example, most covenants permit the lender to call the debt under contingencies defined in accounting terms. Specifying the consequences of certain accounting results in this manner would not be necessary if accounting were merely to inform the contracting parties about the state of the world. The presence of covenants implies that accounting information should induce actions that would otherwise not be taken, rather than purely convey information.

Second, the decision facilitation role can be modeled relatively parsimoniously. All results in this thesis follow directly from the payoff function of the debt contract. No additional assumptions about institutional features are needed to introduce frictions between the contracting parties. In contrast, models of the informational role often rely on exogenously imposed institutional tensions, e.g., the requirement to report the lower of historical cost and market value, to report aggregated balances, or to report with a specific bias.

Third, the decision facilitation problem treats the accounting system as a choice variable that is unconstrained by institutional limitations. In contrast, examining the informational properties of different exogenously prescribed financial reporting regimes amounts to a comparison of institutional features but yields no insight into how an optimal accounting rule for use in debt contracts should be designed. The problem is compounded if the institutional feature is modeled as a statistical property of the accounting variable. For example, the common representation of accounting conservatism as the degree of asym-
metry in the precision of an information signal leaves open the questions: how a statistical property of an exogenous random signal can be a choice variable; why accounting information should be compiled with a bias; why the firm, as its preparer, would not be able to unravel the bias; and why the firm would not manipulate the accounting signal in its favor.10

3. Model Setup

A firm has an investment opportunity with positive expected net present value.11 The project requires a capital outlay of $I$ at time $t = 0$ and yields uncertain cash flows of $c_t$ during the subsequent periods $t = 1, 2$ if continued to the end. Alternatively, the firm’s owners can sell the enterprise after the first period for a liquidation value $v$.12 The total payoffs are therefore either $c_1 + c_2$ under continuation or $c_1 + v$ under liquidation. The cash flows are distributed on the interval $[0, \bar{c}]$ with cumulative probability distribution functions $F_t$ and continuously differentiable density $f_t$. Likewise, the liquidation value is distributed with cumulative probability $F_v$ and continuously differentiable density $f_v$ on the same interval $[0, \bar{c}]$. Cash flows and liquidation values are independent and their distributions are common knowledge. For simplicity, the firm is assumed to undertake only this single project, so that the only business decision to be made subsequent to initiating the project is the binary choice between liquidation and continuation at $t = 1$.

For lack of internal funds, the firm must obtain the requisite capital for the project from an outside investor. The firm is assumed to finance $I$ entirely through debt. This assumption will be relaxed in Section 5. The lender’s payoff function is defined by a maturity value $L$, which the firm must repay upon conclusion of the project or at the time of

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10 Guay & Verrecchia (2006) demonstrate that it is not easy to argue that the informational imbalance implied by the asymmetric recognition of gains and losses under conservative accounting practices is superior to unbiased, full-information accounting. They suggest that the informational inefficiency of conservative accounting may be worth incurring if the cost of reporting verifiable information is higher for gains than for losses.

11 Throughout this text, “the firm” refers to the firm’s owners and managers, who are assumed to seek to maximize the value of the firm’s equity. Intra-firm agency conflicts between owners and managers are not part of this model.

12 The terms ‘continuation’ and ‘liquidation’ are used for concreteness. More generally, the decision alternatives could have been termed ‘high-risk’ and ‘low-risk’ instead.

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liquidation. If the firm’s cash flows or liquidation proceeds are insufficient to cover the maturity value $L$, the lender receives all available payoffs. In addition, the debt contract includes a covenant upon whose violation at $t = 1$ the lender may demand immediate payment of $L$ and thereby force the firm into liquidation. The debt covenant is based on the firm’s accounting book value of equity $BV(\cdot)$, which can be an arbitrary function of any contractible variables. The firm is considered to be in violation of the covenant if its book value falls below some threshold value $BV_0$. The accounting rule $BV(\cdot)$ and the threshold value $BV_0$ are agreed upon as part of the contract at time $t = 0$.

The lender is assumed to break even in expectation, i.e., the firm raises $I$ in a competitive capital market. Both parties are risk-neutral and the risk-free interest rate is normalized to zero. Hence, the terms of the financing contract are set so that the investor’s expected payoff is $I$, after considering the anticipated liquidation or continuation decisions at $t = 1$ across all possible realizations of $BV$. In order to simplify the analysis, realized cash flows must first be used to repay the lender, and the financing contract is assumed to prohibit the firm from issuing additional debt or equity, distributing dividends, and making new investments for the duration of the project. Permitting these decisions would complicate the analysis but not alter the fundamental dynamics of the problem.

The information structure of the problem is as follows. The firm’s cash flows $c_t$ are assumed to be observable and verifiable for contracting purposes after they have been realized, for example via an audit. Prior to the realization of $c_t$, firm and lender only know the prior distribution $F_t$. The liquidation value $v$, in contrast, is assumed to be unverifiable even at time $t = 1$. This lack of contractibility arises in practice when the value of the firm’s assets depends on factors that cannot be described in operational terms ex ante. For example, a manufacturer of board games experience a shift in consumer taste toward electronic games at $t = 1$, which would adversely affect $v$, but this observation

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13 In reality, the contract may specify a payment less than $L$ in case of liquidation at $t = 1$ because the debt is retired before maturity, but for simplicity, it is assumed that the firm does not receive such a discount.
14 For simplicity, the variable $BV$ will refer to both the accounting rule, i.e. the rules according to which book value is computed, and the firm’s actual book value, i.e. the output.
15 One could without loss of generality require that the investor obtain a positive economic profit in expectation.
16 In the language of Christensen & Nikolaev (2011), such restrictions on financing transactions would constitute a ‘capital covenant,’ whereas the covenant based on $BV$ above would be considered a ‘performance covenant.’
could not be anticipated and cast in verifiable terms in a contract at \( t = 0 \). In addition, the liquidation value \( \nu \) is not only unverifiable but also unobservable to both the firm and the lender.

Two information signals correlated with \( \nu \) are realized at \( t = 1 \). The first information signal, denoted \( \hat{s} \), is observable to firm and lender but is not verifiable. For example, \( \hat{s} \) may reflect observations about the shift in consumer taste, trends in sales and inventory costs, creditworthiness of customers, or the entry of new competitors. The second information signal, \( s \), captures only the verifiable components of \( \hat{s} \) and can hence be included in the debt contract. For example, the value a firm’s inventory could be estimated by observable market prices of like goods, the value of its buildings by prevalent real estate prices, or the value of its notes receivable by interest levels and historical default rates. In accounting terminology, \( \hat{s} \) and \( s \) represent estimates of the fair value of the firm’s invested assets in the resale market. The less the firm’s asset values correlate with observable information, the more imprecise these fair value estimates will be. For example, the value of an intangible asset such as some new, proprietary technology can likely only be estimated with substantial error. The information signals \( \hat{s} \) and \( s \) induce posterior distributions with cumulative probability \( G(\nu|s) \) and \( \hat{G}(\nu|\hat{s}) \). The cumulative probability of \( \hat{s} \) conditional on \( s \) will be denoted by \( H(\hat{s}|s) \). Higher realizations of \( \hat{s} \) and \( s \) increase the posterior assessment of \( \nu \) in the sense in the form of first-order stochastic dominance, as specified in the following assumption.

**Assumption 1** (First-order stochastic dominance). For any \( \hat{s} \) and \( s \), \( \frac{\partial G}{\partial \hat{s}} < 0 \) and \( \frac{\partial \hat{G}}{\partial s} < 0 \) at all \( \nu \).

The information in \( s \) is necessarily less precise than the information in \( \hat{s} \) because the former is constrained to come from verifiable sources. In fact, \( \hat{s} \) must be a sufficient statistic for the pair \((\hat{s}, s)\) with respect to \( \nu \), or

\[
\Pr(\nu|\hat{s}, s) = \Pr(\nu|\hat{s})
\]

because verifiable information must necessarily be a subset of the observable information. In other words, all information about \( \nu \) in \( s \) is also reflected in \( \hat{s} \), but \( \hat{s} \) generally contains
additional information that improve its precision over \( s \). Thus, any deviation of \( s \) from \( \hat{s} \) can only be noise and so an observer of \( \hat{s} \) would not revise his beliefs about \( v \) if presented with \( s \) in addition.

One might object that, realistically, new information at \( t = 1 \) should update the observer’s beliefs not only about the liquidation value \( v \) but also about the continuation value \( c_2 \). The simplification chosen here can be justified as follows. Liquidation maximizes firm value given some information signal \( s \) if

\[
E(v - c_2 | s) > 0
\]

In other words, a sufficient statistic for this decision is the expected difference between the liquidation and continuation values. Whether a change in this difference occurs because of a change in the distribution of \( v \) or a change in the distribution of \( c_2 \) is irrelevant with respect to the optimal decision. Therefore, the insights to be gained from this model with respect to contractual efficiency suffer no loss of generality if one sets

\[
E(v - c_2 | s) = E(v | s) - E(c_2 | s) = E(v | s) - \mu_2
\]

for all \( s \). The argument also applies to the properties of \( v \) and \( c_2 \) conditional on \( \hat{s} \).

The agency conflict in the debt contract problem stems from the difference in risk levels between decision options. In the setting described here, these decision options are obtaining the liquidation payoff \( v \) and obtaining the continuation payoff \( c_2 \). As noted in Section 2.3, a clean characterization of the incentive problem requires an operational definition of an ordering of decision options by risk.

**Assumption 2 (Risk-ordering).** Conditional on \( \hat{s} \), the liquidation value \( v \) has a lower risk than the second-period cash flow \( c_2 \) in the sense of Definition 1.

In line with the discussion in Section 2.3, Assumption 2 formalizes the notion that liquidation is the low-risk option by imposing a single-crossing condition on the cumulative probabilities \( \hat{G} \) and \( F_2 \). Assumption 2 is less restrictive than second-order stochastic dominance in that it does not require \( E(v | \hat{s}) \geq \mu_2 \) but more restrictive in that second-order stochastic dominance permits multiple crossing points. The risk-ordering implies that the conditional distribution of \( v \) is less dispersed than the distribution of \( c_2 \) even if \( s \)

\[\text{Equivalently, one could set } E(v | s) \equiv \mu_v \text{ for all } s \text{ and some constant } \mu_v \text{ and let } E(c_2 | s) \text{ vary with } s.\]
is relatively uninformative, and it holds trivially if \( v \) is directly observable, i.e., \( \hat{G} \) is a delta distribution, with \( \Pr(v = \hat{s}) = 1 \).

The economic rationale for declaring \( v \) the low-risk option is that the liquidation value represents an outsider buyer’s expected benefit from using the purchased assets. The buyer will presumably use the assets in period \( t = 2 \) to generate cash flows, and so \( v \) can be interpreted as the buyer’s assessment of \( E(c_2 | s) \equiv \mu_2 \). Thus, the distribution of \( v \) can be viewed as a distribution of expected cash flows to potential buyers of the firm’s assets, which likely shows less variance than the primitive cash flow variable itself.

**Figure 1.** Timeline of the model.

The timeline of the model is summarized in Figure 1. At \( t = 0 \), both parties observe the distributional properties of all variables and agree on a debt contract specifying \( I, L \) and a covenant based on the firm’s book value of equity \( BV \). Firm and lender sign a delegation contract, i.e., if \( BV \) falls below the contractually specified threshold value \( BV_0 \) at

\(^{18}\) This expected cash flow need not coincide with the firm’s own \( \mu_2 \) because the assets may have a different value in the hands of the buyer.
$t = 1$, the firm is considered to have violated the covenant and the lender may call the debt and thus enforce liquidation. Otherwise, decision rights remain with the firm. The question whether a direct decision contract could improve efficiency will be revisited in Section 4.2. If liquidation occurs, proceeds in the amount of $v$ are realized. The lender receives any amount up to $L$ and the firm receives the remainder. If the project is continued instead, $c_2$ is realized in $t = 2$, and the lender receives all payoffs up to $L$ while the firm receives the remainder.

![Control allocation between firm and lender.](image)

**Figure 2.** Control allocation between firm and lender.

Given the above information structure, the book value $BV$ at $t = 1$ can be based on the contractible information variables $c_1$ and $s$. For the sake of brevity, a pair $(c_1, s)$ will be denoted by $\theta$ and the set of all contractible states $\theta$ by $\Theta \subseteq \mathbb{R}^2$. The cumulative probability of $\theta$ will be denoted by $F$ and the joint density by $f$. The accounting rule is a continuous mapping

$$BV : \Theta \to \mathbb{R}$$

The covenant effectively partitions $\Theta$ into the ‘technical default’ subset

$$D \equiv \{\theta : BV < BV_0\}$$

of states in which the firm has violated the covenant and the ‘no-default’ subset

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19 The accounting rule is solely based on verifiable information, i.e. discretionary accounting choices and strategic manipulation of accounting estimates are not part of this model.
of states in which book value is above $BV_0$ and hence no violation has occurred. Figure 2 provides a graphical illustration of a possible partition of $\Theta$ into $D$ and $N$, whereby the path of cutoff states $\theta^*$ along which $BV = BV_0$ identifies the boundary between the two subsets. The debt contract can thus be defined as the set \{I, L, D\}.

The option to renegotiate the contract at $t = 1$ is not considered in this thesis. This restriction may appear to limit the applicability of the results, and indeed the optimal accounting rule would be almost arbitrary in the symmetric information setting considered here if renegotiation were costless because any ex-ante control allocation would yield first-best efficiency ex-post. The results from a setting without renegotiation are nonetheless relevant for several reasons. First, renegotiation is generally infeasible for publicly traded debt obligations because of frictions such as free-rider problems. Public bonds are therefore rarely renegotiated (Smith & Warner, 1979). Second, renegotiation is generally not costless in reality even if it is feasible. Among other things, renegotiation costs may represent direct legal and administrative expenses, which are likely to be high when the debt capital is provided by a large number of lenders, as well as negative externalities. For example, a major customer of the firm may become aware of the renegotiation, interpret the event as a signal that the firm is in financial difficulty, and seek a new supplier that does not have a potential going concern problem. Hence, a strategic, state-contingent allocation of decision rights is meaningful as long as renegotiation costs would outweigh the benefits in at least some states. Finally, a trivial renegotiation to the first-best outcome is generally not attainable when information is asymmetric. While not considered in this thesis, the optimal accounting rule in a setting with asymmetric information is affected by the same incentives as those demonstrated in the following analysis, even if renegotiation is permissible and costless. Thus, this thesis characterizes basic properties of an optimal accounting rule for debt contracts in a simplified scenario but its results should prove useful in extensions of the model to more realistic settings.

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20 Assigning $\{\theta: BV = BV_0\}$ to $N$ rather than $D$ is without loss of generality. All results in this thesis are formulated to hold in either case.

21 The term “renegotiation” refers to a negotiated change in contract terms. A waiver of decision rights by one party without any alteration of the contract terms is assumed to be feasible and costless in all cases.
4. Contract Efficiency and Optimal Accounting Rules

The first part of this section formalizes the notion of inefficiency and derives the contracting parties’ decision incentives. The second part discusses the solution approach and the form of the optimal decision rule. The final part combines these results and sets up the optimization problem.

4.1. Decision Incentives and Efficiency

The contract agreed upon at time $t = 0$ depends on the anticipated liquidation or continuation decisions at $t = 1$. It will therefore be convenient to begin the analysis with the two parties’ decision preferences. The controlling party at time $t = 1$ chooses liquidation or continuation in order to maximize its payoff. The extent to which this payoff maximization deviates from the first-best rule, i.e., to liquidate if and only if $E(v|\hat{s}) > \mu_2$, will ultimately determine the optimal allocation of control rights. The parties’ decision preferences follow from their payoff functions. In particular, the firm maximizes residual claims and therefore prefers liquidation at $t = 1$ whenever

$$x_v > x_2$$

where

$$x_v \equiv E(\max(0, c_1 + \bar{v} - L) | \hat{s})$$

is its expected payoff under liquidation and

$$x_2 \equiv E(\max(0, c_1 + \bar{c}_2 - L))$$

is its expected payoff under continuation.\(^{22}\) It should be noted that $c_1$ has been realized and observed at this time and can therefore be treated as a constant. The firm’s liquidation payoff is zero for $v \leq L - c_1$ and changes one-for-one with $v$ from thereon. Then ceteris paribus, there exists at most one cutoff value $\hat{s} = k^f$ for which (1) holds with equality and above which the firm prefers liquidation because

$$\frac{\partial x_v}{\partial \hat{s}} = -\int_{L-c_1}^{\hat{s}} \frac{\partial \hat{G}}{\partial \hat{s}} dv > 0$$

\(^{22}\) For better readability, random variables inside expectations are denoted by a tilde.
for all $\hat{s}$ by Assumption 1.

