RATE-ADAPTIVE MODULATION AND CODING FOR
OPTICAL FIBER TRANSMISSION SYSTEMS

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Abstract

Modern wireline and wireless communication systems adapt link parameters, such as bandwidth utilization, power allocation, constellation size and forward error correction (FEC) code rate, depending on link quality (often time-varying), trading off bit rate (and thus spectral efficiency) for robustness. By contrast, to date, optical transmission systems have employed fixed bit rates.

Rate-adaptive optical transmission techniques trade off information bit rate for transmission distance and other factors affecting signal quality. These techniques enable increased bit rates over shorter links, while enabling transmission over longer links when regeneration is not available. They are likely to become more important with increasing network traffic and a continuing evolution toward optically switched mesh networks, which make signal quality more variable. Rate-adaptive optical transmission could help improve network robustness, flexibility and throughput.

This study proposes a rate-adaptive transmission scheme using variable-rate FEC codes and variable-size constellations with a fixed symbol rate. To vary the FEC code rate, we use two different schemes: (a) hard-decision decoding (HDD) using serially concatenated Reed-Solomon (RS) codes and inner repetition codes (b) soft-decision decoding (SDD) using outer RS codes and inner low-density parity-check codes. The FEC schemes are combined with single-carrier polarization-multiplexed quadrature amplitude modulation with variable constellation sizes and digital coherent detection. A rate adaptation algorithm uses the signal-to-noise ratio (SNR) or the bit-error ratio (BER) estimated by a receiver to determine the FEC code rate and constellation size that maximizes the information bit rate while satisfying a target decoded BER and an SNR margin. We simulate single-channel transmission through a long-haul fiber system incorporating numerous optical switches, evaluating the impact of fiber nonlinearity and bandwidth narrowing.
In systems with or without inline chromatic dispersion (CD) compensation, we quantify how achievable bit rates vary with distance, and compare the performance of the proposed schemes to an ideal coding scheme. From HDD-based system to SDD-based system, we observed approximately 50% increases in transmission distance, and approximately 2.5-dB reductions in the performance gap from an ideal coding scheme.
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NOTATIONS

Notations

$\alpha$ Attenuation coefficient

$\beta_2$ Dispersion parameter

$\gamma$ Nonlinear coefficient

$k$ Message word length

$n$ Codeword length

$r$ Code rate

$C$ Capacity

$C_{\text{up}}, C_{\text{down}}$ Counters for upward and downward rate change in rate adaptation algorithm

$f_R$ Repetition factor

$F_Q$ Q factor

$\phi_{\text{NL}}$ Nonlinear phase shift

$G$ Gain of amplifier

$L$ Transmission distance or length of fiber

$M$ QAM modulation order

$N_{\text{up}}, N_{\text{down}}$ Counter hysteresis for upward and downward rate change in rate adaptation algorithm

$\Delta$ SNR margin in rate adaptation algorithm

$\mu_{\text{up}}, \mu_{\text{down}}$ SNR penalty for upward and downward rate change in rate adaptation algorithm

$P_{b,\text{in}}, P_{b,\text{out}}$ Bit-error ratio at the input and output of the decoder
\textit{NOTATIONS}

\begin{itemize}
  \item $P_s$ \hspace{1cm} Symbol-error ratio
  \item $R_b$ \hspace{1cm} Information bit rate
  \item $R_s$ \hspace{1cm} Symbol rate
  \item $P_o, P_n$ \hspace{1cm} Signal and noise power
  \item $S$ \hspace{1cm} Power spectral density of ASE noise
  \item $W$ \hspace{1cm} Bandwidth
\end{itemize}
# Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>AC</td>
<td>Adaptive coding</td>
</tr>
<tr>
<td>ACM</td>
<td>Adaptive coding and modulation</td>
</tr>
<tr>
<td>ASE</td>
<td>Amplified spontaneous emission</td>
</tr>
<tr>
<td>ATM</td>
<td>Asynchronous transfer mode</td>
</tr>
<tr>
<td>AWGN</td>
<td>Additive white Gaussian noise</td>
</tr>
<tr>
<td>BCH</td>
<td>Bose/Ray-Chaudhuri/Hocquenghem</td>
</tr>
<tr>
<td>BER</td>
<td>Bit-error ratio</td>
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<tr>
<td>BP</td>
<td>Belief propagation</td>
</tr>
<tr>
<td>CD</td>
<td>Chromatic dispersion</td>
</tr>
<tr>
<td>CSI</td>
<td>Channel state information</td>
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<tr>
<td>DCF</td>
<td>Dispersion compensating fiber</td>
</tr>
<tr>
<td>DVB</td>
<td>Digital video broadcasting</td>
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<tr>
<td>EEPN</td>
<td>Equalization-enhanced phase noise</td>
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<tr>
<td>FEC</td>
<td>Forward error correction</td>
</tr>
<tr>
<td>FWM</td>
<td>Four-wave mixing</td>
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<tr>
<td>GF</td>
<td>Galois field</td>
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<tr>
<td>GVD</td>
<td>Group-velocity dispersion</td>
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<tr>
<td>HDD</td>
<td>Hard-decision decoding</td>
</tr>
<tr>
<td>IP</td>
<td>Internet protocol</td>
</tr>
<tr>
<td>ISI</td>
<td>Inter-symbol interference</td>
</tr>
<tr>
<td>LDPC</td>
<td>Low-density parity-check</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
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<tr>
<td>--------------</td>
<td>--------------------------------------------------</td>
</tr>
<tr>
<td>LLR</td>
<td>Log-likelihood ratio</td>
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<tr>
<td>LO</td>
<td>Local oscillator</td>
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<tr>
<td>LPF</td>
<td>Low-pass filtering</td>
</tr>
<tr>
<td>NLPN</td>
<td>Nonlinear phase noise</td>
</tr>
<tr>
<td>PM</td>
<td>Polarization multiplexing</td>
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<tr>
<td>PMD</td>
<td>Polarization-mode dispersion</td>
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<tr>
<td>PSD</td>
<td>Power spectral density</td>
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<tr>
<td>QAM</td>
<td>Quadrature amplitude modulation</td>
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<tr>
<td>QPSK</td>
<td>Quadrature phase-shift keying</td>
</tr>
<tr>
<td>RDPS</td>
<td>Residual dispersion per span</td>
</tr>
<tr>
<td>ROADM</td>
<td>Reconfigurable optical add-drop multiplexer</td>
</tr>
<tr>
<td>RS</td>
<td>Reed-Solomon</td>
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<tr>
<td>SDD</td>
<td>Soft-decision decoding</td>
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<tr>
<td>SDH</td>
<td>Synchronous digital hierarchy</td>
</tr>
<tr>
<td>SER</td>
<td>Symbol-error ratio</td>
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<tr>
<td>SMF</td>
<td>Single mode fiber</td>
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<tr>
<td>SNR</td>
<td>Signal-to-noise ratio</td>
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<tr>
<td>SONET</td>
<td>Synchronous optical network</td>
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<tr>
<td>SPM</td>
<td>Self-phase modulation</td>
</tr>
<tr>
<td>WDM</td>
<td>Wavelength-division multiplexing</td>
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<tr>
<td>XPM</td>
<td>Cross-phase modulation</td>
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1. Introduction

1.1 State-of-the-art long-haul optical systems

High-capacity long-haul networks, i.e., wide area networks (WANs), based on wavelength-division-multiplexed (WDM) systems are moving from 10 Gbit/s toward 100 Gbit/s in each 50-GHz channel [1]. An example of a WDM-based WAN architecture is shown in Fig. 1.1, where individual wavelength channels are routed optically using optical switches, such as reconfigurable optical add-drop multiplexers (ROADMs) [2]. Modulation and detection techniques have evolved from on-off keying with noncoherent detection to differential phase-shift keying with differentially coherent detection, and are now moving toward polarization-multiplexed (PM) quadrature phase-shift keying (QPSK) with coherent detection, enabling higher spectral efficiencies. Using advanced digital signal processing (DSP) algorithms, coherent receivers can compensate for various impairments arising from long-haul transmission.

The long-haul link consists of multiple single mode fiber (SMF) spans, with a span length of typically 80 to 100 km. Multimode fibers are not suitable for long-distance communication due to their large dispersion [3]. An optical amplifier is placed at the end of each SMF span to compensate its loss. A dispersion-compensating fiber (DCF) may be used to cancel some of the dispersion introduced by the SMF, followed by a second amplifier in order to compensate attenuation introduced by the DCF. In some spans, the lightpath passes through a ROADM. Fig. 1.2 shows a block diagram of a lightpath through a long-haul mesh network. Note that the channel quality of a lightpath can be highly variable, depending on the distance between transmitter and receiver and the number of ROADM’s through which the signal must pass. Along a given lightpath, however, the channel quality is essentially fixed for an indefinitely long period of time.
1. INTRODUCTION

Fig. 1.1 Metropolitan and long-haul networks with wavelength routing, showing two bi-directional lightpaths. For simplicity, only small subsets of metropolitan rings and lightpaths are shown.

Fig. 1.2 Path through a long-haul mesh network showing multiple spans.

Optical fiber telecommunication networks are evolving from legacy systems based on the Synchronous Optical Networking/Synchronous Digital Hierarchy (SONET/SDH) to Ethernet-based systems. As the need for high bandwidth in today’s IP-based Internet grows, IP-over-WDM has been considered as one of the promising solutions for the future optical network architecture [4],[5]. In an IP-over-WDM network, network nodes employ wavelength-routing switches (i.e., ROADMs) and IP routers. Nodes are connected by fibers to form an arbitrary physical mesh topology.
IP-over-WDM can utilize high-bandwidth transport and flexible switching capabilities offered by WDM (meeting the demands for burst and unpredictable IP traffic) and bypass intermediate layers such as Asynchronous Transfer Mode (ATM) and SDH (simplifying the protocol stack and reducing overhead and equipment costs) [6].

1.2 Coherent detection in optical fiber systems

A coherent optical transmission system is shown in Fig. 1.3. The transmitter uses a Mach-Zehnder (MZ) modulator to generate QAM signals in a single polarization and transmit the signal into the WDM system. The receiver uses a local oscillator (LO) that serves as a phase reference and homodyne down conversion in order to coherently detect the received signal.

The relationship of input and output signals for a single-drive MZ modulator is described as [7]
where $E_{\text{in}}(t)$ and $E_{\text{out}}(t)$ are the input and output electric fields of the modulator, respectively, $V(t)$ is the drive signal and $V_\pi$ is the voltage to provide a phase shift of $\pi$ between the two waveguides. Two single-drive MZ modulators can be combined to create a quadrature MZ modulator.

1.3 Characteristics of optical channel

In this section, we review the channel impairments in optical long-haul transmission. We summarize impairments in SMF and their compensation methods and compare the channel characteristics with those in wireline and wireless communication systems. We will address the impact of using multiple optical switches along the signal path in detail in Chapter 2.

1.3.1 Linear impairments in SMF

1.3.1.1 Chromatic dispersion

Chromatic dispersion (CD) occurs because the refractive index of fiber varies linearly with frequency, causing a delay spread proportional to signal bandwidth, and hence intersymbol interference (ISI). This pulse broadening arises from the frequency dependence of the propagation constant $\beta$. In the absence of nonlinearities and polarization-mode dispersion, the equivalent baseband transfer function of a fiber of length $L$ can be taken as [8]

$$H(\omega) = e^{i\beta(\omega)L},$$

(1.1)

where $\omega$ is the angular frequency. Using the Taylor-expansion of $\beta(\omega)$ and noting that the second order term has a dominant effect in dispersion, CD can be described by a baseband transfer function

$$H(\omega) = \exp\left(i \frac{\beta L}{2} \omega^2\right),$$

(1.2)
where $\beta_2$ is the dispersion parameter. CD is a fixed linear impairment and can be compensated optically by using dispersion compensating fiber. DCF introduces loss that must be compensated by optical amplification, leading to an increase in noise. Likewise, the presence of these additional dispersion compensating fibers leads to an increase in fiber nonlinearity which is described in Section 1.3.2. Alternatively, CD can be compensated digitally by a linear equalizer. Since CD is a unitary distortion, i.e., $|H(\omega) = 1|$, no noise enhancement occurs.