Similarly, the lender maximizes debt claims and therefore prefers liquidation at time $t = 1$ whenever

$$u_v > u_2 \quad (2)$$

where

$$u_v \equiv E(\min(c_1 + v, L) | \hat{s})$$

and

$$u_2 \equiv E(\min(c_1 + \hat{c}_2, L))$$

As a mirror image to the firm’s case, the lender’s liquidation payoff increases one-for-one with $v$ when $v \leq L - c_1$ and remains constant once total payoffs exceed the maturity value $L$. Hence, the lender also has at most one indifference point $\hat{s} = k^l$ at which, ceteris paribus, (2) holds with equality because

$$\frac{\partial u_v}{\partial \hat{s}} = -\int_0^{L-c_1} \frac{\partial \hat{G}}{\partial \hat{s}} \, dv > 0$$

is again implied by Assumption 1. The indifference points $k^f$ and $k^l$ will be referred to as liquidation thresholds.

In contrast, the first-best decision strategy at time $t = 1$ maximizes total firm value and hence prescribes liquidation whenever

$$E(v|\hat{s}) > \mu_2 \quad (3)$$

Assumption 1 again implies that

$$\frac{\partial E(v|\hat{s})}{\partial \hat{s}} = -\int_0^c \frac{\partial \hat{G}}{\partial \hat{s}} \, dv > 0$$

and so the first-best indifference point $\hat{s} = m$ at which (3) holds with equality is also unique. The inequalities in (1) and (2) suggest that $k^f$ and $k^l$ do not generally coincide with the first-best benchmark value $m$ because they depend on $c_1$ and on the debt value $L$, whereas $m$ is independent of both. This observation illustrates the link between the incentive conflict and the payoff function of the standard debt contract with a fixed maturity.
value $L$. In a financing contract with state-contingent payoffs, payoffs could be specified such that $k^f = k^l = m$ in all states, which would reduce agency costs to zero.\textsuperscript{23}

The following proposition establishes an important regularity in the behavior of the liquidation thresholds $k^f$ and $k^l$ that holds generically for any binary decision between a high-risk and a low-risk alternative. The distance $|k^i - m|$ for $i \in \{f, l\}$ will be referred to as the inefficiency range. The proof of this and of all subsequent results can be found in the Appendix.

**Proposition 1.** For any state $\theta$ and any maturity value $L$, the firm never prefers inefficient liquidation and the lender never prefers inefficient continuation. The firm’s inefficiency range increases in $L$ and decreases in $c_1$, while the reverse holds for the lender’s inefficiency range.

In essence, Proposition 1 is simply a reflection of the well-known asset substitution problem. The misalignment of preferences is illustrated in Figure 3 for the limit case in which $\Pr(\nu = \hat{s}) = 1$ and hence $m = \mu_2$. The lender’s liquidation payoff $u_\nu$, shown in the left panel, follows the total liquidation value $\nu$ up to the level of the maturity value $L$ and remains constant thereafter. The lender’s continuation payoff $u_2$ is bounded by the lower of $L$ and $\mu_2$, and hence the indifference point at the intersection of $u_\nu$ and $u_\nu$ is bounded from the right by the first-best indifference point $\nu = \mu_2$. The graph of the firm’s liquidation payoff $x_\nu$, shown in the right panel, follows the total liquidation value $\nu$ at a constant distance of $L$ units below, while the distance between the firm’s continuation payoff $x_2$ and the total expected continuation value $\mu_2$ is bounded above by $L$. Hence, the firm’s indifference point at the intersection of $x_\nu$ and $x_2$ is bounded from the left by the first-best indifference point $\nu = \mu_2$. The liquidation thresholds thus describe the fundamental tension in this model: both parties prefer to maximize the investment’s payoff ex ante at $t = 0$ but are unable to implement the maximally efficient outcome because neither can commit to taking the first-best action at $t = 1$.

\textsuperscript{23} As noted in the introduction, non-state-contingent payoffs may also be optimal for other exogenous reasons even if actions are verifiable, e.g. if the firm’s owners and managers can consume private benefits at the expense of outside financiers when payoffs are high (Jensen & Meckling, 1976).
Figure 3. Liquidation thresholds relative to first-best for the lender (left) and the firm (right).  

Proposition 1 is a general result in the following sense. A preference for an inefficient decision necessarily implies that this decision results in a wealth transfer to the decision-maker at the expense of the other contracting party, and that this wealth transfer exceeds the decision-maker’s share of the resulting efficiency loss. For any particular decision, the wealth transfer can only benefit one of the two parties, and so for a given efficiency benchmark, any inefficient action can be preferred by at most one party. Accordingly, if there exists only one possible inefficient action per state of the world, at least one party would make a socially optimal decision. Thus, the two sets of states which the firm and

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24 For ease of exposition, the first-period cash flow has been normalized to $c_1 = 0$ in this diagram.
the lender, respectively, prefer an inefficient decision are disjoint. Proposition 1 shows that the partitioning of the $c_1$-$\hat{s}$-space defining these subsets follows the path of $(c_1, m)$.\footnote{For a general set of decision-relevant random variables in $\mathbb{R}^n$, it stands to reason that the partitioning would follow from a separating hyperplane, provided that the inefficiency ranges can be separated by a single cutoff point along each dimension. The two-dimensional case presented here suffices to illustrate the dynamics of the problem.}

Figure 4. Liquidation preferences as a function of maturity values and past cash flows.

If the contracting parties were to make decisions based on their liquidation thresholds, the firm would cause more inefficiency when the maturity value $L$ is high and $c_1$ is low while the opposite holds for the lender. The reason is that the balance $L - c_1$ of outstanding debt is similar to the strike price of a call option on total firm value held by the firm’s owners: the higher the strike price, the greater the value of potentially inefficient volatili-
ty in firm value, and hence the greater the incentive to forgo efficient but riskless liquidation. The lender’s incentives naturally move in the opposite direction. This observation is an important determinant of the optimal debt covenant and holds under any assumptions about probability distributions. Proposition 1 thus also provides a theoretical motivation for the commonly observed practice of performance pricing (Beatty et al., 2005), i.e., many debt contracts specify lower interest rates \( (L) \) if the borrowing firm has performed well financially \( (c_1) \). Figure 4 shows a graphical illustration.

Relative to the first-best cutoff point \( \hat{s} = m \), delegating the liquidation decision to either the firm or the lender thus incurs inefficiency. In measuring this inefficiency, it will be convenient to consider first the inefficiency incurred under the first-best rule, which is

\[
\int_0^{\mu_2} (\mu_2 - v) \, d\hat{G}
\]

for \( \hat{s} > m \) and

\[
\int_{\mu_2}^\epsilon (v - \mu_2) \, d\hat{G}
\]

for \( \hat{s} < m \).\(^{26}\) Then if the first-best decision is implemented, the expected inefficiency in a given state \( \theta = (c_1, s) \) is

\[
r = \int_0^m \int_{\mu_2}^\epsilon (v - \mu_2) \, d\hat{G} \, dH + \int_{\mu_2}^\epsilon \int_0^{\mu_2} (\mu_2 - v) \, d\hat{G} \, dH
\]

\[
= \int_0^{\mu_2} (\mu_2 - v) \, dG + \int_0^m (E(v|\hat{s}) - \mu_2) \, dH
\]

One should observe that \( r \) is an unavoidable loss that arises because the information available at \( t = 1 \) is imperfect. Naturally, \( r = 0 \) when \( \hat{s} \) is a perfect signal of \( v \).

The agency cost of delegating the liquidation decision to the firm or the lender is the inefficiency incurred incremental to \( r \) and thus takes the form

\[
q_f = \int_0^{\mu_2} (\mu_2 - v) \, dG + \int_0^{k_f} (E(v|\hat{s}) - \mu_2) \, dH - r
\]

\[
= \int_m^{k_f} (E(v|\hat{s}) - \mu_2) \, dH
\]

\(^{26}\) For brevity, \( dG(v|\hat{s}) \) will be written as \( d\hat{G} \) from hereon, with analogous abbreviations for \( \hat{G}(v|\hat{s}) \) and \( H(\hat{s}|\hat{s}) \).
in a given state $\theta$ if the decision is delegated to the firm and

$$q^l = \int_{k^l}^m \left( \mu_2 - E(\nu|\hat{s}) \right) dH$$

(6)

if the decision is delegated to the lender. Similarly, the payoff to the lender given an allocation of decision rights to party $i \in \{f, l\}$ in a given state $\theta$ is

$$u^i = \int_0^{k^l} u_2 \, dH + \int_{k^l}^{l^e} u_\nu \, dH$$

(7)

The firm’s payoff $x^l$ is given by the same expression if $u_2$ and $u_\nu$ in (7) are replaced by $x_2$ and $x_\nu$. The inefficiencies in (5) and (6) represent a latent agency cost incurred in a delegation contract. In contrast, if liquidation and continuation were contractible actions, one could alternatively devise a direct decision contract, under which the decision is prescribed rather than left to the firm or the lender. The following section compares the two approaches.

4.2. Delegation Contracts and Direct Decision Contracts

The purpose of this section is to determine whether the delegation contract assumed so far could be improved upon by permitting a direct mapping from $\Theta$ to liquidation or continuation if these decisions were contractible. Intuitively, the advantage of a direct decision contract is the absence of any agency cost incurred as a result of the contracting parties’ misaligned decision incentives. The reason is that both parties prefer to implement the first-best action ex ante when the contract is concluded at $t = 0$. The agency cost arises under the delegation contract because neither party can commit at $t = 0$ to following the first-best decision rule ex post at $t = 1$, and a direct decision contract would circumvent this commitment problem. Yet, only the imprecise information signal $s$ is contractible and can be used to determine whether to liquidate the firm or not, whereas a delegation contract can take advantage of the more precise information signal $\hat{s}$, albeit at the cost of letting firm and lender follow an inefficient decision rule. The following result confirms that the delegation contract assumed so far is indeed optimal.
Proposition 2. For any direct decision contract, there exists a delegation contract that achieves a more efficient outcome.

Proposition 2 states that the decision usefulness of $\hat{s}$ vis-à-vis that of $s$ always outweighs the agency cost of the delegation contract. The logic behind this result rests on the perfect misalignment of decision incentives established in Proposition 1. If $E(v|s) < \mu_2$, prescribing liquidation yields an inefficient outcome if $\hat{s} < m$ because $\hat{s}$ is a sufficient statistic for the pair $(s, \hat{s})$ with respect to $v$. Conversely, delegating the decision to the lender instead only yields an inefficient outcome if $k^l < \hat{s} < m$. Hence, although firm and lender do not always use their information to implement the first-best action, the perfect asymmetry of the liquidation thresholds shown in Proposition 1 can be exploited to ensure that the outcome is never worse than under a direct decision contract. Naturally, Proposition 2 holds vacuously if liquidation is not a contractible action, and it holds as an equivalence when $s$ and $\hat{s}$ are equally precise.

The dominance of state-contingent delegation over direct decision contracts may appear to stand in contrast to results from the managerial accounting literature, which generally show that delegation is inferior to a properly designed direct decision rule. The optimal mechanism in these studies typically elicits non-contractible private information by specifying appropriate transfer payments to or from the reporting party. The reason this solution is not applicable in this model is that no such transfer payments occur under a standard debt contract because the firm’s payment to the lender is not contingent on any state or action unless bankruptcy occurs. The exogenously given payoff function in the debt contract setting thus creates an important difference to the standard principal-agent problem.

An important caveat about Proposition 2 is that the superiority of the delegation contract is only unambiguous if the business decision is binary. Proposition 2 holds because for each $\hat{s}$, at most one party has an incentive to make an inefficient decision. If the decision were not binary, even the perfect misalignment of incentives shown in Proposition 1 would not guarantee an efficient delegation contract because neither party might prefer the first-best action. For example, if the business decision were to involve an inefficient low-risk option, an inefficient high-risk option, and an efficient intermediate option, the
lender may have an incentive to choose the first and the firm may have an incentive to choose the second one. Then a pure delegation scheme would not necessarily achieve the best outcome, e.g., if the difference in the precision of $\hat{s}$ and $s$ is sufficiently small. More importantly, multiple action choices also broaden the set of possible decision mechanisms substantially. For example, delegation rules could then be combined with a restriction on the set of permissible actions, a scheme that, in view of Proposition 1, would serve no purpose in the binary case.27

In practice, direct decision contracts may not be feasible because many decision options are not contractible. The set of possible actions in any given state of the world is likely to be very large in reality and may not be known until the state is realized. For example, it may be known ex ante that a movie production company typically has a choice between a ‘safe’ production, such as a movie in a currently popular and well-established genre that is very likely to be adequately profitable but has to share the market with many competitors, and a risky production in a yet undeveloped genre that may turn out be a either a blockbuster or a complete failure. However, since viewers’ tastes change over time, it may not be possible to identify and describe the content of risky or safe productions ex ante at time $t = 0$, and hence a direct mapping from contractible information to decisions would be infeasible in this setting. The optimality of the delegation contract therefore applies not only to settings in which the set of contractible decisions contains known, binary choices but also when it contains options unknown at the inception of the contract. Overall, Proposition 2 is consistent with the prevalence of covenants in practice that assign state-contingent decision rights to one of the contracting parties.

4.3. Design and Uniqueness of the Optimal Delegation Contract

Whether inefficiency and payoffs are realized according to the liquidation thresholds $k^I$ or $k^L$ depends on the accounting rule $BV$. Specifically, the expected inefficiency at time $t = 1$ in a given state $\theta$ and under a given accounting rule $BV$ can be written as

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27 Nonetheless, the binary decision structure of the liquidation option is not overly limiting if one considers the large variety of high-level strategic decisions that follow this pattern, e.g., the decision to acquire a competitor, replace an executive, enter a new product line, expand overseas etc. As noted in Aghion & Bolton (1992), contractible decisions are likely to be of this type.
\[ q(BV) = I_D(BV) \cdot q^I + \left(1 - I_D(BV)\right) \cdot q^f \]  
\[ \text{where } I_D \text{ is the technical default indicator function} \]

\[ I_D(BV) = \begin{cases} 
0 & \text{if } BV(\theta) \geq BV_0 \\
1 & \text{if } BV(\theta) < BV_0 
\end{cases} \]

Since raising or lowering the default threshold is equivalent to adding or subtracting a constant from \( BV \), no loss of generality is incurred by normalizing the former to \( BV_0 = 0 \).

The term \( q(BV) \) reflects the agency cost of the contract incurred in a particular state \( \theta \). Analogously, the firm’s and lender’s payoffs \( u(BV) \) and \( x(BV) \) in a given state for a given accounting rule are defined by replacing \( q \) in (8) with \( x \) and \( u \), respectively. The relationship between \( q, u \) and \( x \) can be summarized by the following identity. In each state \( \theta \), the total payoff from the firm’s investment must equal

\[ w - q(BV) = x(BV) + u(BV) \]  
\[ \text{where the left-hand side is the first-best payoff} \]

\[ w \equiv c_1 + \max(E(v|S), \mu_2) \]  
\[ \text{less the agency cost } q, \text{ and the right-hand side is the allocation of this payoff to the two parties.} \]

One can interpret (9) as a type of balance sheet equation, where assets are shown on the left and liabilities and equity are shown on the right. The value of the firm’s equity, conditional on \( \theta \), can therefore be written as

\[ x(BV) = w - q(BV) - u(BV) \]  
\[ \text{by rearranging (9). When the contract is negotiated at time } t = 0, \text{ the state } \theta \text{ is yet unrealized and so the firm seeks to maximize its equity value } x \text{ in expectation across all } \theta, \text{ subject to the lender’s break-even constraint. The firm’s optimization problem at } t = 0 \text{ is therefore} \]

\[ \max_{BV(.)} \int_\theta \left[w(\theta) - q(BV(\theta), \theta) - u(BV(\theta), \theta)\right] dF \]  
\[ \text{subject to} \]

\[ \int_\theta u(BV(\theta), \theta) dF \geq I \]
The firm’s optimization program is a simple form of an optimal control problem with a static constraint. For ease of exposition, it will be convenient to write the objective function in (12) in Lagrange form as

$$Y(BV, L, \lambda) = \int y(\theta, BV, L, \lambda) \, dF$$

(14)

where

$$y(\theta, BV, L, \lambda) = w(\theta) - q(BV(\theta), \theta) + (\lambda - 1) \cdot u(BV(\theta), \theta) - \lambda I$$

(15)

and \(\lambda \geq 0\) is the Lagrange multiplier for the lender’s break-even constraint in (13). Determining when this constraint binds is the first step in setting up the necessary conditions for an optimal contract.