1.3.1.2 Polarization-mode dispersion

Polarization-mode dispersion (PMD) occurs because random asymmetry of fiber causes random birefringence, which is a random difference between the refractive indices for light in different polarizations. As a result, a signal is received with random, time-varying, frequency-dependent variations in both polarization and group delay. In an SMF with PMD, input and output electric fields can be described by two-component Jones vectors in the frequency domain, which are related by a frequency-dependent $2 \times 2$ Jones matrix as follows

$$
\begin{bmatrix}
E_{out,x}(\omega) \\
E_{out,y}(\omega)
\end{bmatrix} = U(\omega)
\begin{bmatrix}
E_{in,x}(\omega) \\
E_{in,y}(\omega)
\end{bmatrix},
$$

(1.3)

where $[E_{in,x}(\omega) E_{in,y}(\omega)]^T$ and $[E_{out,x}(\omega) E_{out,y}(\omega)]^T$ are the input and output Jones vectors, respectively, in $x$- and $y$-polarizations, and $U(\omega)$ is a Jones matrix. PMD is subject to time variation on the millisecond scale and, typically, must be compensated adaptively in the receiver. PMD can be compensated optically, but this requires complex adaptive optical systems. PMD also can be compensated by linear digital signal processing. Since $U(\omega)$ is a unitary matrix, no noise enhancement occurs.

1.3.2 Nonlinear impairments in SMF

In silica optical fibers, the fiber refractive index increases slightly in proportion to the local intensity at a point in space and time, a phenomenon called the Kerr effect. Although silica is intrinsically not a highly nonlinear material, nonlinear effects become significant because signals are confined within the small cross-sectional area
1. INTRODUCTION

of a fiber and propagate over very long distances [3]. If the signal power is increased beyond a certain level, nonlinear effects cause an increase in signal distortion and noise. Hence, nonlinearity is considered a fundamentally limiting factor to the information-theoretic capacity of optical fiber communications.

Signal propagation in the presence of nonlinearity is governed by the nonlinear Schrödinger equation

\[
\frac{\partial E}{\partial z} + \frac{i}{2} \beta_2 \frac{\partial^2 E}{\partial t^2} + \frac{\alpha}{2} E = j\gamma |E|^2 E, \tag{1.4}
\]

where \( E(z,t) \) is the electric field, \( \alpha \) is the attenuation coefficient, \( \beta_2 \) is the dispersion parameter, \( \gamma \) is the nonlinear coefficient, and \( z \) and \( t \) are the propagation direction and time, respectively. Few analytical solutions for (1.4) exist, but it can be solved numerically using the split-step Fourier method, which divides fiber into small steps of size and solves each step iteratively.

Self-phase modulation (SPM) is a nonlinear effect caused by the intensity of a signal itself, and results in a multiplicative phase noise. Four-wave mixing (FWM) and cross-phase modulation (XPM) are caused by interaction of a signal with other signals at adjacent optical carrier frequencies, and result in additive noise and multiplicative phase noise, respectively. In the absence of amplified spontaneous emission (ASE) noise, all nonlinear effects are deterministic, so it is possible in principle to pre-compensate for them at the transmitter or the receiver [9].

In amplified long-haul systems, ASE noise interacts with the signal and induces nonlinear phase noise (NLPN) via the Kerr effect [10], limiting performance of phase-sensitive detection schemes. NLPN can be induced by the interactions between ASE noise and signals of co-channel or adjacent channels. In the linear regime, phase noise is manifested as a linear sum of signal and noise components, whereas in nonlinear regime phase noise is proportional to the signal power as shown in Fig. 1.4, where \( E_I \) and \( E_Q \) denote in-phase and quardrature-phase electric fields respectively, \( E_s \) and \( E_n \) denote signal and noise electric fields respectively, \( E_{tot} \) and \( E'_{tot} \) denote total electric
1. INTRODUCTION

fields due to linear and nonlinear phase noises respectively, and $\phi_{NL}$ is the phase shift caused by nonlinear phase noise.

![Diagram of phase noise mechanisms in two regimes](image)

**Fig. 1.4** Mechanism of phase noise in two regimes (a) linear regime (b) nonlinear regime.

Since optical amplifiers add noise, intensity fluctuates randomly, causing random phase shifts. Nonlinear phase shift is correlated with received intensity, and the received constellation acquires spiral shape. Modulation formats with large amplitude changes are more susceptible to nonlinear phase noise. **Fig. 1.5** shows the

![Impact of phase noise on BPSK signal transmission](image)

**Fig. 1.5** Impact of phase noise on the transmission of BPSK signal in two regimes (a) linear regime (b) nonlinear regime.
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Impact of phase noise in the linear and nonlinear regimes for a BPSK modulated signal. In a coherent receiver, nonlinear phase noise can be compensated digitally by applying a phase shift proportional to the received intensity. However, the nonlinear interactions between signal and noise are non-deterministic, and thus nonlinear phase noise cannot be perfectly compensated.

1.3.3 Comparison with other transmission media

Table 1.1 compares the characteristics of optical channels (using SMF and coherent detection) with the characteristics of other transmission media.

<table>
<thead>
<tr>
<th>Medium</th>
<th>Distortion</th>
<th>Noise</th>
<th>Interference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wireline</td>
<td>• Frequency-dependent attenuation (fixed, linear)</td>
<td>• Additive thermal noise</td>
<td>• Coupling between wire pairs (fixed, linear)</td>
</tr>
<tr>
<td></td>
<td>• Imperfections in transmission line (fixed, linear)</td>
<td>• Quantization noise</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Crosstalk between wire pairs (fixed coupling)</td>
<td></td>
</tr>
<tr>
<td>Wireless</td>
<td>• Multipath fading (time-varying, linear)</td>
<td>• Additive thermal noise</td>
<td>• Co-channel interference (linear, time-varying coupling)</td>
</tr>
<tr>
<td>Fiber</td>
<td>• CD: frequency dependent phase distortion (fixed, linear, unitary)</td>
<td>• Additive optical amplifier noise</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• PMD: frequency dependent 2×2 matrix (time-varying, linear, unitary)</td>
<td>• FWM: additive noise (fixed or time-varying, nonlinear)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• SPM: phase shift proportional to intensity (signal-dependent, fixed coupling, nonlinear)</td>
<td>• NLPN: phase rotation correlated with received intensity (signal-dependent, fixed coupling, nonlinear)</td>
<td>• Adjacent/co-channel crosstalk (fixed or time-varying, linear, typically weak)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>• XPM: adjacent channel (fixed, nonlinear, strong)</td>
</tr>
</tbody>
</table>

Table 1.1 Comparison of channel characteristics between optical and other communication systems.
1. INTRODUCTION

1.4 Evolution of FEC in optical fiber systems

Forward error correction (FEC) schemes in optical communications have evolved over several generations, starting from single block codes with hard-decision decoding (HDD), followed by concatenated block codes with HDD and then various types of codes with soft-decision decoding (SDD), as demands for higher data rates have increased and advanced ASIC technologies have become available [11]. In recent years especially, intensive research has addressed iterative SDD of various codes, including block turbo codes [12] and low-density parity-check codes (LDPC) [13], seeking the highest possible coding gain. Table 1.2 summarizes the history of FEC technique evolution in optical fiber systems.

<table>
<thead>
<tr>
<th>Generation</th>
<th>First (1990s)</th>
<th>Second (2000s)</th>
<th>Third (since 2005)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Code type</strong></td>
<td>Single RS code</td>
<td>Concatenated block codes, Concatenated block and convolutional codes</td>
<td>Turbo product codes, LDPC codes</td>
</tr>
<tr>
<td><strong>Code rate</strong></td>
<td>0.93</td>
<td>~0.9</td>
<td>~0.85</td>
</tr>
<tr>
<td><strong>Decoding type</strong></td>
<td>Hard decision</td>
<td>Hard decision, soft decision</td>
<td>Soft decision</td>
</tr>
<tr>
<td><strong>Coding gain at BER = 10^{-13}</strong></td>
<td>5.8 dB</td>
<td>&gt; 7.5 dB</td>
<td>&gt; 10 dB</td>
</tr>
</tbody>
</table>

Table 1.2 Evolution of FEC techniques in optical fiber systems.

1.5 Motivations

Early wireline and wireless communication systems transmitted typically at fixed information bit rate, using a fixed forward error correction (FEC) code, modulation order and transmitted signal power. In such schemes, which do not exploit any channel knowledge, either link throughput is limited by the worst-case channel state or a robust link connection is not guaranteed. Many recent non-optical communication systems adapt link parameters based on channel conditions (often time-varying), trading off bit rate (and thus spectral efficiency) for robustness. For
example, asymmetric digital subscriber lines transmit data over multiple subcarriers, maximizing link capacity by dynamically loading bits per frequency bin depending on the varying level of intersymbol interference (ISI), crosstalk, and signal-to-noise ratio (SNR) in each bin [14]. Similarly, many mobile wireless communication systems, e.g., high-speed packet access (HSPA) and third-generation partnership project long term evolution (LTE), use adaptive coding and modulation to enhance performance over fading channels, based on channel quality feedback from the receiver [15] or channel estimation in the transmitter [16]. These systems also use hybrid automatic repeat request techniques to improve the throughput of the radio link, by combining variable-rate FEC codes and automatic repeat query functionalities in the physical layer [16],[15].

Previous optical transmission systems, such as those based on synchronous optical networking and synchronous digital hierarchy, transmit at fixed bit rates. As Ethernet-based systems are deployed in large-scale IP networks [17], transceivers using coherent detection and digital signal processing will enable higher bit rates and spectral efficiencies, and could also support rate-adaptive transmission, with bit rates negotiated between routers and transceivers. Optical switches, such as reconfigurable optical add-drop multiplexers (ROADMs), make it possible to route signals optically over longer distances but add loss and can limit signal bandwidth, all of which can make signal quality more variable. Rate-adaptive transmission could help improve network robustness, flexibility and throughput. Although present standardization efforts for optical transport networks are concentrated on transmitting client data at fixed rates [18], rate-adaptive modulation and coding may merit consideration for future networks.

Variable-rate transmission in optical fiber has been studied previously, but only in the context of code-division multiple access [19], or with a limited range of variation in code rate or modulation order in highly idealized optical channels [20],[21],[22]. In this thesis, we study specific rate-adaptive transmission schemes in detail and propose practical solutions that can be implemented in future deployed optical fiber systems.
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1.6 Thesis organization

This work is presented in three stages: adaptive coding (AC) with a fixed modulation and an HDD FEC scheme, adaptive coding and modulation (ACM) with an HDD FEC scheme, and ACM with a SDD FEC scheme.

In Chapter 2, we describe a rate-adaptive transmission scheme for long-haul systems using a family of variable-rate hard-decision codes with a fixed modulation format (PM-QPSK) at a fixed symbol rate. Using simulation, we study achievable information bit rates as a function of transmission distance in a model terrestrial long-haul network.

In Chapter 3, we extend the system of Chapter 2 to variable-size constellations and variable-rate FEC codes with HDD. In systems with or without inline CD compensation, we quantify the achievable information bit rate vs. distance and estimate the performance gap between the proposed scheme and an ideal coding scheme that achieves information-theoretic limits.

In Chapter 4, we propose a rate-adaptive transmission scheme using variable-size constellations and variable-rate FEC codes with SDD. We quantify the achievable information bit rate vs. distance and estimate the performance gap from an ideal coding scheme in systems with or without inline CD compensation. We also quantify performance improvements between HDD and SDD FEC schemes.

In Chapter 5, we conclude the thesis and discuss directions for future work.
2. Adaptive Hard-Decision Coding and Fixed Modulation

2.1 Introduction

Variable-rate transmission in optical fiber has been studied to date in the context of code-division multiple access [19], or with limited available rate variation [20],[21]. In this chapter, we describe a rate-adaptive transmission scheme for long-haul systems using a fixed modulation format (PM-QPSK) at a fixed symbol rate, with a family of hard-decision codes permitting rate variation over a wide range of code rates. Using simulation, we study achievable information bit rates as a function of transmission distance in a model terrestrial long-haul network.