**Lemma 1.** The lender’s break-even constraint binds under any optimal contract.

Lemma 1 rules out the possibility that the firm may ever choose to overpay the lender. One might suspect that such an overpayment could be optimal if it improves the lender’s decision incentives and thereby reduces inefficiency by more than the amount of the overpayment. As equation (11) shows, the firm would reap the entire benefit of this efficiency gain. The reason why such an overpayment is never optimal from the firm’s perspective is that whenever the lender’s expected payoff is above the required level \(I\), the firm could increase its equity value by either reducing the maturity value \(L\) or expanding the no-default set \(N\).\(^{28}\)

An optimal accounting rule \(BV\) induces a violation of the debt covenant, and hence a transfer of decision rights to the lender, in all states \(\theta\) in which \(y\) is higher under \(\theta \in D\) than under \(\theta \in N\). This condition is equivalent to requiring that

$$\Gamma(L, \lambda, \theta) \equiv q^l(\theta) - q^f(\theta) + (1 - \lambda) \cdot \left( u^l(\theta) - u^f(\theta) \right) = 0$$

(16)

in all critical states \(\theta = \theta^*\), i.e., states in which \(BV(\theta) = BV_0 = 0\). The formal derivation of \(\Gamma\) can be found in the Appendix under the proof of Lemma 2. Equation (16) states that in any state \(\theta^*\), the firm is indifferent between retaining and ceding control if and only if the net effect of transferring control to the lender on the firm’s equity value is zero. In

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\(^{28}\) The argument underlying Lemma 1 also applies to any form of financing agreement other than debt contracts.
economic terms, $\Gamma$ thus represents the net cost or benefit of technical default, which always accrues to the firm because the lender breaks even by Lemma 1. It should be noted that equation (16) imposes no restrictions on $BV$ for $\theta \neq \theta^*$. This observation is intuitive: if the book value is not at the default threshold, a small increase or decrease in $BV$ does not change whether the firm has violated the covenant or not and hence does not affect control rights, so the value of $Y$ would remain unchanged.

The first-order necessary condition (16) implies that $BV = BV_0$ and $\Gamma = 0$ must hold simultaneously in an optimal contract or, equivalently, control allocation to either party is equally efficient if and only if the book value is at the technical default threshold. In all other states, $\Gamma$ must either be strictly positive or negative. In particular, $\Gamma < 0$ means that the firm is better off ceding control to the lender, while $\Gamma > 0$ means that the equity value is higher when the firm retains control over the liquidation decision. Given the normalization $BV_0 = 0$, it is therefore optimal to specify the accounting rule so that $BV < 0$ in the former and $BV > 0$ in the latter case, which yields the following result.

**Lemma 2.** An accounting rule $BV(\cdot)$ is optimal if and only if $BV(\theta) = z(\theta) \cdot \Gamma$ for some function $z(\theta) > 0$ at all $\theta$.

The solution given in Lemma 2 holds for any optimal contract, but the result does by itself not yet identify the globally optimal contract terms. The reason is that $\Gamma$ is a function of the choice variable $L$, and a further necessary condition for an optimal contract is that $\frac{\partial Y}{\partial L} = 0$. Hence, $BV = z(\theta) \cdot \Gamma$ states the solution function only in implicit form. In order to conclude that Lemma 2 identifies the global optimum, it remains to be shown that (16) and $\frac{\partial Y}{\partial L} = 0$ can only hold simultaneously for a unique value of $L$ and that this critical point indeed attains a maximum. The following proposition resolves this problem.

**Proposition 3.** The optimal debt contract $\{I, L, D\}$ is unique.

The uniqueness result applies to $L$ and to the partition $\{D, N\}$ that $BV$ implements, but not to the accounting rule $BV$ itself because $z(\theta)$ can take arbitrarily many functional
forms as long as $z > 0$ for all $\theta$. But even though the accounting rule that implements a given allocation of control rights is not unique, this multiplicity is inconsequential because the objective function $Y$ takes the same values under all such accounting rules, i.e., contractual efficiency and the payoffs to both parties are the same. The requirement that $D$ and $N$ remain unchanged under all optimal accounting rules implies that the critical level set $\{\theta^*\} \equiv \{\theta: BV(\theta) = 0\}$ must be the same for all possible solution functions $BV$. In particular, the normalization $z(\theta) \equiv 1$ for all $\theta$ is without loss of generality with respect to the properties of $BV$ on $\{\theta^*\}$ and will be assumed hereafter for expositional convenience.

The following heuristic provides some intuition why the optimal control allocation and maturity value must be unique. The level of $L$ and the size of $D$ are substitutes with respect to the lender’s payoff, i.e., one can raise the expected debt payments either by increasing the interest rate on the debt or by enlarging the number of states in which the lender has control. Control rights are valuable because the two parties’ decision incentives are misaligned, as shown in Proposition 1. Since the lender’s break-even constraint binds by Lemma 1, one can evaluate the set of all possible solutions systematically by raising the maturity value $L$ and reducing the default set $D$ such that the lender’s expected payoff is always equal to the investment cost $I$. By Proposition 1, raising $L$ exacerbates the firm’s incentives but improves the lender’s, so reducing $D$ at the same time gives more control to the party whose incentives are deteriorating. This monotonicity ensures that there exists a unique point at which the rate of increase in the agency cost caused by the firm equals the rate of decrease in the agency cost caused by the lender.

Figure 5 below illustrates how the sets $D$ and $N$ are chosen optimally. The state space $\Theta = \{c_1, s\}$ is two-dimensional in this problem, and so the $c_1$-$s$-plane covers all possible contractible realizations of $\theta$. For each point in this plane, one can plot on the $z$-axis the inefficiency incurred if the firm or the lender, respectively, were in control of the liquidation decision. The distance between the resulting two surfaces at any point $\theta = (c_1, s)$ is $|q^{r} - q^{l}|$. The level set $\{\theta^*\}$ identifies the boundaries between region $N$, in which the inefficiency given by the firm’s surface is incurred, and region $D$, in which the inefficiency given by the lender’s surface is incurred. If the choice of $\{\theta^*\}$ were unconstrained, this boundary would optimally follow the intersection of the two surfaces, but the binding of
the lender’s break-even constraint generally prevents this outcome. Yet, \( \{ \theta^* \} \) is not arbitrary. Given that each \( \theta^* \) is a solution to \( \Gamma = 0 \), the multiplier

\[
\lambda - 1 = \frac{q^f(\theta^*) - q^l(\theta^*)}{u^f(\theta^*) - u^l(\theta^*)}
\]

must be constant across all \( \theta^* \). In economic terms, its value can be interpreted as the firm’s cost of reducing inefficiency. The intuition for this observation arises from the requirement that an optimal contract may not permit any rearrangement of \( D \) and \( N \) such that the firm’s payoff increases. In particular, if the ratio were not constant, there would exist states in \( D \) and \( N \) over which firm and lender could swap control rights such that the break-even constraint is maintained but inefficiency decreases.

![Figure 5](image)

**Figure 5.** Agency costs by state under control allocation to either the firm or the lender.

The preceding discussion has emphasized the role of tradeoffs between the parties’ respective decision rights and the maturity value of the debt \( L \). The latter reflects the interest rate the firm pays on the debt. Hence, the existence of a continuum of \( L \) and \( D \) along which the lender receives the same (break-even) payoff implies that the interest rate paid on a loan is by itself not indicative of the cost of the debt. While this caveat has
been raised in prior research, the uniqueness result in Proposition 3 adds an important qualification: in the presence of an agency conflict involving potential asset substitution, the tradeoff between \( L \) and \( D \) that minimizes the agency cost is not an arbitrary one. It is this agency cost, rather than the rate of interest, that constitutes the firm’s actual cost of debt.

4.4. Properties of Optimal Accounting Rules in Debt Contracts

It has yet to be determined whether precise statements are possible about how the input variables \( c_1 \) and \( s \) determine the properties of the optimal accounting rule \( BV \). In order to improve tractability when answering this question, the distribution \( h(\hat{s}|s) \) is hereafter assumed to have a monotone likelihood ratio, as stated in Assumption 3 below. The monotone likelihood ratio property operationalizes the intuitive idea that \( s \) and \( v \) should be positively correlated and ensures that higher realizations of \( s \) shift the posterior distribution of \( v \) to the right in a consistent manner. A monotone likelihood ratio implies first-order stochastic dominance but is more restrictive because it requires a monotonic shift in the density \( h(\hat{s}|s) \) on any subinterval of \([0, \bar{c}]\).

**Assumption 3** (Monotone likelihood ratio). The ratio \( l(\hat{s}|s) \equiv \frac{\partial h(\hat{s}|s)}{\partial s} \cdot \frac{1}{h(\hat{s}|s)} \) is increasing in \( v \).

The perhaps most elementary question about \( BV \) is whether book value is, in its optimal configuration, an increasing or decreasing function of the firm’s past cash flows \( c_1 \) and of the estimate \( s \) of the firm’s liquidation value. Taken together, Proposition 1 and Assumption 3 permit an unambiguous answer to this question. Under standard accounting practices, one might expect the firm’s book value to increase in both of these inputs, but as the following result shows, optimal accounting in the debt contract setting differs from this conjecture.
Proposition 4. In any critical state $\theta^*$, the optimal accounting rule $BV$ is increasing in past cash flows $c_1$ and decreasing in the liquidation value estimate $s$.

The rationale for the cash flow part of Proposition 3 follows directly from the dependence of the inefficiency ranges on $c_1$, as shown in Figure 4. Higher past cash flows lessen the firm’s incentive to seek inefficiently high risk and thus decrease its inefficiency range. At the same time, the lender’s incentive to seek inefficiently low risk is exacerbated. An efficient debt contract therefore assigns decision rights to the firm when realized cash flows are high and to the lender when realized cash flows are low, consistent with the observation that firms tend to violate covenants more often in practice if they performed poorly in the past.

That book value should be lowered when the firm’s estimated liquidation value $s$ increases may seem counterintuitive at first but follows economic logic. Liquidation is efficient when $v$ is high. Given Assumption 1, a high realization of $s$ then indicates that the probability that the firm will forgo efficient liquidation is larger than the probability that the lender will enforce inefficient liquidation because $k^f \geq m \geq k^l$ for all $\theta$ and $L$ by Proposition 1. In other words, technical default is more likely to result in an efficient decision for high $s$. One can thus view the firm’s book value as a reflection of the continuation value, or ‘value-in-use,’ of its assets, net of the opportunity cost of disposing of them via liquidation. The accounting treatment of $s$ is therefore consistent with the going concern presumption of standard financial reporting, augmented by the inclusion of opportunity costs in the computation of an asset’s carrying amount.\footnote{Book value would naturally be increasing in $s$ if $s$ were instead defined to be informative about $c_2$ rather than $v$.}

The monotonicity of $BV$ in both $c_1$ and $s$ on the level set $\{\theta^*\}$ implies that there exists a bijective mapping between the critical values $c_1^*$ and $s^*$, i.e., for each $c_1$, there exists a unique cutoff value $s^*$ above which the firm is in technical default, and for each $s$, there exists a unique cutoff value $c_1^*$ below which the firm is in technical default. The path of $\{\theta^*\}$ is therefore increasing monotonically in the $c_1$-$s$-space, consistent with the illustration in Figure 2. This monotonicity permits the construction of a useful alternative representation of the optimal accounting rule in the form of a \textit{separated solution}, i.e., $BV$ can
be written as a sum of two functions that only depend on either $c_1$ or $s$ but not both variables jointly.

**Corollary to Proposition 4.** The optimal accounting rule can be written as a separated solution

$$BV = bv_0 + bv_c(c_1) + bv_s(s)$$

where $bv_0$ is a constant, $bv_c(\cdot)$ is bounded and everywhere positive and increasing, and $bv_s(\cdot)$ is bounded and everywhere negative and decreasing.

The separated solution identified in the preceding corollary shows similarity to the familiar accrual accounting system, in which cash flows from past transactions are aggregated and adjustments in the form of accruals are made. The constant $bv_0$ can be interpreted as scheduled accruals that do not depend on the realization of cash flows or other economic inputs, e.g., depreciation, amortization, or accrued interest. Further, any differences between $bv_c(c_1)$ and $c_1$ or between $bv_s(s)$ and $s$ can be viewed as variable accruals computed based on realized economic inputs. For example, the difference between $bv_c$ and $c_1$ may reflect accruals related to units-of-production depreciation, warranty costs, or inventory obsolescence, and the difference between $bv_s(s)$ and $s$ may reflect impairments.

Notwithstanding the above interpretation of $BV$ in terms of regular financial accounting principles, the economic nature of accrual accounting in the debt contract setting differs from general-purpose financial reporting in one important respect. The objective of general-purpose financial reporting is the recognition of revenues and expenses in accordance with economic resource flows rather than cash flows, for example as reflected in the matching principle. In contrast, the accounting rule in the debt contract problem considered here produces accruals that reflect the firm’s contingent losses from asset substitution. In other words, high accrued expenses do not necessarily imply a high rate of usage of the firm’s invested assets during the period in question, but rather indicate a strong incentive for the firm to seek inefficiently high business risk.

Despite the different economic motivation for the recognition rules in the accounting process underlying $BV$, the book value $BV$ is built on the principles of historical cost and
fair value accounting familiar from regular financial reporting. For example, when $s$ is a perfect estimate of $\hat{s}$, one can readily verify that the separated solution becomes

$$BV = m - s$$  \hspace{1cm} (18)

where $E(v|m) = \mu_2$ as defined previously, and leads to a straightforward implementation of the first-best outcome because a perfect estimate of $\hat{s}$ effectively yields a complete contract. Given the result of Proposition 1, partitioning $\Theta$ at $s = m$ then removes all agency cost. The optimal accounting rule in this scenario can be viewed as an example of fair value accounting based on a current estimate $s$ of the firm’s asset values.

In the converse extreme case when $s$ is entirely uninformative about $\hat{s}$, the posterior distribution of $\hat{s}$ is equal to the prior distribution for all $s$, and so the first-order condition $\Gamma = 0$ from equation (16) is invariant under $s$. The separated solution of the optimal accounting rule is then reduced to

$$BV = bv_0 + c_1$$  \hspace{1cm} (19)

where $bv_0 \leq 0$.\(^{30}\) Given its independence of the liquidation value estimate $s$, this accounting rule can be viewed as a form of pure historical cost accounting because book value is solely a reflection of transactions concluded in the past. For all intermediate precision levels of $s$, the accounting rule $BV$ therefore always has characteristics of both fair value and historical cost accounting.

In terms of economic fundamentals, the solution in (18) for precise $s$ is solely calibrated toward the first-best benchmark rule $\hat{s} = m$ and is therefore independent of the potential agency cost reflected in the contracting parties’ inefficiency ranges $|k^f - m|$ and $|k^l - m|$. In contrast, the optimal accounting rule in (19) for uninformative $s$ is calibrated to partition the state space at the set of points where the probability-weighted inefficiency ranges are equal, i.e., the optimal $BV$ for uninformative $s$ is determined by the need to balance the agency cost. The solution for intermediate precision levels of $s$ must therefore reflect two optimality benchmarks that are in part determined independently.

\(^{30}\) One might conjecture that adding or omitting an uninformative $s$ from $BV$ is a matter of indifference because both parties are risk-neutral, but the control allocation induced by this $s$ would be uncorrelated with $\hat{s}$ and hence with the contracting parties’ incentives. Therefore, $s$ could at most not overturn a control allocation that is efficient ex ante based on knowledge of $c_1$, but it will do so in some cases and is hence optimally excluded from $BV$.  

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The accounting rules (18) and (19) are also related to the concepts of relevance and reliability of accounting information. The cash flow value \( c_1 \) is measured without error and therefore highly reliable because it represents past transactions that have already been concluded and whose payments have been settled, but it has no bearing on the first-best decision rule because the latter requires forward-looking information about \( \nu \) and \( c_2 \). Nonetheless, past cash flows are pertinent to the problem indirectly through their influence on the contracting parties’ incentives, as shown in Proposition 1. In contrast, the fair value estimate \( s \) is forward-looking and therefore inherently uncertain but highly relevant because it provides a direct estimate of the critical variable \( \hat{s} \) (and hence of \( \nu \)) needed to make the optimal liquidation decision. The relative sensitivity of \( BV \) to \( c_1 \) and \( s \) is thus a reflection of a relevance-reliability tradeoff.

A simple illustration of the relevance-reliability tradeoff arises in the special case when the sensitivities of \( \Gamma \) to changes in \( c_1 \) and to changes in \( s \) are in constant proportion

\[
- \frac{\partial \Gamma}{\partial c_1} \cdot \left( \frac{\partial \Gamma}{\partial s} \right)^{-1} = \gamma
\]

for all \( \theta^* \) and some constant \( \gamma > 0 \). In this case, the separated solution of the optimal accounting rule, as detailed in the proof of the corollary to Proposition 3, is reduced to the linear function

\[
BV = b\nu_0 + \frac{\gamma}{\gamma + 1} c_1 - \frac{1}{\gamma + 1} s
\]

Large values of \( \gamma \) imply that \( s \) is relatively uninformative and hence has little impact on \( \Gamma \), while changes in \( c_1 \) result in substantial shifts in agency costs. Then the coefficient on \( c_1 \) approaches 1 while the coefficient on \( s \) shrinks toward zero. Conversely, a low value of \( \gamma \) indicates that \( s \) is highly correlated with \( \nu \) while the agency cost implications of changes in \( c_1 \) are relatively minor. In this case, the coefficient on \( c_1 \) is reduced toward zero while the coefficient on \( s \) increases toward unity. A decrease in \( \gamma \) can thus be viewed as a shift from historical cost toward fair value accounting. Graphically, an increase in the information content of \( s \) can be represented as a clockwise rotation of the graph of \( \{\theta^*\} \) in the \( c_1-s \)-plane in Figure 2.

These observations about \( \gamma \) are consistent with the simple intuition that more informative input data should have a greater impact on the accounting measure. One would
therefore expect the accounting rules underlying debt covenants in industries with relatively more verifiable inputs to asset valuation to exhibit greater reliance on fair value measures, e.g., among financial services firms whose assets and liabilities are mainly in the form of securities with verifiable market prices. A highly informative $s$ permits a debt contract with nearly first-best efficiency, so it stands to reason that these firms would rely heavily on debt financing. Conversely, firms with proprietary intangibles that are difficult to value based on verifiable information are expected to deemphasize fair value measures in their debt covenants and use debt financing to a lesser degree.\textsuperscript{31}

5. **Contract Terms, Accounting Conservatism and Capital Structure**

The uniqueness of the optimal debt contract and hence of the tradeoff between interest rates and control rights raises a number of interesting questions. For example, what firm characteristics determine whether this tradeoff involves low interest rates and extensive decision rights for the lender, or the reverse? When is conservative accounting an optimal design choice for $BV$? What firm characteristics make debt financing more or less costly, in the sense of contractual efficiency? This section investigates the tradeoffs involved in the choice of $L$ and $BV$, the determinants of the optimal degree of accounting conservatism in $BV$, and the resulting implications for the firm’s capital structure and the efficiency of the contract. In order to facilitate the discussion, the following definition of accounting conservatism will be adopted.

**Definition 2** (Accounting conservatism). *An accounting rule $BV$ is said to be more conservative than an accounting rule $BV'$ if $BV(\theta) \leq BV_0$ whenever $BV'(\theta) = BV_0$ in at least some settings, and never $BV(\theta) \geq BV_0$ whenever $BV'(\theta) = BV_0$.\textsuperscript{32}*

\textsuperscript{31} The decrease in the usefulness of accounting measures that include imprecise fair value estimates seems consistent with the empirical evidence in Demerjian (2011), who documents a decline in the use of balance sheet-based debt covenants, concurrent with standard setters’ efforts to expand the application of fair value accounting to assets and liabilities whose values are inherently difficult to estimate.

\textsuperscript{32} A ‘setting’ refers to a given set of distributional parameters for $c_1$, $c_2$, $v$, $\delta$ and $s$. 

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More conservative accounting according to Definition 2 means an increased incidence of low book values $BV$, consistent with the general notion that conservatism means a greater tendency to recognize losses than to recognize gains.\textsuperscript{33} Definition 2 is intended to be descriptive of a fundamental and empirically observed effect of conservatism, namely, low book values of equity. A covenant based on more conservative accounting rules is thus more easily violated and hence assigns more control rights to the lender. In comparison to other models of accounting conservatism, e.g., in Gigler et al. (2009) or in Göx and Wagenhofer (2010), this definition does not require any specific changes to the informational properties of $BV$. It should be emphasized that Definition 2 does not impose any restrictions or assumptions on $BV$ but merely establishes terminology to characterize and compare different $BV$.

5.1. Financial Leverage and Profitability

It will be convenient to begin the analysis with the investment cost parameter $I$. The firm so far has been assumed to raise $I$ entirely in the form of debt, but firms in practice finance their capital needs by a variety of combinations of debt and equity. The following discussion will therefore adopt the representation

$$I = I^X + I^U$$  \hspace{1cm} (20)

of the firm’s capital structure immediately following the conclusion of the debt contract, where $I^X$ is the amount of paid-in equity capital and $I^U$ is the principal amount of the debt. The analysis so far has assumed that $I^U = I$ and $I^X = 0$, but replacing $I$ by $I^U$ in the break-even constraint (13) would not alter the structure of the problem because an equity investment of $I^X > 0$ does not give rise to any additional constraints or alter the problem in other ways. The debt financing amount $I^U$ can thus serve as a measure of financial leverage, which yields the following result.

\textsuperscript{33} It is not generally possible to meet the stricter definition of a decrease in $BV$ at all $\theta^*$ in all possible settings unless $\{\theta^*\}$ is a singleton. The latter case can only arise if the state space is one-dimensional, e.g., as in Sridhar & Magee (1997). A sufficient condition under which the stricter definition can be met even if $\Theta$ is not one-dimensional is $\frac{\partial^2 y}{\partial BV a l} \propto \frac{\partial^2 y}{\partial BV c}$ on $\{\theta^*\}$, i.e., the effects of $BV$ on the first-order necessary conditions with respect to $L$ and $\lambda$ are proportional across $\theta^*$. 

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Proposition 5. For firms with higher leverage, optimal debt covenants are based on more conservative accounting rules and higher maturity values.

Proposition 5 appears to be consistent with empirical findings. For example, firms with higher leverage have been found to recognize losses in a timelier manner (Ahmed et al., 2002; Nikolaev, 2010), use restrictive covenants more frequently (Nikolaev, 2010) and experience more frequent covenant renegotiations (Nikolaev, 2012). It is, however, important to note that this result has nothing to do with the informational properties of conservative accounting practices, such as providing an ‘early warning’ to lenders. Rather, an increase in the amount of outstanding debt means a greater likelihood that the firm will ultimately default and the lender receives less than the full maturity value $L$. The debt is therefore more at risk when $I^u$ increases and requires a higher maturity value $L$ to compensate the lender. By Proposition 1, the lender’s inefficiency range shrinks as a result, so that contract efficiency improves if the lender receives additional decision rights at the same time.

The result in Proposition 5 is stated in terms of financial leverage and hence implicitly assumes that an increase in $I^u$ is offset by a decrease in $I^x$ while the total investment cost $I$ is held constant. Yet, one can readily observe that an increase in $I^u$ might alternatively have been the result of an increase in $I$ that the firm decided to finance, at least partly, by raising additional debt. In contrast to the leverage interpretation, this scenario implies a decrease in the firm’s profitability rather than a mere shift in capital structure. To illustrate this claim, one can note that the expected return on the firm’s investment over the two contract periods can be written as

$$\frac{W - Q}{l} = \frac{E_\theta(w - q)}{l}$$

where $W \equiv E_\theta(w)$ is the expected first-best firm value, $Q \equiv E_\theta(q)$ is the expected inefficiency, $w$ is the first-best firm value for a given $\theta$ as defined in (10), and $q$ is the inefficiency as defined in (8). When $I$ increases, the expected return declines by

$$\frac{d}{dl} \left( \frac{W - Q}{I} \right) = -\frac{W - Q - (1 - \lambda) \cdot I}{I^2} < -\frac{W - Q - I}{I^2} < 0$$
where the first inequality follows from Lemma 1 and the final inequality from the requirement that the investment has positive net present value.\textsuperscript{34} Mathematically, this alternative interpretation of an increase in $I^U$ does not alter the problem. Proposition 5 thus has a natural corollary.

**Corollary to Proposition 5.** For firms with lower profitability, optimal debt covenants are based on more conservative accounting rules and higher maturity values.

The intuition for why the two seemingly unrelated interpretations of $I^U$, as either a measure of profitability or as a measure of financial leverage, have the same effects on the terms of the optimal debt contract is as follows. In both cases, the firm’s payment obligation to the lender, and thus its incentive to make decisions that yield inefficiently high risk, have increased relative to total firm value. In response, an efficient contract shifts decision rights to the lender, who now has relatively less detrimental incentives. This observation suggests that conservatism in accounting, as defined in this context, is neither unconditionally beneficial nor unconditionally disadvantageous. Rather, its optimal degree is determined by the economic situation of the firm and its capital structure. A direct empirical implication is that firms in competitive industries with low profit margins should tend to follow more conservative accounting practices under their debt covenants.

Given the possible leverage interpretation of $I^U$, it may seem natural to make $I^U$ and $I^K$ choice variables of the firm, but as noted in the introduction, an endogenous capital structure would require a full model of capital costs across all types of financing options. Nonetheless, the model can make a limited contribution to solving the firm’s capital structure problem in the following sense. In the frictionless setting of Modigliani and Miller (1958), the firm should not be able to improve its return, net of financing costs, by changing its capital structure, and so the firm’s return less its cost of equity financing would be the same regardless of $I^U$. Then if one were to introduce the agency cost from

\footnote{If $\frac{dQ}{dI} < -1$, the firm would voluntarily overpay the lender because the resulting reduction in inefficiency would exceed the amount of the overpayment, but Lemma 1 shows that this situation can never arise in an optimal contract.}
asset substitution as the sole friction, ceteris paribus, the firm’s owners would choose \( I^U \)
to maximize their net return

\[
Y - I^X = W - Q + (\lambda - 1) \cdot (U - I) + I^U
\]  
(21)

where the equality follows from (20).

Given the lender’s binding break-even constraint, maximizing (21) is equivalent to maximizing total firm value. The corresponding first-order condition would require that

\[
\frac{dY}{dI^U} + 1 = -\lambda + 1
\]
is zero at an optimal interior value of \( I^U \), where the equality follows from the envelope theorem. As noted in the Introduction, the corner solution \( I^U = 0 \) would attain the first-best outcome because the agency conflict is eliminated under pure equity financing. But if \( I^U \) were bounded away from zero for exogenous reasons and \( \lambda \) evolved in a non-monotonic manner, one might conjecture that this stylized capital structure problem might also have a local interior maximum under some contract for which \( \lambda = 1 \). The following result demonstrates that this is not the case.

**Proposition 6.** Firm value is convex in leverage.

Proposition 6 seems to stand in contrast to the common perception that a higher degree of debt financing exacerbates the agency conflict between firm and lender (Green & Talmor, 1986; Décamps & Faure-Grimaud, 2002). The reason for the disparity lies in the introduction of the debt covenant. Absent state-contingent control allocation, the lender’s break-even constraint can only be maintained for higher amounts of debt financing \( I^U \) by raising the maturity value \( L \), which exacerbates the firm’s decision incentives monotonically, as shown in Proposition 1. Including a covenant in the contract permits a gradual transfer of decision rights to the lender, whose incentives improve as \( L \) increases. The agency conflict then naturally disappears in the corner cases when all decision rights and payoff claims are vested with only one party. Intuitively, the reason why intermediate levels of leverage are inefficient is that they require the most costly compromises between the contracting parties’ conflicting decision incentives.
Proposition 6 implies that a maximization of firm value or a minimization of inefficiency would yield a corner solution in the simplified setting considered here. Then even if the trivial optimum $I_U = 0$ is infeasible or undesirable for exogenous reasons, one would still expect firms, ceteris paribus, either to minimize their debt as much as possible or to seek the highest possible amount of debt. Indeed, Propositions 5 and 6 are consistent with the finding by Rajan & Zingales (1995) that firms with higher profitability tend to have lower leverage. Since profitability and leverage have additively inverse effects on contract efficiency, the opposing directional alignment observed in practice may in part be intended to reduce agency costs. One should, however, bear in mind that the convexity result is obtained in a setting in which asset substitution is the only financing cost that varies with leverage, while firms’ leverage choices in practice are likely guided by a number of other factors.

![Figure 6](image-url)  
**Figure 6.** Debt and equity values as a function of leverage.

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35 For example, a corner solution need not coincide with the extreme value $I_U = 0$ if the firm only has limited access to equity capital, in which case $I_U$ is bounded below. On the opposite extreme, $I_U$ may be bounded above because the firm can only make a credible commitment to pay out some maximum amount to an outside financier, e.g., because of the opportunity to consume private benefits at the financier’s expense.
Figure 6 provides a graphical illustration of Proposition 6 by plotting the values of debt and equity as functions of $I^U$. The first-best firm value $W$ provides an upper bound on actual firm value and is independent of the amount of debt financing $I^U$, consistent with the capital structure irrelevance principle. The lender’s payoff $U \equiv E_\theta(u)$ increases one-for-one with $I^U$ because the break-even constraint $U \geq I^U$ binds under any optimal contract by Lemma 1. Hence, the value of the firm’s equity $X \equiv E_\theta(x)$ is bounded above by $W - I^U$. One can readily observe that even if the firm’s choice of $I^U$ were constrained to interior values for exogenous reasons, the value of $I^U$ that minimizes inefficiency would always coincide with one of the endpoints of the feasible interval.\(^{36}\)

The slope of $X$ in Figure 6 is $-\lambda$ by the envelope theorem. The convexity result therefore implies that the Lagrange multiplier $\lambda$ can serve as an index of leverage and profitability, i.e., firms with higher $\lambda$ are more profitable and have lower financial leverage. In order to obtain more precise analytical statements, it will be convenient to examine the ‘median’ case $\lambda = 1$ in subsequent results.\(^{37}\) A debt contract with $\lambda = 1$ will be referred to as representative. All subsequent results are qualitatively valid for cases $\lambda \neq 1$, but clean analytical statements would require additional technical assumptions on $G$ and $H$ without adding economic insight.

5.2. Debt Cost and Tradeoffs between Interest Rates and Control Rights

The preceding discussion still leaves unanswered the question how tradeoffs between control rights and interest rates are determined. Proposition 5 establishes that an increase in $I^U$ must coincide with both an increase in the degree of conservatism of $BV$ and an increase in the maturity value $L$.\(^{38}\) Hence, differences in leverage or profitability do not im-

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\(^{36}\) Naturally, $I^U$ is bounded above by the total investment cost $I$, and so the corner solution $I^U = W$ is only feasible in case of a zero-NPV investment, in which case the lender would receive all payoffs in all states and effectively become a full equity-owner.

\(^{37}\) One can see from equation (17) that $\lambda = 1$ is indeed the median case if, among all possible optimal debt contracts that could arise in reality, $\Pr(q > q\forall \theta') = \Pr(q < q\forall \theta')$. In the case $\lambda = 1$, the optimal partitioning line in the $c_s$-$s$-space follows the intersection of the two surfaces in Figure 5.

\(^{38}\) Strictly speaking, the interest rate is reflected in the ratio $\frac{L}{I^U}$ rather than just the maturity value $L$, but this distinction does not affect the conclusions. In a first-best scenario, the break-even constraint $U = I^U$ implies that $\frac{dL}{dI^U} = \frac{1}{1 - \Pr(L)}$, where $\Pr(L)$ is the probability that total payoffs are less than $L$. The interest rate
ply that firms trade more conservative accounting rules for a lower interest rate (or the reverse). Rather, leverage and profitability play an important role as covariates of the optimal contract terms $BV$ and $L$. In other words, differences in interest rates and accounting conservatism between two firms do not yield meaningful insights about tradeoffs unless leverage and profitability are held constant. The following definition formalizes the notion of controlling for leverage and profitability, which will be useful for subsequent results.

**Definition 3.** Let $\beta$ denote an arbitrary parameter, let $\nabla Y$ denote the gradient of $Y$ with respect to $\lambda$, $L$ and $BV$, and let $\nabla_i Y$ denote its partial derivative with respect to $i$. A change in the optimal contract terms $\{L, D\}$ induced by $\left(\nabla_\beta Y - a \nabla_i Y\right)$ is said to be controlled for leverage and profitability if

$$\left(\nabla_\beta Y - a \nabla_i Y\right) \perp a \nabla_i Y$$

for some $a \in \mathbb{R}$.

Definition 3 addresses the question: what part of an observed difference between the contract terms of two firms can be explained by differences in their leverage and profitability? Mechanically, this means adjusting $I^U$ for one firm until any remaining differences in $BV$ and $L$ between the two firms are orthogonal to further changes in $I^U$, i.e., constructing an orthogonal decomposition of the difference. This approach is analogous to the use of control variables in a linear regression and has the important empirical implication that testing the relationship between accounting conservatism and interest rates may not yield meaningful results unless the role of the firm’s capital structure and profitability as covariates of the optimal contract terms is reflected in the research design.

The intuition for Definition 3 is best illustrated by example. First, suppose that the generic parameter $\beta$ in the definition were to do nothing but to scale all cash flows and liquidation proceeds by the same constant. The effect of an increase in $\beta$ on the optimal contract terms $BV$ and $L$ would then be exactly proportional to the effect of a decrease in $I^U$ then increases monotonically by $\frac{d}{d I^U} \left(\frac{L}{I^U}\right) = \frac{1}{I^U} \cdot \left(\frac{dL}{dI^U} - \frac{L}{I^U}\right) > 0$, where the inequality follows from the observation that $(1 - Pr(L)) \cdot L < I^U$. The second-best setting analyzed in this thesis is similar, although the interest rate need not increase monotonically at all levels of $I^U$. 

54
In other words, the implications of changing $\beta$ for $BV$ and $L$ could be fully explained by Proposition 5, i.e., changes in leverage and profitability. In this case, $\beta$ has no effect on the tradeoff between control rights and interest rates. It is instructive to contrast this simple scenario with the following case.