This chapter is organized as follows. In Section 2.2, we review the evolution of FEC techniques in optical fiber systems. We then describe our variable-rate FEC scheme and a rate adaptation algorithm that uses measured SNR or FEC decoder input BER to determine the maximum FEC code rate that can be supported. In Section 2.3, we describe simulations of the rate-adaptive scheme in a model terrestrial network. In Section 2.4, we present simulation results, including achievable information bit rates as a function of distance. In Section 2.5, we discuss the observed trend of SNR vs. distance and compare the performance of the proposed scheme to information-theoretic limits.

2.2 Rate-Adaptive Coding Scheme

2.2.1 Evolution of FEC in Optical Fiber Systems

The default FEC scheme of ITU-T G.709 is a Reed-Solomon (RS) code with parameters \( (n, k) = (255, 239) \) [18], where \( n \) and \( k \) indicate the length in bytes of the code word and message word, respectively. This code, adopted during the 1990s, is
2. ADAPTIVE HARD-DECISION CODING AND FIXED MODULATION

sometimes classified as first-generation FEC in optical fiber systems [12], and yields a net coding gain\(^1\) of approximately 5.8 dB at a decoded BER of \(10^{-13}\).

Short component codes can be concatenated in a serial or parallel manner to build a longer code and hence achieve a higher coding gain [23]. Linear block codes, e.g., RS and Bose/Ray-Chaudhuri/Hocquenghem (BCH) codes, are commonly used as component codes. Concatenated codes with several different types of component codes were recommended by ITU-T G.975.1 during the early 2000s [24], and are sometimes classified as second-generation FEC in optical fiber systems. The net coding gains of these codes are typically greater than 7.5 dB at a decoded BER of \(10^{-13}\).

In recent years, intensive research has addressed iterative soft-decision decoding of various codes, e.g., block turbo codes [12] and low-density parity-check codes [25], seeking further increases in coding gain. These types of codes are sometimes called third-generation FEC in optical fiber systems, and their target net coding gain is above 10 dB at a decoded BER of \(10^{-13}\).

### 2.2.2 Variable-Rate FEC Coding Scheme

We consider serially concatenated systematic RS codes over \(GF(2^8)\) that offer coding gains comparable to second-generation FEC codes. We refer to these as RS-RS codes throughout the remainder of this chapter. Fig. 2.1 shows a high-level block diagram of the encoding and decoding chain for RS-RS codes. The encoder chain consists of an outer RS code, a linear block interleaver, an inner RS code, and a repetition code as the innermost code. The RS codes are nonbinary cyclic linear block codes that have strong capability to correct both random and burst errors, and are decoded using the Berlekamp-Massey algorithm or the generalized Euclidean algorithm [26],[27]. An innermost repetition code is added to compensate for the diminishing coding gain of RS-RS codes as the code rate is decreased in the low-SNR

---

\(^1\) We define *net coding gain* as the reduction in SNR per bit enabled by a code at a specified decoder output BER as compared to an uncoded system, assuming an additive white Gaussian noise (AWGN) channel. The net coding gain takes account of an increased noise variance in the coded system.
region. In the receiver, de-repetition is performed first, involving soft accumulation of repeated symbols, followed by a hard decision. De-repeated hard decisions are fed into the inner RS decoder, followed by a de-interleaver and the outer RS decoder. It would be possible to replace the repetition code by a stronger innermost code to enhance performance at low rates. However, our goal is to keep the scheme as simple as possible while ensuring that it works well in the high-rate regime near 100 Gbit/s, and reasonably well at much lower rates.

![FEC chain for encoding and decoding serially concatenated RS-RS and repetition codes.](image)

The proposed rate-adaptive coding scheme is intended to be used with a fixed constellation and fixed symbol rate, which is intended to simplify implementation of analog and digital hardware. As the SNR changes, the rates of the RS-RS and repetition codes are adjusted to maximize the overall rate. There are several options for constructing variable-rate block codes. One approach is simply to use entirely different codes at different rates, but this comes at the expense of increased encoder and decoder complexity. The alternative approaches are to puncture or shorten a single code, called a mother code, to generate derived codes of various rates. Puncturing removes parity bits from a codeword after encoding, thus increasing the code rate. The decoder treats the punctured parity bits as erasures. In puncturing, performance loss increases as the code rate increases, as compared to codes having the same code rate but having codeword length equal to that of the mother code. Shortening deletes information bits from a codeword, decreasing the code rate. The decoder knows the locations and pattern of the deleted bits. In shortening, performance loss increases as the code rate decreases.
2. ADAPTIVE HARD-DECISION CODING AND FIXED MODULATION

Fig. 2.2 Hybrid method of constructing variable-rate codes by puncturing and shortening a single mother code, which is selected from near the middle of the code rate range.

$$\text{Mother code rate} = \frac{k_0 n_0}{n n_0}$$

$$\text{Derived code rate} = \frac{k' n_0}{n' n_0}$$

We employ a hybrid method of creating multi-rate codes, which represents a tradeoff between complexity and performance. First, a mother code is selected, having a rate lying in the middle of the range of interest. Higher-rate codes are obtained by puncturing the mother code, and lower-rate codes are generated by shortening the mother code. This hybrid code construction method is shown schematically in Fig. 2.2. The numbers of bits punctured or shortened in the various derived codes are shown in Table 2.1. The parameter $r_C = k/n$ is the rate of the concatenated RS-RS code, with the mother code having a rate $r_C = 0.6359$. Fig. 2.3 shows the performance of the five constructed RS-RS codes, in terms of the relationship between $P_{b,\text{in}}$ and

<table>
<thead>
<tr>
<th>$n_o$</th>
<th>255</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_o$</td>
<td>239</td>
</tr>
<tr>
<td>$n_i$</td>
<td>255</td>
</tr>
<tr>
<td>$k_i$</td>
<td>173</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$r_C$</th>
<th>$\Delta n_i^0$</th>
<th>$\Delta n_i^0 (= \Delta k_i^0)$</th>
<th>$n'_i$</th>
<th>$k'_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8765</td>
<td>70</td>
<td>0</td>
<td>185</td>
<td>173</td>
</tr>
<tr>
<td>0.7542</td>
<td>40</td>
<td>0</td>
<td>215</td>
<td>173</td>
</tr>
<tr>
<td>0.6359</td>
<td>0</td>
<td>0</td>
<td>255</td>
<td>173</td>
</tr>
<tr>
<td>0.5218</td>
<td>0</td>
<td>70</td>
<td>185</td>
<td>103</td>
</tr>
<tr>
<td>0.3998</td>
<td>0</td>
<td>112</td>
<td>143</td>
<td>61</td>
</tr>
</tbody>
</table>

Table 2.1 Parameters for puncturing and shortening a mother code
2. ADAPTIVE HARD-DECISION CODING AND FIXED MODULATION

$P_{b,\text{out}}$, the BERs at the input and output of the RS-RS decoder, respectively (when combined with repetition coding, $P_{b,\text{in}}$ is measured at the output of the de-repeater). The curves in Fig. 2.3 have been calculated analytically assuming independent errors and no decoding failures [28]. The highest-rate code, with code rate $r_C = 0.8765$, offers a net coding gain of about 7.6 dB at a decoded BER of $10^{-13}$.