**Definition 4.** A parameter $\eta \in \mathbb{R}$ provides an ordering of the firm’s liquidation value if the likelihood ratio $j(\hat{s}|s) \equiv \frac{\partial h(\hat{s}|s)}{\partial \eta} \cdot \frac{1}{h(\hat{s}|s)}$ is increasing in $\hat{s}$ and decreasing in $s$ for all $\hat{s}$, $s$ and $\eta$.

Similar to Assumption 3, the definition of $\eta$ utilizes the monotone likelihood ratio property to operationalize the notion of an increase in $\hat{s}$. Firms with higher $\eta$ have higher expected liquidation values, while the distribution of the continuation value $c_2$ is held fixed. In contrast to the previous example with $\beta$, $\eta$ has two distinct effects on $BV$ and $L$ in this case. First, $\eta$ increases firm value because the liquidation option has become more valuable, which implies that the ratio of $I^U$ to total firm value has changed. This effect can be explained by the result in Proposition 5. Second, $\eta$ shifts the relative weights of cash flows and the liquidation value $v$ in the contracting parties’ payoff functions in favor of $v$. If one were interested in inferring the implications of the second effect on the tradeoff between $BV$ and $L$, a comparison of the firm’s optimal debt contract terms before and after an increase in $\eta$ would not be ceteris paribus because leverage and profitability are no longer the same. By applying Definition 3, one can separate the two effects to obtain the following result.

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39 Scaling all contract terms and payoff variables by a constant $\beta$ changes the debt value by the same factor to $\beta I^U$ because decision incentives are unaltered. The scaled contract would thus be optimal if $I^U$ were also scaled by $\beta$ so that the break-even constraint still holds. Hence, the optimal contract terms are linear in the ratio of $\beta$ to $I^U$.

40 The likelihood ratio of $\eta$ is assumed to decrease in $s$ because, as one can verify graphically, a likelihood ratio $j$ that is constant or increasing in $s$ is difficult to sustain: if $s$ increases, $h(\hat{s}|s)$ is shifted to the right, and hence the likelihood ratio in $\eta$ must generally be negative for a larger range of $\hat{s}$.

41 One should note that the contracting parties’ decision incentives are invariant under $\eta$ because $\hat{s}$ is realized before the decision whether to liquidate the firm or not is made. This would not apply if $\eta$ were instead defined with respect to $G(\hat{s}|s)$ or the continuation value $c_2$, but subject to a technical assumption on $h(\hat{s}|s)$, the implications are the same.
Proposition 7. If leverage and profitability are controlled, the optimal representative
debt contract for firms with higher liquidation values is based on lower interest rates and
more conservative accounting rules.

The intuition for Proposition 7 follows from the incentive misalignment laid out in
Proposition 1. As \( \eta \) increases, liquidation becomes the first-best decision in more states
of the world. Decision rights are optimally vested with the party that most likely prefers
the first-best decision, and hence the allocation of control rights to the lender is expanded,
i.e., the optimal accounting rule is more conservative. A second, less apparent but im-
portant factor is that lowering \( L \) becomes more beneficial with higher \( \eta \) because the lend-
er’s inefficient liquidation decisions now occur in states with a lower ex ante probability.
Despite the intuitive rationale, one should note the caveat that Proposition 7 is stated for
the ‘representative’ contract, i.e., \( \lambda = 1 \). As noted in the previous section, the reason is
mainly analytical convenience, but without further technical assumptions, it is possible
that the marginal benefit of raising \( L \) might increase in \( \eta \) for sufficiently large or small \( \lambda \)
and thus create a countervailing effect.\(^42\) One should therefore bear in mind that the pre-
dictions identified in Proposition 7 may not be observable in firms around the tails of the
leverage and profitability distributions.

In practice, one would expect firms with high values of \( \eta \) to hold assets with a high
degree of fungibility, which would be valuable to a broad set of potential outside buyers
in liquidation and be well-suited as loan collateral. Tangible assets, such as plant, proper-
ty and equipment, are likely to fall into this category, and hence firms with high \( \eta \) likely
operate in industries that require extensive investment in these assets. Proposition 7 is
thus consistent with the notion that collateralized debt carries a lower interest rate. By
contrast, firms with low \( \eta \) are likely to derive more of their value from assets that are less
likely to have much value in the hands of a different owner. Firm-specific intangibles
such as human capital or a well-functioning organization are likely to fall into this cate-
gory. One would therefore expect the latter firms to be more prevalent in industries char-

\(^{42}\) For very high \( \lambda \), the firm has efficient incentives in most states and bankruptcy is highly unlikely. Rais-
ing \( L \) would incur little additional inefficiency in this case but create substantial slack in the break-even
constraint, so that additional control allocation to the firm can increase efficiency. Raising \( \eta \) increases this
marginal benefit. A symmetric argument applies when \( \lambda \) is close to zero.
acterized by a high intensity of research and development activity, product innovation, and intellectual capital. The finding by Nikolaev (2012) that firms with a lower book-to-market ratio and a higher intensity of research and development spending have a lower likelihood of renegotiating their debt covenants is thus consistent with Proposition 7.43

Proposition 7 calls for the natural follow-up investigation whether $\eta$ has any predictable impact on the efficiency of the contract, i.e., on the firm’s cost of debt. The impact on contract efficiency of an increase in $\eta$, controlled for leverage and profitability, is

$$
- \frac{\partial Q}{\partial \eta} = \int_N \int_m^{k_f} (E(v|\hat{s}) - \mu_2) \cdot j(\hat{s}|s) \, dH \, dF \\
+ \int_D \int_k^m (\mu_2 - E(v|\hat{s})) \cdot j(\hat{s}|s) \, dH \, dF
$$

(22)

where $j(\cdot)$ denotes the likelihood ratio as given in the definition of $\eta$.44 One may conjecture that firms with more valuable liquidation options enter into more efficient debt contracts because they can pledge collateral of higher expected value and pay a lower rate of interest. Yet, inspection of (22) reveals no particular directional bias. For small $I^U$, the set $N$ is large and the firm’s inefficiency range $|k^f - m|$ is small, while the technical default set $D$ is small and the lender’s inefficiency range $|k^l - m|$ is large. The converse applies to large $I^U$. Further, the monotonic increase in $j(\cdot)$ suggests that the first term in (22) is generally positive and the second term is generally negative. Taken together, these observations do not permit the conclusion that (22) is systematically different from zero, which implies that firms with a high proportion of tangible assets and the ability to post loan collateral do not enter into more efficient debt contracts, even though Proposition 7 predicts such firms to pay a lower rate of interest on their debt. This conclusion again emphasizes that the interest rate level is by itself not indicative of the firm’s actual cost of debt.

A similarly cautionary commentary also applies to accounting conservatism. Proposition 7, like Proposition 5 and its corollary, identifies circumstances under which conservatism is part of an optimal debt contract. However, the optimality of conservatism is nei-

43 Renegotiations are not necessarily preceded by a covenant violation but tend to coincide with either a violation or financial results that are likely to trigger a violation in the near future.

44 The total change in inefficiency is $\frac{\partial \theta}{\partial \eta} = \frac{\partial \theta}{\partial \eta} + (1 - \lambda) \cdot \frac{\partial \mu}{\partial \eta}$ by the envelope theorem. Controlling for leverage and profitability in the sense of Definition 4 eliminates the second term.
ther unconditional nor does the inclusion of conservative accounting rules imply greater contractual efficiency and hence a lower cost of debt. Proposition 7 thus provides an illustration why the common empirical finding that firms applying more conservative accounting practices pay lower interest rates on their debt (Ahmed et al., 2002; Zhang, 2008; Sunder, Sunder & Zhang, 2011) is moot with respect to the benefits of conservative accounting.

Proposition 7 effectively examines the implications of a shift in the mean of the liquidation value relative to expected cash flows. A natural complement to this result is an analysis of changes in the variance of payoffs. The following discussion will focus on the firm’s operating risk, defined as the variance of the its second-period cash flows $c_2$.

Higher operating risk is modeled as a mean-preserving spread of the distribution of $c_2$.

**Definition 5 (Operating risk).** A parameter $\sigma \in \mathbb{R}$ provides an ordering of the firm’s operating risk if

$$
\int_0^t \frac{\partial F_2(c_2)}{\partial \sigma} dc_2 \geq 0
$$

for any $t \in [0, \bar{c}]$, with equality when $t = \bar{c}$.

Mean preservation implies that $\sigma$ has no bearing on the first-best decision rule because $\frac{\partial \mu_2}{\partial \sigma} = 0$ by construction, and hence the first-best firm value $W$ is the same for all $\sigma$. However, this does not imply that firm value is invariant under $\sigma$ because operating risk affects the contracting parties’ decision incentives. In particular, the firm adjusts its liquidation threshold upward by

$$
\frac{\partial k^f}{\partial \sigma} = \frac{\partial x_2}{\partial \sigma} \cdot \left(\frac{\partial u_\psi}{\partial \delta}\right)^{-1} = -\int_{L-c_1}^{\bar{c}} \frac{\partial F_2}{\partial \sigma} dc_2 \cdot \left(\frac{\partial u_\psi}{\partial \delta}\right)^{-1} \geq 0 \tag{23}
$$

while the lender adjusts its liquidation threshold downward by

$$
\frac{\partial k^l}{\partial \sigma} = \frac{\partial u_2}{\partial \sigma} \cdot \left(\frac{\partial u_\psi}{\partial \delta}\right)^{-1} = -\int_0^{L-c_1} \frac{\partial F_2}{\partial \sigma} dc_2 \cdot \left(\frac{\partial u_\psi}{\partial \delta}\right)^{-1} \leq 0 \tag{24}
$$

in response to an increase in $\sigma$. This observation is consistent with the rationale that the firm benefits from inefficiently high risk to begin with, so that an increase in $\sigma$ should reinforce its incentives further. On the other hand, higher $\sigma$ also reinforces the lender’s
incentive to avoid risk. Taken together, (23) and (24) and the constant first-best cutoff value \( m \) suggest that efficiency decreases with \( \sigma \) because \( k^l \leq m \leq k^f \) by Proposition 1, i.e., both parties’ liquidation thresholds move away from the efficient benchmark. The next result formalizes this reasoning.

**Proposition 8.** *Higher operating risk reduces the efficiency of the representative debt contract.*

The decrease in efficiency implies that debt financing is costly for firms with high operating risk. Ceteris paribus, one would therefore expect a lesser prevalence of debt financing in volatile industries and among firm with high-risk business models. Two qualifications about the implications of Proposition 8 are in order. First, a higher \( \sigma \) exacerbates the agency conflict not because it increases the total risk of the business but because it increases the risk differential between the low-risk liquidation option and the high-risk continuation option. To appreciate this point, one may consider an alternative definition of \( \sigma \) by which the variance of \( v \) conditional on \( \hat{s} \) increases in \( \sigma \). One can readily verify that the signs in (23) and (24) would be reversed in this case, i.e., the parties’ decision incentives would move closer to the first-best value.\(^{45}\)

The second observation concerns the capital structure effects of \( \sigma \). Proposition 8 presents the convenient case of the ‘representative’ debt contract with \( \lambda = 1 \), which, as discussed earlier, reflects a medium level of leverage and profitability. It is, however, instructive to note how \( \lambda \) affects the outcome. Ceteris paribus, an increase operating risk not only reduces contract efficiency but also leads to a wealth transfer from the lender to the firm, i.e., the lender’s expected payoff \( U \) decreases. Since the break-even constraint \( U = I^U \) must hold, this wealth transfer requires a higher rate of interest and a more conservative accounting rule in order to compensate the lender, in line with Proposition 5.\(^{46}\) Given the convexity result in Proposition 6, this adjustment to the contract terms leads to

\(^{45}\) An increase in the variance of \( v \) conditional on \( \hat{s} \) would still likely reduce overall contract efficiency because a higher conditional variance would imply a reduction in the information content of \( \hat{s} \). However, this aspect is not germane to debt financing as less precise information would reduce the efficiency of *any* form of financing.

\(^{46}\) The wealth transfer effect would be the same even if \( \sigma \) were defined with respect to \( v \).
an additional decrease in efficiency for firms with high \( \lambda \) (i.e., low leverage and high profit rates), while the efficiency loss is mitigated in firms with low \( \lambda \). Hence, not only is debt more costly for firms with high operating risk, but this cost increase is more pronounced in highly profitable firms with low leverage. Figure 7 provides a graphical illustration of the decrease in the firm’s equity value \( X \) after an increase in \( \sigma \) as a function of \( I^U \).

**Figure 7.** Firm value as a function of operating risk.

A remaining question is how operating risk affects the optimal accounting rule \( BV \). One should first observe that the symmetry in the response of the contracting parties’ decision incentives to changes in \( \sigma \) implies that \( BV \) should, on average, be unaffected by \( \sigma \) if leverage and profitability are controlled for in the sense of Definition 3. In other words, predictable changes in \( BV \) only arise because \( \sigma \) reduces profitability and increases leverage. As noted already, this effect implies an increase in the degree of conservatism of \( BV \). One might therefore be inclined to predict firms with high operating risk to apply more conservative accounting practices under their debt covenants. However, it is important to bear in mind that financial leverage is a choice variable, and firms with high operating
risk may find it optimal to set their amount of debt financing at a low level, which would imply a low optimal level of conservatism.

6. Results with Project Selection

The firm’s investment opportunity has been assumed to be fixed and exogenous to this point, but in practice, businesses tend to face a range of possible investment opportunities, or the investment in question can be implemented with various marketing strategies, product designs, degrees of operating leverage etc. These project selection choices likely yield different levels of risk and return. Ex ante, both firm and lender would prefer to implement the project with the highest expected return. However, if the project choice is not contractible but must be made at the discretion of the firm after the contract has been signed, the firm may have an incentive ex-post to select a project with low return but high risk. The following definition operationalizes this idea.

**Definition 6.** A parameter \( \varphi \in \mathbb{R} \) implements a risk-return ordering of investment opportunities if

\[
\int_0^t \frac{\partial c_2}{\partial \varphi} \, dc_2 \geq 0
\]

for any \( t \in [0, \bar{c}] \).

The risk-return index \( \varphi \) establishes an ordering of cash flows in \( t = 2 \) by second-order stochastic dominance. Higher values of \( \varphi \) thus identify projects of higher variance and (weakly) lower return, i.e., \( \frac{\partial \mu_2}{\partial \varphi} \leq 0 \), with equality in the limit case when \( \varphi \) is a mean-preserving spread. A project choice is therefore efficient if the investment with the lowest available \( \varphi \) is implemented. Yet, the firm’s equity value \( X \) may increase in \( \varphi \) if

\[
\frac{\partial X}{\partial \varphi} = \int_N H(k^f) \cdot \frac{\partial x_2}{\partial \varphi} \, dF
\]

\[
+ \int_D \left( H(k^l) \cdot \frac{\partial x_2}{\partial \varphi} + \left( \mu_2 - E(v|k^l) \right) \cdot h(k^l) \cdot \frac{\partial k^l}{\partial \varphi} \right) \, dF > 0
\]

\[ (25) \]
because
\[
\frac{\partial x_2}{\partial \varphi} = - \int_{L-c_1}^{c} \frac{\partial F_2}{\partial \varphi} \, dc_2
\]
is positive if \( L \) is sufficiently high or the effect of higher \( \varphi \) is sufficiently close to a mean-preserving spread. An immediate implication of this observation is that the firm’s liquidation threshold \( k' \) increases in \( \varphi \) when (25) holds. In contrast, \( \frac{\partial u_2}{\partial \varphi} < 0 \) and so \( \frac{\partial k'}{\partial \varphi} < 0 \). In order for (25) to hold, the increase in \( x_2 \) across \( \theta \) must therefore be large enough to outweigh the firm’s loss incurred by exacerbating the lender’s decision incentives. In particular, the firm will implement an inefficient project if \( \frac{\partial x}{\partial \varphi} > 0 \) at the lowest, first-best level of \( \varphi \), normalized to \( \varphi = 0 \) hereafter. Then if implementing the project with \( \varphi = 0 \) is desired, the firm’s optimization program includes an additional project selection (or incentive compatibility) constraint and thus takes the form
\[
\max_{BV, L, \lambda, \kappa} \left( X + \lambda \cdot (U - I) + \kappa \cdot (X_{|\varphi=0} - X_{|\varphi>0}) \right)
\]  
(26)

It is instructive to examine the solution to (26) graphically. Consider the set of all pairs \((L, BV)\) of maturity values and book value rules for the first-best project such that the lender’s break-even constraint is met and \( BV \) is optimal conditional on \( L \). Proposition 3 states that, given the original optimization program (12) without the project selection constraint, this set contains a unique optimal maturity value and a unique optimal allocation of control rights, implemented by \( BV \).\footnote{As noted previously, the optimal accounting rule \( BV = z(\theta) \cdot \Gamma \) has been normalized by setting \( z(\theta) \equiv 1 \) for all \( \theta \).} The project selection constraint truncates the set by eliminating some conditionally optimal contracts under which the firm would have an incentive to implement a project other than the first-best one. The objective of the following analysis is to characterize this truncation and determine the implications for the accounting rules \( BV \) can be implemented under (25) relative to the original problem in (12).

In order to visualize this approach, it will be useful to construct a mapping \( BV \rightarrow \psi \), where \( \psi \in \mathbb{R} \), such that i) for any \( \psi > \psi' \), \( D(\psi) \subset D(\psi') \), i.e., \( BV \) increases such that the partitioning frontier in Figure (2) moves upward uniformly, and ii) each \( \psi \) identifies a \( BV \) that is conditionally optimal for some \( L \). One should observe that \( \psi' \) is more con-
servative than $\psi$. As noted previously, the optimal accounting rule $BV = z(\theta) \cdot \Gamma$ has been normalized, without loss of generality, by setting $z(\theta) \equiv 1$ for all $\theta$ and is therefore unique for each $L$. One can now graph the set of conditionally optimal contracts

$$COC \equiv \{(L, \psi): BV(\psi) = \arg \max_{BV \lambda} (X + \lambda \cdot (U - I)) \mid_{L} \}$$

as a continuum of pairs $(L, \psi)$ in $\mathbb{R}^2$, as shown in Figure 8. The proof of Proposition 9 below demonstrates that the mapping $L \to \psi$ must be monotonically increasing.

![Figure 8](https://example.com/figure8.png)

**Figure 8.** Feasible contracts in a project selection problem.

In the presence of the project selection constraint in (26), contracts from $COC$ at which the firm would prefer to select an inefficient project instead of the first-best one are not implementable. In order to identify which subset of $COC$ is eliminated by the project selection constraint, one can examine the set of contracts under which the firm has a constant incentive to increase $\varphi$, i.e., contracts under which $\frac{\partial x}{\partial \varphi}$ is constant. Since $\frac{\partial x}{\partial \varphi} > 0$ at the lowest $\varphi$ implies that the firm will select an inefficient project, the frontier beyond which the first-best project can be implemented is given by the set of contracts $PS \equiv \left\{(L, \psi): \frac{\partial x}{\partial \varphi} \bigg|_{\varphi = 0} = 0 \right\}$. Figure 8 provides an illustration. The following discussion as-

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sumes that it is optimal to implement the project with the lowest \( \varphi \), so that the optimization problem can be written as

\[
\max_{BV, \lambda, \kappa} \left( X + \lambda \cdot (U - I) - \kappa \cdot \left. \frac{\partial X}{\partial \varphi} \right|_{\varphi=0} \right)
\]

(27)

if the firm can choose \( \varphi \) from a continuum.\(^{48}\)

It is important to note that the firm does not consider the lender’s break-even constraint when choosing \( \varphi \) because the contract has already been concluded. In characterizing the set \( PS \), one can therefore restrict attention to the effects of \( L \) and \( \psi \) on the firm’s equity value \( X \) alone. Specifically, a control transfer to the lender reduces the firm’s benefit from increasing \( \varphi \) by

\[
\frac{\partial^2 X}{\partial \varphi \partial L} = \left( \mu_2 - E(v|k^l) \right) \cdot h(k^l) \cdot \frac{\partial k^l}{\partial \varphi} \cdot \int_k \frac{\partial x_2}{\partial \varphi} dH < 0
\]

(28)
in any given state \( \theta \). This outcome is sensible because an increase in risk increases the lender’s incentive to choose the low-risk liquidation option. The firm can therefore only benefit from inefficiently high risk if it has control and can enforce the high-risk continuation option. Ceteris paribus, the firm therefore chooses a project with lower \( \varphi \) if it has more control rights, i.e., for higher \( BV \) and thus higher \( \psi \).

In contrast, raising the maturity value \( L \) generally increases the firm’s marginal benefits of increased risk. The intuition follows from the call option analogy: higher volatility becomes more valuable in a call option the further the current price of the underlying is below the strike price. In a debt contract, equity value is the call option value and the maturity value of the debt is the strike price. Therefore, the equity value increases more from raising \( \varphi \) when \( L \) is high.

One should, however, note the caveat that the strict relationship of an everywhere monotonic interaction between \( L \) and \( \varphi \) would require technical regularity conditions, as the following heuristic discussion demonstrates. Formally, the effect \( L \) on the firm’s project selection incentives is given by

\(^{48}\) Implementing a project with a higher \( \varphi \) or a project from a discrete set of investment opportunities would yield the same insights.
\[ \frac{\partial^2 X}{\partial \varphi \partial L} = \int_N \left( h(k^f) \cdot \frac{\partial x_2}{\partial \varphi} \cdot \frac{\partial k^f}{\partial L} + H(k^f) \cdot \frac{\partial^2 x_2}{\partial \varphi \partial L} \right) dF \\
+ \int_D \left( h(k^l) \cdot \frac{\partial x_2}{\partial \varphi} \cdot \frac{\partial k^l}{\partial L} + H(k^l) \cdot \frac{\partial^2 x_2}{\partial \varphi \partial L} \right) dF \\
+ \frac{\partial}{\partial \varphi} \left( \left( \mu_2 - E(v|k^l) \right) \cdot h(k^l) \cdot \frac{\partial k^l}{\partial L} \right) dF \]  

(29)

The first term in each integral is positive for all \( \theta \). The second-order stochastic dominance criterion in Definition 6 further implies that integrating

\[ \frac{\partial^2 x_2}{\partial \varphi \partial L} = \frac{\partial F_2(L - c_1)}{\partial \varphi} \]

over high values of \( c_1 \) generally yields positive values and integrating the term over low values of \( c_1 \) generally yields negative values. The requirement that \( \frac{\partial x_2}{\partial \varphi} > 0 \) for sufficiently many \( \theta \) in order for (25) to hold then implies that positive values should dominate. Further, i) \( \inf \ k^f \geq \sup \ k^l \) and ii) \( H(\cdot) \) is decreasing in \( s \), so that the weighting by \( H \) is higher for low values of \( c_1 \) and hence for positive values of \( \frac{\partial^2 x_2}{\partial \varphi \partial L} \) overall. These observations suggest that the second terms generally yield a positive value when integrated over \( \Theta \). For the final term in the integral over \( D \), one should note that \( k^l \) decreases in \( \varphi \) and \( \frac{\partial k^l}{\partial L} \) increases in \( \varphi \) for low \( c_1 \), i.e., the range over which the lender has control under an optimal contract. The preceding observations suggest that \( \frac{\partial^2 x}{\partial \varphi \partial L} \) is positive. A clean formal result that \( \frac{\partial^2 X}{\partial \varphi \partial L} > 0 \) must hold under all circumstances would require technical regularity conditions, but imposing such assumptions would not yield any further economic insight.\(^{49}\) Instead, the discussion will hereafter assume directly that \( \frac{\partial^2 X}{\partial \varphi \partial L} > 0 \).

The above arguments imply that the firm is held indifferent between implementing the first-best project and implementing an inefficient project if an increase in \( BV \), and hence in the firm’s decision rights, is compensated by a decrease in \( L \). The frontier, given by \( PS \), that separates feasible from infeasible contracts in the \( L-\psi \)-space is therefore

\(^{49}\) As a simple example, a uniform distribution \( F(\cdot) \) would be sufficient.
monotonically decreasing and thus intersects the set $COC$ of conditionally optimal break-even contracts exactly once, as shown in Figure 8. If the global optimum without the project selection constraint lies below the frontier given by $PS$, the project selection constraint does not bind and $\kappa = 0$. In the converse case, this global optimum is infeasible and the new constrained optimum that solves (26) coincides with the intersection of the two curves. The following proposition formalizes this result.

**Proposition 9.** The optimal accounting rule becomes (weakly) more conservative after the addition of a project selection constraint.

Proposition 9 predicts a prevalence of conservative accounting rules and strict covenants in practice, particularly in businesses and industries where the scope for discretionary, non-contractible managerial decisions that increase risk beyond the efficient level is large. The outcome is consistent with the notion that covenants serve to counteract borrowers’ incentives to increase risk at the expense of lenders. The mechanism is not one by which the lender adjusts or reverts the borrower’s previous actions after a covenant violation. Rather, an allocation of decision rights that favors the lender discourages the firm from making inefficient decisions in the first place because of the lender’s threat to revert them. It is, however, important to note the complementary effect of $L$. Since interest rates and control rights are substitutes with respect to the lender’s payoff and the break-even constraint must bind, the optimality of greater accounting conservatism in the presence of a project selection problem also rests on the attenuating effect of lower maturity values on the firm’s incentives for asset substitution.

Proposition 9 agrees with the conclusion by Caskey & Hughes (2012), who examine the use of several alternative fair value measures in a covenant and determine that a valuation method by which the lowest value is recognized yields the most efficient debt contract because it allocates the most decision rights to the lender and thus generates the deterrent effect mentioned above. However, Proposition 9 does not arise from a replication of the setup in Caskey & Hughes. First, unlike the model in Caskey & Hughes, the model

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50 The intersection point is optimal because firm value must be monotonically increasing on the curve of $COC$ toward the frontier given by $PS$. The reason is that the global optimum is unique by Proposition 3.
in this thesis does not rely on an exogenous, reduced-form representation of the agency conflict. Rather, the agency cost arises directly from the properties of the debt contract. This aspect is critical because the agency cost is a function of the endogenously determined contract terms, as the interaction between $\varphi$, $BV$ and $L$ demonstrates. In a reduced-form setting, this interaction is absent and hence not reflected in the solution. Second, the optimal accounting function in Proposition 9 is the outcome of an unconstrained optimization, whereas Caskey & Hughes restrict the choice set to five predetermined functions. Whether the optimal accounting rule among these five permits the same inferences as an unconstrained solution is not clear ex ante.

7. Conclusion

Accounting measures frequently serve as a basis for debt covenants. This thesis examines how the underlying accounting rules are designed optimally to minimize inefficiency in a setting with incomplete contracts and asset substitution. Accounting can serve two broad purposes in this context. First, it can provide information to the contracting parties and thus reduce the cost of decision errors. Second, it can act as a decision mechanism in the presence of an incentive misalignment problem, e.g., by triggering technical default events under a covenant. The focus of this thesis is on the second role. Accounting is modeled as an information aggregation function that implements an efficient resolution of decision problems involving risk-return tradeoffs. The analysis shows that this efficient resolution takes the form of an allocation of decision rights between borrower and lender.

This setup permits several observations about the properties of the optimal accounting rule. Both past transaction data in the form of realized cash flows and forward-looking data in the form of fair value estimates are valuable in the debt contract problem. The former affects the contracting parties’ decision incentives and hence the severity of the asset substitution problem, whereas the latter provides information about the first-best decision. The relative usefulness of these two types of information determines whether the optimal accounting rule has a historical cost focus or a fair value focus. Furthermore,
accounting conservatism, defined as the tendency of the accounting rule to yield values below the technical default threshold, is neither unambiguously beneficial nor detrimental. In particular, the empirically observed negative correlation between measures of conservatism and interest rate levels may only reflect the substitutive nature of the two with respect to the lender’s payoff.

The properties of the optimal accounting rule give rise to a number of empirical predictions. Debt financing is expected to be more prevalent in industries in which reliable estimates of the value of the borrowers’ assets are available, and these debt contracts should rely more on fair value measures. Firms with high leverage or low profit margins are expected to follow more conservative accounting practices than highly profitable firms and firms predominantly financed by equity. Conservative accounting is also optimal when the borrowing firm has incentives and discretion to make inefficiently risky investments after the contract has been signed. Moreover, debt contracts are least efficient at medium levels of leverage, at which compromises between the conflicting incentives of the firm and the lender are most costly. Ceteris paribus, firms are therefore expected to align their degree of debt financing inversely to their profitability, consistent with the observation in Rajan & Zingales (1995). A high liquidation value relative to the value of the firm as a going concern increases the optimal degree of accounting conservatism and decreases the optimal interest rate level. Finally, high volatility of operating cash flows increases the cost of debt, particularly for highly profitable firms, and is therefore expected to correlate with a high degree of equity financing.

These results are subject to several qualifications. First, the predictions concerning the firms’ capital structure are only partial in the sense that the model does not include the cost of alternative financing sources, specifically equity. Empirical results may be difficult to obtain if debt and equity costs are too highly correlated. In addition, debt financing is costly relative to equity in this model, and hence there must exist benefits outside the model that compel firms to choose positive leverage ratios to begin with. Their correlation with the forces in the model may again complicate empirical tests. Similarly, cash flows, liquidation values, the precision of fair value information, profitability, operating risk and other variables in the model are assumed to be independent, but the correlation structure is unlikely to be so simple in reality. Lastly, the firm is assumed to borrow from
a single lender, but in practice, debt financing is often provided by multiple lenders, whose claims may be ranked by seniority. It may be important to consider that the severity of the incentive misalignment likely depends on the seniority of the lender’s claim, whereby unsecured lender’s incentives are more closely aligned with those of equityholders.

A further commentary concerns the extent to which the properties of the optimal accounting rule derived in this thesis are useful attributes of accounting in general. In practice, accounting serves multiple purposes, and firms generally seem to rely on a basic, general accounting system and make purpose-specific adjustments to the output information, e.g., for external financial reporting, performance evaluation, or contracts. The model in this thesis examines the role of accounting for a single, specific purpose and therefore cannot identify a clear boundary between contract-specific accounting adjustments and general-purpose accounting rules. Providing insight into which aspects of the mapping from the state space to the allocation of decision rights are optimally part of general-purpose accounting rules and which aspects should be specific to debt contracts would require a more general model in which accounting has multiple roles.

Finally, the discrimination between accounting rules and covenant provisions in this model is to some extent ambiguous. The debt covenant is assumed to take the simple form of a fixed cutoff value of the accounting measure $BV$ while all computational mechanisms are absorbed into the accounting rule, but one could alternatively specify a floating threshold whose calculation assumes some of the properties of $BV$ without altering the outcome. An example from practice may illustrate this point: an income escalator clause, i.e., a prescription that less than one-hundred percent of the borrower’s positive income in a given period is credited to owner’s equity as computed under the covenant, can be viewed either as a covenant provision superimposed on the accounting rule or as an accounting rule by itself that prescribes the accrual of an expense related to agency costs. Another manifestation of this ambiguity is the possible substitution between accounting conservatism and covenant tightness. One should therefore bear in mind that some of the predicted properties of the optimal accounting rule may manifest themselves in the form of covenant provisions rather than accounting numbers from in public financial reports.
Appendix

Proof of Proposition 1. The indifference point between liquidation and continuation under the efficient decision rule, given the firm’s information signal, solves

\[ E(v|m) - \mu_2 = \int_0^{\xi} \left( F_2(t) - \hat{G}(t|m) \right) dt = 0 \]

Then the risk-ordering property in Assumption 2 implies that

\[ x_v(m) - x_2 = \int_{L-c_1}^{\xi} \left( F_2(t) - \hat{G}(t|m) \right) dt \leq 0 \quad \text{(A1)} \]

for \( L - c_1 \geq 0 \). In order to satisfy the firm’s indifference condition \( x_v = x_2 \), (A1) must hold with equality, which requires increasing the conditioning signal to some level \( k^f \geq m \) in view of Assumption 1. A symmetric argument shows that the lender’s indifference point must be some value \( k^l \leq m \).

By the implicit function theorem, the firm’s liquidation threshold changes with \( c_1 \) by

\[ \frac{\partial k^f}{\partial c_1} = \left( \hat{G}(L - c_1|k^f) - F_2(L - c_1) \right) \cdot \left( \frac{\partial x_v}{\partial s} \bigg|_{s=k^f} \right)^{-1} \]

If \( \hat{G}(L - c_1|k^f) \geq F_2(L - c_1) \), this inequality would have to hold for all outcomes higher than \( L - c_1 \) by Assumption 2, but then \( x_v = x_2 \) could not hold. The denominator is positive in view of Assumption 1, and therefore \( \frac{\partial k^f}{\partial c_1} \leq 0 \). The argument that the lender’s liquidation threshold changes by \( \frac{\partial k^l}{\partial c_1} \leq 0 \) is symmetric.

Proof of Proposition 2. In an optimal direct decision contract, liquidation is undertaken for any realization of \( s \) such that \( E(v|s) > m \). The lender’s break-even constraint can then be met by setting \( L \) appropriately. The inefficiency incurred in this case is

\[ \int_0^{\mu_2} (\mu_2 - v) dG \]

Alternatively, one can consider a delegation contract by which the lender receives the right to call the debt and hence enforce liquidation whenever \( E(v|s) > \mu_2 \) and the firm
retains control otherwise. The lender will choose liquidation for any \( \hat{s} > k^l \), and hence the inefficiency incurred under this delegation contract is

\[
 r + q^l = \int_0^{k_l} \int_{\mu_2}^{c} (v - \mu_2) \, dG \, dH + \int_{k_l}^{c} \int_{\mu_2}^{c} (\mu_2 - v) \, dG \, dH \\
= \int_0^{\mu_2} (\mu_2 - v) \, dG + \int_0^{k_l} (E(v|\hat{s}) - \mu_2) \, dH
\]  

(A2)

where \( r \) is defined in (4) and the simplification \( E(v|\hat{s}, s) = E(v|\hat{s}) \) arises because the lender’s information signal \( \hat{s} \) is a sufficient statistic for the pair \((\hat{s}, s)\) with respect to \( v \).