![Fig. 2.3 Performance of serially concatenated RS-RS codes with five different rates, in terms of input and output BERs of the RS-RS decoder. When combined with an innermost repetition code, the $P_{b,\text{in}}$ is measured at the output of the de-repeater.](image)

Since we use repetition coding as an innermost code, the overall FEC code rate is $r_C r_R = r_C f_R$, where $r_R = 1/f_R$ is the rate of the repetition code. The repetition factor $f_R$ denotes the number of repetitions of each bit from the output of the inner RS encoder, and ranges from 1 to 4. Given a symbol rate $R_s$, a line code rate $r_L$, and a modulation order $M$, the information bit rate $R_b$ can be calculated as

$$R_b = 2 r_L r_C r_R R_s \log_2 M,$$

assuming PM transmission.

2.2.3 Rate Adaptation Algorithm

We assume that there exists a feedback link by which the receiver can communicate channel state information (CSI) to the transmitter, and that a controller at the transmitter can adjust the FEC code rate depending on the reported CSI. We
consider two possible choices for CSI: (a) \(\text{SNR}\), which is the SNR per symbol estimated at the de-repeater soft input, or (b) \(P_{b,\text{in}}\), which is the BER of hard decisions measured at the de-repeater output (RS-RS decoder input). A rate adaptation algorithm using \(\text{SNR}\) as CSI is straightforward in terms of mapping a CSI value to an expected transmission rate, but may not reliably yield the highest possible information bit rate when \(\text{SNR}\) is not an accurate predictor of \(P_{b,\text{in}}\), in particular when \(P_{b,\text{in}}\) is a function of \(\text{SNR}\) as well as other factors (e.g., noise distribution, nonlinearities and other impairments). On the other hand, an algorithm using \(P_{b,\text{in}}\) as CSI requires a more complicated CSI-to-rate mapping, in particular when repetition codes are used, but may be a better parameter to adapt to since it captures the effects of noise as well as other impairments, and hence reliably yields the highest information bit rates even when \(\text{SNR}\) is not an accurate predictor of \(P_{b,\text{in}}\). Obtaining \(P_{b,\text{in}}\) is straightforward under RS decoding because RS decoder can find the number of corrected bits [27].

We first describe rate adaptation using \(\text{SNR}\) as CSI. Assuming AWGN, the SNR per symbol averaged over the two polarizations estimated empirically at the de-repeater soft input is related to the empirically estimated Q factor\(^2\) as:

\[
\text{SNR} = F_Q^{-2}. \tag{2.2}
\]

The BER of received channel bits (corresponding to hard decoding the de-repeater input) is [3]:

\[
P_{b,\text{in}} = \frac{1}{2} \text{erfc} \left( \frac{F_Q}{\sqrt{2}} \right) = \frac{1}{2} \text{erfc} \left( \sqrt{\frac{\text{SNR}}{2}} \right), \tag{2.3}
\]

where \(\text{erfc}(\cdot)\) is the complementary error function. Also assuming AWGN, with repetition factor \(f_R\) and soft decision decoding of the repetition code, the SNR and Q factor at the de-repeater output become [29]:

\(^2\) The SNR per symbol is estimated empirically as the ratio between estimates of the received energy per symbol and the noise variance, both measured in two polarizations. The Q factor is estimated empirically as the ratio between the mean level separation and sum of the estimated noise standard deviations, averaging ratios computed for the inphase and quadrature components in two polarizations.
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\[ SNR_{rep} = f_R SNR = f_R F_Q^2 = F_{Q,rep}^2, \]  

and the BER of hard decisions at the de-repeater output (RS-RS decoder input) becomes:

\[ P_{b,in} = \frac{1}{2} \text{erfc} \left( \frac{F_{Q,rep}}{\sqrt{2}} \right) = \frac{1}{2} \text{erfc} \left( \sqrt{\frac{SNR_{rep}}{2}} \right). \]  

Fig. 2.4 Achievable information bit rate \( R_b \) vs. SNR per symbol using RS-RS codes and repetition factor \( f_R \) for PM-QPSK on AWGN channel. Information bit rates are computed using (2.1), while SNR values are computed by combining (2.4), (2.5) and Fig. 2.3. The set of 13 filled \((f_R,\text{code})\) combinations represent a possible choice of modes for rate-adaptive transmission.

In order to determine the transmission rate, the controller requires information on the minimum \( SNR \) required for each \((f_R,r_C)\) pair to achieve a target decoder output BER \( P_{b,out,req} \) and the resulting \( R_b \). This information is shown in Fig. 2.4, which assumes AWGN. Although the optical channel is not strictly Gaussian, it is commonly assumed that dispersion can be completely compensated and the remaining uncompensated nonlinearity is similar to white noise. Hence the AWGN model is universally used for optical channels [30]. Information bit rates are computed using (2.1) and required SNR values are computed by combining (2.4), (2.5) and Fig. 2.3 with \( P_{b,out,req} = 10^{-15} \) (a typical target BER range in optical fiber transmission is \( 10^{-15} \) - \( 10^{-12} \)). The curves for \( f_R \geq 2 \) in Fig. 2.4 are computed from the curve for \( f_R = 1 \)
by dividing $R_b$ by $f_R$ and subtracting $10\log_{10}(f_R)$ from $SNR$. A practical rate adaptation algorithm would quantize $SNR$ values into discrete segments, and select a single combination $(f_R, r_C)$ that achieves the highest $R_b$ within each quantized segment. These predefined transmission parameters are commonly referred to as modes. In Fig. 2.4, the 13 filled-in $(f_R, r_C)$ combinations can be used as transmission modes in a rate adaptation algorithm.

![Diagram](image)

Fig. 2.5 (a) Transmission modes with $SNR$ margin of $\mu = 5$ dB using $SNR$ as CSI, taking into account $SNR$ penalties, $\Delta_1 = 0$, $\Delta_2 = 0.25$, $\Delta_3 = 0.5$, $\Delta_4 = 0.75$ in dB. (b) Example of rate adaptation algorithm using $SNR$ as CSI. The transmission modes for margins $\mu_{up} = 5$ dB and $\mu_{down} = 1$ dB are computed using (a).

We propose an algorithm that uses reported $SNR$ as CSI to determine transmission rate. Assuming there are $N_s$ segments of quantized $SNR$, i.e., there are $N_s$
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modes, we define \((fR_rC)\) as the optimal code combination for segment \(i\), and \(SNR_{th,i}\) as the minimum SNR (in dB) with which \((fR_rC)_i\) can achieve \(P_{b,\text{out,req}}\) on an AWGN channel, where \(SNR_{th,i} < SNR_{th,i+1}, \ i = 1, 2, \ldots, N_s - 1\). The information bit rate of segment \(i\) is denoted by \(Rb_i\), where \(Rb_i < Rb_{i+1}, \ i = 1, 2, \ldots, N_s - 1\). For example, in Fig. 2.4, the number of transmission modes is 13, \((fR_rC)_9 = (1,14579/36465), SNR_{th,9} = 5.8 \text{ dB}, \text{ and } Rb_9 = 45.6162 \text{ Gbit/s.}\)

In the presence of fiber nonlinearity, residual ISI and laser phase noise, the receiver noise is not necessarily AWGN, and thus (2.2)-(2.5) are not necessarily valid, i.e., SNR may not be an accurate predictor of \(P_{b,\text{in}}\). In this case, we could modify all of the threshold SNRs (the \(SNR_{th,i}\)). Updating the threshold SNR for mode \(i\) would require searching for the SNR that achieves target \(Rb_i\) (satisfying \(P_{b,\text{out,req}}\)) that is predicted by the threshold SNR with AWGN assumption. In the interest of simplicity, we introduce penalty parameters \(\Delta f_{f}\) (in dB) for \(fR = 1, 2, 3, 4\), which quantify the increase in SNR required to achieve the desired \(P_{b,\text{in}}\) at repetition factor \(fR\), as compared to the AWGN case described by (2.4) and (2.5). We have introduced different \(\Delta f_{f}\) for each \(fR\), which can be necessary when performing repetition in a suboptimal manner, but is unnecessary when performing repetition optimally (see Section 2.4). Also, to allow for some performance margin to trade off the information bit rate for robustness, we introduce SNR margin parameters \(\mu_{\text{up}}\) and \(\mu_{\text{down}}\) (in dB) that are added to the \(SNR_{th}\) when changing the rate upward and downward, respectively. The actual margin function can be implemented in the transmitter depending on several factors such as channel states, packet type (either control or data), etc. Choosing \(\mu_{\text{up}} > \mu_{\text{down}}\) provides hysteresis, such that SNR fluctuations of magnitude less than \(\mu_{\text{up}} - \mu_{\text{down}}\) will not cause rate changes. Transmission modes with \(\mu \text{ dB} \) margin, and incorporating SNR penalty parameter \(\Delta f_{f}\), can be defined by shifting the curves of Fig. 2.4 to the right by \(\mu + \Delta f_{f} \text{ dB},\) as illustrated in Fig. 2.5 (a).

In order to determine the transmission rate, starting from the current \(SNR_{th}\), the controller monitors the reported SNR. If SNR exceeds the \(SNR_{th}\) corresponding to the next-higher rate for \(N_{\text{up}}\) consecutive observations, then the controller raises \((fR_rC)\) by
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one level. A similar logic is applied to when lowering the rate, using a maximum downward threshold counter $N_{\text{down}}$. This counter-based approach can prevent unnecessary rate changes when the channel undergoes a brief degradation.

A pseudocode for the algorithm is given as follows:

1. Initialize parameters to the highest-rate mode (assuming that short-distance transmission is the dominant regime).
   - Initialize codes: $(f_R, r_C)_i \rightarrow (f_R, r_C)_{N_s}$
   - Initialize up/down counters: $C_{up} = C_{down} = 0$

2. Check if rate change is necessary.
   - if $\text{SNR} < (\text{SNR}_{th,i} + \Delta f + \mu_{\text{down}})$
     - $C_{\text{down}} = C_{\text{down}} + 1$
     - if $C_{\text{down}} \geq N_{\text{down}}$, $(f_R, r_C)_i \rightarrow (f_R, r_C)_{i-1}$
   - elseif $\text{SNR} \geq (\text{SNR}_{th,i+1} + \Delta f + \mu_{\text{up}})$
     - $C_{\text{up}} = C_{\text{up}} + 1$
     - if $C_{\text{up}} \geq N_{\text{up}}$, $(f_R, r_C)_i \rightarrow (f_R, r_C)_{i+1}$
   - else
     - $C_{\text{up}} = C_{\text{down}} = 0$
     - $(f_R, r_C)_i \rightarrow (f_R, r_C)_i$

3. Go to step 2.

The described algorithm is illustrated by Fig. 2.5 (b), which shows the hysteresis obtained when $\mu_{\text{up}} > \mu_{\text{down}}$. An example of rate adaptation using the proposed algorithm is shown in Fig. 2.6, which assumes $\mu_{\text{up}} = 3$ dB, $\mu_{\text{down}} = 2$ dB, $N_{\text{up}} = N_{\text{down}} = 3$, $\Delta f_1 = 0$ dB, $\Delta f_2 = 0.25$ dB, $\Delta f_3 = 0.5$ dB, $\Delta f_4 = 0.75$ dB, and the mode quantization shown in Fig. 2.4. It is observed that the algorithm tracks the reported $\text{SNR}$ and adapts the rate as desired.
We now describe an algorithm using the de-repeater output BER (RS-RS decoder input BER) $P_{b,\text{in}}$ as CSI. This BER can be estimated by the decoder from the number of bit errors corrected in decoding. In this case, the receiver would report the measured $P_{b,\text{in}}$, and the controller would run the algorithm given above using BER thresholds $P_{b,\text{in,th,i}}$, computed by substituting $\text{SNR}_{\text{th},i}$ into (2.4) and (2.5). The resulting transmission modes are shown in Fig. 2.7. Measuring $\text{SNR}_{\text{th},i}$ in dB and including an SNR margin of $\mu$ dB, the threshold $P_{b,\text{in,th,i}}$ can be computed as \[ P_{b,\text{in,th,i},M} = \frac{1}{2} \text{erfc} \left( \sqrt{\frac{f_p 10^{(\text{SNR}_{\text{th},i} + M)/10}}{2}} \right), \] (2.6)
and the corresponding transmission modes with margin are defined as in Fig. 2.8 (a).