The first term in (A2) is equal to the inefficiency incurred under the direct decision rule. The second term is negative because \( E(v|\hat{s}) < \mu_2 \) for \( \hat{s} \leq m \) by construction of \( m \) and because \( k_l \leq m \) by Proposition 1. Hence, the delegation contract achieves greater efficiency than the direct decision contract when \( E(v|s) > \mu_2 \). The argument that delegating the decision to the firm when \( E(v|s) \leq \mu_2 \) achieves greater efficiency than prescribing continuation of the business contractually is symmetric.

**Proof of Lemma 1.** In any given state \( \theta \in N \), the firm maximizes its payoff by solving

\[
\max_{k^f} x^f(k^f)
\]

where \( k^f \) is the firm’s liquidation threshold. Similarly, the lender solves

\[
\max_{k^l} u^l(k^l)
\]

in any state \( \theta \in D \). Then for a given state \( \theta \), it must be that

\[
x^f(k^f) \geq x^l(k^l)
\]

with equality only when

\[
\arg \max_{k^f} x^f(k^f) \supseteq \arg \max_{k^l} u^l(k^l)
\]

It follows that the firm’s total expected payoff is

\[
X(N') \geq X(N)
\]

for any partition \( \{D, N\} \) and \( \{D', N'\} \) with \( N' \supseteq N \), i.e., the firm’s payoff increases with its control rights. A symmetric argument applies to the lender.

Then for any partition \( \{D, N\} \) such that \( U > I \), the firm could choose some \( \{D', N'\} \) with \( N' \supseteq N \) so that its payoff increases without violating the lender’s break-even constraint, and so \( \{D, N\} \) could not be part of an optimal contract. If \( U > I \) and \( D = \emptyset \), i.e.,
the firm cannot expand its space of control, it can lower the maturity value $L$ until $U = I$ because states in which payoffs depend on $h^i$ (and hence adverse incentive effects could reduce $X$) are reached with probability zero when $D = \emptyset$. Hence, $U = I$ under any optimal contract.

**Proof of Lemma 2.** Under an optimal accounting rule, the first variation
\[
\frac{d}{d\epsilon} \int \eta(BV + \epsilon j(\theta)) \, dF \bigg|_{\epsilon=0} = \int j(\theta) \cdot \Gamma(L, \lambda, \theta) \cdot \delta(BV) \, dF
\]
(A3)
must be zero for any continuous test function $j(\cdot)$, where $\Gamma(\cdot)$ is given by (16) and $\delta(\cdot)$ is the Dirac delta distribution. By the fundamental lemma of calculus of variations, (A3) is zero if and only if the Euler-Lagrange equation
\[
y_B \equiv \frac{\partial y}{\partial BV} = \Gamma(L, \lambda, \theta) \cdot \delta(BV) = 0
\]
(A4)
holds for all $\theta$. Since the delta distribution is zero at any $BV \neq 0$ but assigns point mass when $BV = 0$, (A4) holds if and only if $\Gamma = 0$ whenever $BV = 0$. The accounting rule $BV = z(\theta) \cdot \Gamma$ is therefore sufficient to set (A3) to zero. It remains to be shown that this solution maximizes $Y$. Indeed, $\Gamma > 0$ implies that the firm’s equity value is higher if $\theta \in N$, while $\Gamma < 0$ implies the opposite, which corresponds to the control allocation that $BV = z(\theta) \cdot \Gamma$ implements as long as $z(\theta) > 0$ for all $\theta$. In order to establish necessity, it suffices to note that an accounting rule $BV'$ cannot be written in the form $BV = z(\theta) \cdot \Gamma$ only if $\text{sgn}(BV') \neq \text{sgn}(BV)$ for at least some $\theta$. However, then the control allocation under $BV'$ is not optimal for any such $\theta$, and hence $BV'$ cannot be an optimal accounting rule.

**Proof of Proposition 3.** A sufficient condition for the contract to achieve a local maximum is
\[
\int j^2(\theta) \cdot \text{det}(M) \, dF > 0
\]
(A5)
where
\[
M = \begin{bmatrix}
0 & U_L & U_B \\
U_L & Y_{LL} & Y_{BL} \\
U_B & Y_{BL} & Y_{BB}
\end{bmatrix}
\]
(A6)
is the bordered Hessian matrix at a given $\theta$ and $j(\theta)$ is an arbitrary, continuous test function. Then (A5) holds if and only if
\[
\det(M) = -U_L^2 Y_{BB} + 2u_B U_L Y_{BL} - u_B^2 Y_{LL} \geq 0
\]
for all $\theta$, with strict inequality for at least some $\theta$. The elements of $M$ are defined as follows.

As argued in the proof of Lemma 1, the lender’s payoff decreases whenever a state $\theta$ is transferred from $D$ to $N$. Hence, the change in the lender’s payoff from increasing book value
\[
u_B \equiv \frac{\partial u}{\partial BV} = (u_f - u_i) \cdot \delta(BV)
\]
must be non-positive for all $\theta$. Conversely, the effect of raising the maturity value $L$ given by
\[
u_L \equiv \frac{\partial U}{\partial L} = \int \frac{\partial u}{\partial L} dF
\]
must be positive in view of the following argument. Let $L^*$ denote the optimal maturity value for a given level of $I$. For $I = 0$, $D = \emptyset$ and $L^* = 0$ because $U = I$ by Lemma 1, and so
\[
u_L |_{L = 0} = \int_\theta \left( \int_{k^f}^{k^c} \frac{\partial u_2}{\partial L} dH + \int_{k^c}^{c} \frac{\partial u_\nu}{\partial L} dH \right) dF > 0
\]
If one were to raise $L$ and dynamically adjust $D$ and $I$ such that $L = L^*$ everywhere, the resulting path would cover all possible optima. If $\nu_L < 0$ for any $L_1 > 0$ on this path, continuity implies that $\nu_L = 0$ for some optimal $L_0 < L_1$ because $\nu_L |_{L = 0} > 0$. Then $Y_L = 0$ at $L_0$ if and only if $Q_L = 0$, which implies $X_L = 0$ because
\[
X_L + U_L = -Q_L
\]
in view of (9). Hence, $L_0$ must maximize both $X$ and $U$, which implies that both would decrease if $L$ were increased beyond $L_0$. Therefore, $L^* = L_1$ cannot be part of an optimal contract and so $\nu_L > 0$ at any $L^*$.

The second partial derivative $Y_{LL} \equiv \frac{\partial^2 Y}{\partial L^2}$ must be negative. Otherwise, the firm could raise $L$ and obtain a higher equity value without violating the lender’s break-even constraint because $\nu_L > 0$ as noted above, but the break-even constraint must bind by Lem-
ma 1. Hence, no maturity value \( L \) such that \( Y_{LL} > 0 \) can meet the necessary condition \( Y_L = 0 \). The second-order condition with respect to the accounting rule is

\[
y_{BB} = \frac{\partial^2 y}{\partial BV^2} = \Gamma \cdot \frac{d\delta(BV)}{dBV} = \Gamma \cdot \delta'(z(\theta) \cdot \Gamma) = -\frac{1}{z(\theta)} \cdot \delta(z(\theta) \cdot \Gamma)
\]

when evaluated at the optimum \( BV = z(\theta) \cdot \Gamma \), where \( y \) is defined in (15). Then \( y_{BB} < 0 \) for all \( \theta^* \) because \( z > 0 \) by Lemma 2.\(^{51} \) Finally, the cross-partial derivative with respect to \( BV \) and \( L \) is

\[
y_{BL} = \frac{\partial^2 y}{\partial BV \partial L} = \frac{\partial \Gamma}{\partial L} \cdot \delta(BV) = -\frac{\partial \Gamma}{\partial c_1} \cdot \delta(BV)
\]

where the second equality follows from the observation that \( \Gamma \) only depends on \( L \) and \( c_1 \) through the joint term \( L - c_1 \). The delta distribution is zero for \( BV \neq 0 \), and hence it suffices to examine \( y_{BL} \) for \( \theta = \theta^* \). When \( \lambda = 1 \), the \( u \) terms are eliminated and one obtains

\[
-\frac{\partial \Gamma}{\partial c_1} = \left(E(v|k^f) - \mu_2\right) \cdot h(k^f|s) \frac{\partial k^f}{\partial c_1} + \left(\mu_2 - E(v|k^l)\right) \cdot h(k^l|s) \frac{\partial k^l}{\partial c_1} \tag{A7}
\]

By Proposition 1, both liquidation thresholds decrease in \( c_1 \) and \( k^f \geq m \geq k^l \), which yields \( \frac{\partial \Gamma}{\partial c_1} < 0 \). Equation (A7) implies that the marginal benefit of giving control rights to the lender for a given \( s \) is monotonically decreasing in \( c_1 \), and hence the critical value \( c_1^* \) at which \( \Gamma = 0 \) is unique when \( \lambda = 1 \). If \( c_1^* \) were not unique for some \( \lambda \neq 1 \), there would have to exist some interval on the support of \( C_1 \) whose states are in \( D \) and whose neighboring interval to the left is in \( N \). But then the two parties could exchange control rights over states from these two intervals such that the break-even constraint \( U = I \) is maintained. This control transfer is feasible because \( u_B \leq 0 \) for all \( \theta \) as noted above and would decrease inefficiency. Hence, an accounting rule that yields multiple \( c_1^* \) for a given \( s \) cannot meet condition (A4), and so \( y_{BL} < 0 \) for any \( \theta^* \).

The above observations imply that \( \det(M) \) must be non-negative for all \( \theta \) and strictly positive for all \( \theta^* \). Hence, any accounting rule and maturity value that satisfy the first-order condition attain a maximum. Multiple local maxima would require the existence of saddle points or minima, and so the maximum must be unique.

\(^{51}\) Equivalently, one could define \( \frac{\partial \Gamma}{\partial BV} = \nabla \Gamma \cdot \iota \) as the directional derivative of \( \Gamma \), where \( \nabla \) is the gradient with respect to \( \theta = (c_1, s) \), and \( \iota \) is any vector in \( \Theta \) that corresponds to a unit increase in \( BV \). Graphically, \( \iota \) thus points from default to no-default states.
Proof of Proposition 4. Let $s^*$ and $c_1^*$ denote the critical values at which $BV = BV_0 = 0$. The path of $\{\theta^*\}$ is one-to-one if the Euler-Lagrange equation

$$z(\theta) \cdot \Gamma = 0$$

holds for a unique $c_1^*$ given $s$ and for a unique $s^*$ given $c_1$, where $BV = z(\theta) \cdot \Gamma$ is the optimal accounting rule from Lemma 2. Since $\Gamma = 0$ at all $\theta^*$ and $z > 0$ for all $\theta$ by Lemma 2, the type of stationary point at $\theta^*$ for a given $c_1$ is identified by the sign of

$$\frac{\partial \Gamma}{\partial s} = \int_{k_1}^{k_f} (x_2 - x_\theta) \cdot l(\hat{s}|s) \, dH + \lambda \int_{k_1}^{k_f} (u_2 - u_\theta) \cdot l(\hat{s}|s) \, dH$$

which is identical to $\Gamma$ except for the likelihood ratio factor

$$l(\hat{s}|s) \equiv \frac{\partial h(\hat{s}|s)}{\partial s} \cdot \frac{1}{h(\hat{s}|s)}$$

By Assumption 1, $l$ is increasing in $\hat{s}$. Further, the parenthetical terms in both integrands are increasing on $[k_1, k_f]$, as shown in the proof of Proposition 1. Since $\Gamma = 0$ for any $\theta^*$, the previous observations imply that $\frac{\partial \Gamma}{\partial s} < 0$. Control is therefore optimally transferred from the firm to the lender at any $s^*$. The existence of multiple $s^*$ for a given $c_1$ would require that $\frac{\partial \Gamma}{\partial s} > 0$ for at least one $s^*$, and hence the critical value must be unique. The uniqueness of the critical value $c_1^*$ follows from $\frac{\partial \Gamma}{\partial c_1} > 0$ at any $\theta^*$, as shown in the proof of Proposition 2 above.

Proof of Corollary to Proposition 4. Let $\Gamma_c \equiv \frac{\partial \Gamma}{\partial c_1}$ and $\Gamma_s \equiv \frac{\partial \Gamma}{\partial s}$. The level set

$$\{\theta^*\} = \{\theta: BV(\theta) = 0\} = \{\theta: \Gamma = 0\}$$

is characterized by the directional derivative

$$\nabla BV \cdot \iota = \Gamma_c \cdot dc_1^* + \Gamma_s \cdot ds^* = 0$$

(A8)

where $\nabla$ denotes the gradient of $BV$ with respect to $\theta$, and $\iota$ is a vector in the $c_1$-$s$-space tangent to the path of $\{\theta^*\}$. Equation (A8) must hold for all $\theta^*$ and states the well-known result that the gradient of $BV$ must be orthogonal to the level set $\{\theta^*\}$. Dividing (A8) through by $\Gamma_c - \Gamma_s$ and integrating yields the separated solution

$$BV = bv_0 + bv_c(c_1) + bv_s(s)$$
where \( b\nu_0 \) is a constant,
\[
\nu_c(c_1) = \int_0^{c_1} \frac{\Gamma_c(t, s^*(t))}{\Gamma_c(t, s^*(t)) - \Gamma_s(t, s^*(t))} \, dt
\]
is positive and increasing for all \( c_1 \) because \( \Gamma_c > 0 \) and \( \Gamma_s < 0 \) at all \( \theta^* \) by Proposition 3, and
\[
\nu_s(s) = \int_0^s \frac{\Gamma_s(t, c_1^*(t))}{\Gamma_c(t, c_1^*(t)) - \Gamma_s(t, c_1^*(t))} \, dt
\]
is negative and decreasing for all \( s \). \(^{52}\) The cutoff values \( s^* \) and \( c_1^* \) are uniquely identified for given \( c_1 \) and \( s \), respectively, by Proposition 3. By construction, \(|\nu_i| \in [0, 1]\) for \( i = c, s \) and all \( \theta \), and hence both terms are bounded.

**Proof of Proposition 5.** By Proposition 2, the optimal accounting rule \( BV \) is identified by its first-order condition \( y_B = 0 \). Then \( BV \) is still optimal after an increase in \( I \) if and only if
\[
\frac{dy_B}{dBV} = y_{BB}BV_I + y_{BL}L_I + u_B\lambda_I = y_{BB}BV_I + y_{BL} \cdot \left( L_I + \frac{u_B}{y_{BL}}\lambda_I \right) \tag{A9}
\]
is zero for all \( \theta^* \), where \( BV_I \equiv \frac{\partial BV}{\partial I} \), \( L_I \equiv \frac{\partial L}{\partial I} \) and \( \lambda_I \equiv \frac{\partial \lambda}{\partial I} \) denote the contract term adjustments and all other terms are defined as in the proof of Proposition 2. Similarly, the adjustments must solve
\[
\frac{dY_L}{dI} = Y_{LL}L_I + \int_\theta y_{BL}BV_I \, dF + U_L\lambda_I = 0 \tag{A10}
\]
and
\[
\frac{dU}{dI} = U_LL_I + \int_\theta u_BBV_I \, dF = 1 \tag{A11}
\]
If \( BV_I \geq 0 \) for all \( \theta^* \), (A11) implies that \( L_I > 0 \), which in turn yields \( \lambda_I > 0 \) by (A10) because \( Y_{LL} < 0 \) and \( U_L > 0 \) under any optimal contract by Proposition 2. But then (A9) cannot hold for any \( \theta^* \) because \( y_{BB} < 0 \), \( y_{BL} < 0 \) and \( u_B < 0 \) for all \( \theta^* \) by Proposition 2,

\(^{52}\) If \( s^* \) is undefined for some \( c_1 \) because either the lender or the firm optimally receives control for all \( s \), one can set \( \Gamma_s \) to zero so that the integrand of \( \nu_c \) becomes \( \frac{\Gamma_c}{\Gamma_c - \Gamma_s} = 1 \) for these \( c_1 \). This normalization is without loss of generality because the constant term \( b\nu_0 \) can be adjusted accordingly. An analogous argument applies to values of \( s \) for which \( c_1^* \) is undefined in \( \nu_s \). Graphically, these two cases correspond to regions beyond the points at which the path of \( \{ \theta^* \} \) shown in Figure 2 intersects the \( c_1 \)-axis or the \( s \)-axis, respectively.
and hence \( BV_i \geq 0 \) for all \( \theta^* \) is impossible. Conversely, in order to demonstrate that \( BV_i \leq 0 \) for all \( \theta^* \) is feasible, one can consider the scenario \( y_{BL} \propto u_B \) on \( \{ \theta^* \} \). Then \( BV_i \) must be of the same sign for all \( \theta^* \) because

\[
L_i + \frac{u_B}{y_{BL}} \lambda_i = C
\]

for some constant \( C \) at all \( \theta^* \) in (A9). The case \( C < 0 \) implies \( BV_i \geq 0 \), which has been ruled out as a solution, and hence \( BV_i \leq 0 \) for all \( \theta^* \) in this scenario. Therefore, the conservatism of \( BV \) increases in \( I \). The conclusion that \( L_I > 0 \) follows from Proposition 6 below.