Although (2.4) and (2.5) assume AWGN, once the $P_{b,\text{in,th,i},\mu}$ are computed, the algorithm is intended to work even on non-AWGN channels without the need for SNR penalty parameters since satisfying $P_{b,\text{in,th,i}}$ guarantees achieving $P_{b,\text{out,req}}$ and $R_{b,i}$. A pseudocode for the algorithm using $P_{b,\text{in}}$ as CSI is given as follows:

1. Initialize parameters to the highest-rate mode.
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- Initialize codes: \((f_R, r_C)_i \rightarrow (f_R, r_C)_{N_s}\)
- Initialize up/down counters: \(C_{up} = C_{down} = 0\)

2. Check if rate change is necessary.
- if \(P_{b,in} > P_{b,in,th,i,\mu_{down}}\)
  - \(C_{down} = C_{down} + 1\)
  - if \(C_{down} \geq N_{down}\), \((f_R, r_C)_i \rightarrow (f_R, r_C)_{i-1}\)
- elseif \(P_{b,in} \leq P_{b,in,th,i+1,\mu_{up}}\)
  - \(C_{up} = C_{up} + 1\)
  - if \(C_{up} \geq N_{up}\), \((f_R, r_C)_i \rightarrow (f_R, r_C)_{i+1}\)
- else
  - \(C_{up} = C_{down} = 0\)
  - \((f_R, r_C)_i \rightarrow (f_R, r_C)_i\)

3. Go to step 2.

The described algorithm is depicted in Fig. 2.8 (b).

Note that, in either algorithm, as an alternative to using counter parameters \(N_{up}\) and \(N_{down}\), there exist other possible mechanisms to guard against rate changes due to short transients in CSI, e.g., comparing a moving average of CSI to a threshold CSI, i.e., \(SNR_{th,i}\) or \(P_{b,in,th,i}\), trading off computing complexity for performance.
Also note that, when using either algorithm, a controller at the transmitter should set the launched power $P_t$ to optimize the CSI. Optimal launched power values may be stored in a look-up table or determined on the fly, based on CSI reported by the receiver. When using $SNR$ as CSI, at each transmission distance, by definition, a single $P_t$ is optimal for all values of the repetition factor $f_R$. When using $P_{b,in}$ as CSI, at each transmission distance, the optimal $P_t$ may or may not depend on $f_R$, as illustrated in Section 2.4.

Fig. 2.8 (a) Transmission modes with SNR margin of $\mu = 3$ dB, using $P_{b,in}$ as CSI. (b) Example of rate adaptation algorithm using $P_{b,in}$ as CSI. The transmission modes for margins $\mu_{up} = 4$ dB and $\mu_{down} = 1$ dB are computed using (a).
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2.2.4 Implementation Considerations

In simulating our model long-haul system, we use a repetition code block size of 66, i.e., repetitions are separated by 66 symbols, to make the scheme compatible with the 64b/66b line coding used in 10 Gbit Ethernet and proposed for 100 Gbit Ethernet [31]. In order to be consistent with these standards, we also assume that line encoding occurs first and FEC encoding follows.

We assume that line, RS-RS, and repetition encoding are performed on a single serial bit stream, after which, blocks of $66f_R$ bits are permuted and mapped in round-robin fashion to the four dimensions of the PM-QPSK signal ($x$ and $y$ polarizations, inphase and quadrature components). This particular implementation of encoding, permutation, and symbol mapping is illustrated in Fig. 2.9. In order to minimize “noise” correlation between repeated symbols caused by fiber nonlinearity, when $f_R > 1$, the second, third and fourth repetition of a block are permuted using the different...
matrices shown. We observed that these permutations do not significantly modify the desirable correlation properties of pseudorandom binary sequence, such as those used for line coding in [31]. The performance benefits obtained by permutation will be described in Section 2.4.

If it is necessary in practice to implement parallel FEC encoders, it would seem natural to parallelize the input information bit stream into four tributaries, perform line and RS-RS encoding on each tributary independently, and map each encoded tributary to one particular signal dimension. We expect that in practice, performance should essentially be independent of the implementation of encoding and symbol mapping, provided that residual ISI and “noise” correlation caused by fiber nonlinearity are not significant.

Fig. 2.10 Model long-haul optical system that consists of line and FEC encoder/decoder, equalizer, PM-QPSK modulator/demodulator, dispersion precompensation fiber, and multiple spans. The dashed line encloses the FEC encoding and decoding chains, which are detailed in Fig. 2.1.

2.3 System Simulations

We have evaluated the rate-adaptive transmission scheme in the model long-haul system shown in Fig. 2.10. System design parameters are summarized in Table 2.2.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
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</thead>
<tbody>
<tr>
<td>Center wavelength</td>
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<td>Nm</td>
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<td>Nominal channel bandwidth</td>
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<td>GHz</td>
</tr>
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<td>Symbol rate $R_s$</td>
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<td>Gsymb/s</td>
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<tr>
<td>Bandwidth containing 99% of modulated signal power</td>
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<td>GHz</td>
</tr>
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<tr>
<td>Repetition factor $f_R$</td>
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<tr>
<td>Repetition code block size</td>
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<td>bit</td>
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<td>Target decoder output BER $P_{b, \text{out}req}$</td>
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</tr>
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<td>SMF length per span</td>
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<tr>
<td>