**Proof of Proposition 6.** Since \( \frac{dY}{dI} = 1 - \lambda \) by the envelope theorem, firm value is convex in \( I \) if \( \frac{\partial \lambda}{\partial I} < 0 \). Given the uniqueness of the optimal contract terms, the contract term adjustments \( \lambda_i = \frac{\partial \lambda}{\partial I}, L_i = \frac{\partial L}{\partial I} \) and \( BV_i = \frac{\partial BV}{\partial I} \) in response to an increase in \( I \) are optimal if and only if

\[
\int_\Theta \mathbf{M} \cdot \left[ \begin{array}{c}
\lambda_i \\
L_i \\
BV_i
\end{array} \right] dF = \left[ \begin{array}{c}
1 \\
0 \\
0
\end{array} \right]
\]

(A12)

where \( \mathbf{M} \) is as defined in Proposition 2. Then in each state \( \theta^* \), the integrand of (A12) is

\[
\mathbf{M} \cdot \left[ \begin{array}{c}
\lambda_i \\
L_i \\
BV_i
\end{array} \right] = \left[ \begin{array}{c}
1 + \epsilon_1 \\
\epsilon_2 \\
0
\end{array} \right]
\]

(A13)

for some \( \epsilon_i \), where \( E_{\theta}(\epsilon_i) = 0 \) for \( i = 1,2 \). The third element on the right-hand side of (A13) is always zero because the first-order condition \( y_B = 0 \) with respect to \( BV \) must hold for every \( \theta \). Solving (A13) yields

\[
\lambda_i \equiv (1 + \epsilon_1) \cdot (y_{LL}y_{BB} - y_{BL}^2) + \epsilon_2 \cdot (u_By_{BL} - U_Ly_{BB})
\]

and

\[
L_i \equiv (1 + \epsilon_1) \cdot (u_By_{BL} - U_Ly_{BB}) - \epsilon_2 u_B^2
\]

for all \( \theta^* \), where the definitions and signs of all terms on the right-hand sides are as given in the proof of Proposition 2. Since \( E_{\theta}(\epsilon_2) = 0 \), continuity implies that \( \epsilon_2 = 0 \) for some \( \theta \). Then if \( L_I > 0 \), it must be that \( \epsilon_1 > -1 \) at this \( \theta \), in which case \( \lambda_i \) is negative if

\[
y_{LL}y_{BB} - y_{BL}^2 < 0
\]

(A14)
It remains to be shown that both \( L_I > 0 \) and \( (A14) \) hold for all \( I \). First, \( L = 0 \) when \( I = 0 \) but \( L > 0 \) when \( I > 0 \), and so by continuity, \( L_I < 0 \) can only hold if \( L_I = 0 \) for some \( I \). The multiplier of \( \varepsilon_1 \) in \( L_I \) is always nonzero, and so \( L_I = 0 \) can only hold if \( \varepsilon_1 = -1 \) when \( \varepsilon_2 = 0 \), which in turn implies \( \lambda_I = 0 \). Then \( (A9) \) can only hold if \( BV_I = 0 \) for all \( \theta^* \), but then all contract terms remain unchanged while \( I \) has increased, in which case the lender’s break-even constraint is no longer met. Hence, \( L_I \leq 0 \) cannot hold for any level of \( I \).

In order to establish \( (A14) \), one can note that its left-hand side is equal to the determinant of the Hessian matrix in the unconstrained optimization program

\[
\max_{BV,L} \left( w - q(BV) + (\lambda - 1) \cdot u(BV) \right)
\]

(A15)

over \( L \) and \( BV \), which differs from the original program (12) in that \( \lambda \) has been fixed, \( \Theta \) is reduced to a single state and \( u \) is unconstrained. The unique maximum of this program is always a corner solution with either \( L = 0 \) and \( BV > 0 \) when \( \lambda \leq 1 \), or \( L = 2\tilde{c} \) and \( BV < 0 \) when \( \lambda \geq 1 \) because i) \( q = 0 \) in both cases, ii) \( w \) does not depend on the contract terms, iii) \( u \) attains its minimum or maximum when all payoffs and control rights are allocated to one party only, and iv) the interior stationary point given by \( L \) and \( BV \) must be unique by Proposition 2, and so local interior maxima cannot exist. Further, \( Y_{LL} < 0 \) and \( y_{BB} < 0 \) imply that \( L \) and \( BV \) cannot yield a minimum either, and therefore \( (A15) \) must be at a saddle point, which implies that \( (A14) \) holds.

**Proof of Proposition 7.** By Definition 4, controlling for profitability and leverage requires that

\[
\left( \nabla_\eta Y - a \nabla_{i\nu} Y \right) \perp a \nabla_{i\nu} Y
\]

or

\[
\int \Theta \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} U_\eta \\ Y_{L\eta} \\ y_{B\eta} \end{bmatrix} - a \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \right) dF = 0
\]

(A16)

which implies that \( a = -U_\eta \). The parenthetical term in (A16) is the effect of \( \eta \) after profitability and leverage are controlled. Then given the uniqueness of the optimal contract, contract term adjustments in response to an increase in \( \eta \) are optimal if and only if
\[
\frac{dy_B}{d\eta} = y_{B\eta} + y_{BB} B V_{\eta} + y_{BL} L_{\eta} + u_B \lambda_{\eta} = 0 \quad (A17)
\]
for all \( \theta^* \), where \( BV_{\eta} \equiv \frac{\partial B}{\partial \eta} \), \( L_{\eta} \equiv \frac{\partial L}{\partial \eta} \) and \( \lambda_{\eta} \equiv \frac{\partial \lambda}{\partial \eta} \). Further, these adjustments must solve
\[
\frac{dY_L}{d\eta} = Y_{L\eta} + Y_{LL} L_{\eta} + \int_{\Theta} y_{BL} B V_{\eta} dF + U_L \lambda_{\eta} = 0 \quad (A18)
\]
and
\[
\frac{dU}{d\eta} = U_L L_{\eta} + \int_{\Theta} u_B B V_{\eta} dF = 0 \quad (A19)
\]
The definitions and signs of all terms not involving \( \eta \) follow from the proof of Proposition 2.

First, assume that \( y_{B\eta} < 0 \) for all \( \theta^* \) and \( Y_{L\eta} < 0 \). Then if \( BV_{\eta} \geq 0 \) for all \( \theta^* \), (A19) can only hold if \( L_{\eta} > 0 \), which implies that \( \lambda_{\eta} > 0 \) in order for (A18) to hold. But then (A17) cannot hold for any \( \theta^* \), and so \( BV_{\eta} \geq 0 \) for all \( \theta^* \) is impossible. Next, consider the scenario \( u_B \propto y_{BL} \propto y_{B\eta} \) on \( \{ \theta^* \} \). Then
\[
\frac{y_{B\eta}}{y_{BL}} + L_{\eta} + \frac{u_B}{y_{BL}} \lambda_{\eta} = C
\]
for some constant \( C \) at all \( \theta^* \) in (A17). The case \( C < 0 \) implies \( BV_{\eta} \geq 0 \) for all \( \theta^* \), which has been ruled out, and so it must be that \( C > 0 \) and hence \( BV_{\eta} \leq 0 \) for all \( \theta^* \).

The above arguments imply that the conservatism of \( BV \) increases in \( \eta \) if it can be shown that \( y_{B\eta} < 0 \) and \( Y_{L\eta} < 0 \). Since \( y_B \) is given by (A4), the effect of \( \eta \) on \( y_{B\eta} \) takes the same sign as
\[
\frac{\partial \Gamma}{\partial \eta} = \int_{k^f}^{k^l} \left( x_2 - x_v + \lambda \cdot (u_2 - u_v) \right) \cdot j(\hat{s}|s) dH \quad (A20)
\]
where
\[
j(\hat{s}|s) \equiv \frac{\partial h(\hat{s}|s)}{\partial \eta} \cdot \frac{1}{h(\hat{s}|s)}
\]
as in the definition of \( \eta \). The term is equivalent to \( \Gamma \) except for the weighting factor \( j \).

Then (A20) is negative for all \( \theta^* \) because i) \( j \) is increasing in \( v \) by Assumption 1, ii) \( x_2 - x_v \) and \( u_2 - u_v \) are decreasing in \( v \), and iii) \( \Gamma = 0 \) by the first-order condition. Then \( y_{B\eta} \leq 0 \) for all \( \theta^* \), with strict inequality for \( \theta^* \). The cross-partial derivative with respect to \( L \) and \( \eta \) is
\[ Y_{L\eta} = \int_D \left( \mu_2 - E(v|k^l) \right) \cdot \frac{\partial k^l}{\partial L} \cdot h(k^l|s) \cdot j(k^l|s) \, dF \]
\[ + \lambda \int_N \left( \mu_2 - E(v|k^f) \right) \cdot \frac{\partial k^f}{\partial L} \cdot h(k^f|s) \cdot j(k^f|s) \, dF \]
\[ + (\lambda - 1) \int_D \left( \int_0^{k^f} \frac{\partial u_2}{\partial L} \cdot j(\xi|s) \, dH + \int_0^{\xi} \frac{\partial u_2}{\partial L} \cdot j(\xi|s) \, dH \right) \, dF \]
\[ + (\lambda - 1) \int_D \left( \int_0^{k^l} \frac{\partial u_2}{\partial L} \cdot j(\eta|s) \, dH + \int_0^{\eta} \frac{\partial u_2}{\partial L} \cdot j(\eta|s) \, dH \right) \, dF \]  
(A21)

For \( \lambda = 1 \), the last two terms are eliminated. Then (A21) must be negative because i) the integrand of the first term is always positive and the integrand of the second term is always negative in view of \( k^l \leq m \leq k^f \), ii) any \( s \) in \( N \) is lower than any \( s \) in \( D \) for a given \( c_1 \) by Proposition 3, and iii) \( j \) is increasing in \( v \) and decreasing in \( s \), and so the inequality
\[ j(k^f|s) \geq j(k^l|s) \]  
(A22)

must hold both for all \( s \) given any \( c_1 \) and for all \( c_1 \) given any \( s \). Then since \( Y_L = 0 \) must hold at the optimum and \( Y_{L\eta} \) is equivalent to \( Y_L \) except for the weighting terms (A22), it must be that \( Y_{L\eta} < 0 \) for any \( \lambda \) sufficiently close to 1.

**Proof of Proposition 8.** By the envelope theorem, the firm’s equity value changes with \( \sigma \) by
\[ \frac{dY}{d\sigma} = \int_D \left( -\frac{\partial q(BV)}{\partial \sigma} - (\lambda - 1) \cdot \frac{\partial u(BV)}{\partial \sigma} \right) \, dF \]
where
\[ \frac{\partial q(BV)}{\partial \sigma} = I_D(BV) \cdot (E(v|k^l) - \mu_2) \cdot h(k^l|s) \cdot \frac{\partial k^l}{\partial \sigma} + \left( 1 - I_D(BV) \right) \cdot (E(v|k^f) - \mu_2) \cdot h(k^f|s) \cdot \frac{\partial k^f}{\partial \sigma} \]

must be positive regardless of control allocation because \( \frac{\partial k^l}{\partial \sigma} \leq 0 \) and \( \frac{\partial k^f}{\partial \sigma} \geq 0 \) as shown in (23) and (24) and \( k^l \leq m \leq k^f \) by Proposition 1. The change in the lender’s payoff is
\[
\frac{\partial u(BV)}{\partial \sigma} = I_D(BV) \cdot \int_0^{k^l} \frac{\partial u_2}{\partial \sigma} dH + \left( 1 - I_D(BV) \right) \\
\cdot \left( \left( \mu_2 - E(v|k^f) \right) \cdot h(k^f|s) \cdot \frac{\partial k^f}{\partial \sigma} + \int_0^{k^f} \frac{\partial u_2}{\partial \sigma} dH \right)
\]

which must be negative because

\[
\frac{\partial u_2}{\partial \sigma} = -\int_0^{L-c_1} \frac{\partial F_2}{\partial \sigma} dc_2 \leq 0
\]

Since the lender’s break-even constraint binds for all \( \sigma \) by Lemma 1, \( \frac{\partial Y}{\partial \sigma} \) must also equal the change in total firm value, and so an increase in \( \sigma \) results in a decrease in both equity value and efficiency through \( \frac{\partial q(BV)}{\partial \sigma} \). The decrease is larger for \( \lambda > 1 \) but mitigated for \( \lambda < 1 \) because \( \frac{\partial u(BV)}{\partial \sigma} < 0 \).

**Proof of Proposition 9.** Let \( D^*(L) \) denote the unique optimal set of states in which the firm violates the covenant, conditional on \( L \). Consider the two conditional optimization problems

\[
\max_{L,\lambda_1} Y_1 \equiv \max_{L,\lambda_1} (X + (\lambda_1 - 1) \cdot U)
\]

and

\[
\max_{D,\lambda_2} Y_2 \equiv \max_{D,\lambda_2} (X + (\lambda_2 - 1) \cdot U)
\]

In \( Y_1, L \) is a choice variable but the control allocation is given by some \( D^* \). Conversely, \( D \) is a choice variable from the set of all \( D^* \) in \( Y_2 \) but \( L \) is given. Consider the set of solutions to \( Y_1 \) and \( Y_2 \) conditional on all possible \( D^* \) and \( L \), respectively. Then for a given \( D^*_0 \), the solution \( L^*_0 \) in \( Y_1 \) must be such that \( D^*_0 \) solves \( Y_2 \) conditional on \( L^*_0 \). Otherwise, continuity implies that there would exist some \( L \) for which two different \( D^* \) constitute the optima in \( Y_1 \) and \( Y_2 \), which is impossible because \( U \) increases in \( D \), and hence the break-even constraint would not bind in at least one of the problems.

The global optimum identified in Proposition 3 coincides with the unique point at which \( \lambda_1 = \lambda_2 \). Uniqueness implies that along the path of conditionally optimal \( D^* \) and
$L^*$, firm value is monotonically increasing toward the global optimum if one were to start with $L = 0$ and raise $L$ toward its maximum value. The necessary condition for a conditionally optimal contract in $Y_1$ is

$$\frac{\partial Y_1}{\partial L} = \frac{\partial X}{\partial L} + \lambda_1 \frac{\partial U}{\partial L} = 0$$

An increase in $L$ at the same optimum in $Y_2$ yields

$$\frac{dY_2}{dL} = \frac{\partial Y_2}{\partial L} > 0$$

by the envelope theorem as long as the global optimum has not been reached. Since $X$ is decreasing in $L$ and $U$ is increasing in $L$ around $L = 0$ and since $\lambda_1 > 0$, a sign reversal from $\frac{\partial U}{\partial L} > 0$ to $\frac{\partial U}{\partial L} < 0$ can only occur when $\frac{\partial X}{\partial L} = \frac{\partial U}{\partial L} = 0$ at some point. The latter implies that $\frac{\partial Y_2}{\partial L} = 0$, and hence this point would have to constitute the global optimum.

Therefore, $\frac{\partial U}{\partial L} > 0$ for all $L$ below the global optimum.

The preceding argument implies that a mapping $BV \to \psi$, with $\psi \in \mathbb{R}$, exists such that $\psi > \psi' \iff D \subset D'$ and each $BV(\psi)$ maximizes $Y_2$ for some $L$. Then $\frac{\partial U}{\partial L} > 0$ implies that the mapping $L \to \psi$ from $L$ to the conditionally optimal $\psi$ is monotonically increasing. Conversely, $\frac{\partial^2 X}{\partial \varphi \partial L} < 0$ and $X_{\varphi L} > 0$ imply that

$$\frac{\partial L}{\partial \varphi} = -\frac{X_{\varphi \psi}}{X_{\varphi L}} < 0$$

characterizes the path of contracts $(L, \psi)$ along which $X_{\varphi} = 0$, i.e., the firm is indifferent between implementing the first-best project and implementing a less efficient project.

Contracts with $L$ and $\psi$ above this path thus violate the project selection constraint.

Hence, a global optimum $(L, \psi)$ in a setting without a project selection constraint that lies below the path remains unchanged if project selection is introduced, but a global optimum above the path must be adjusted to a lower level of $\psi$ in order to meet the project selection constraint. Conservatism decreases in $\psi$, and hence the optimal level of accounting conservatism increases weakly if project selection is introduced.
References


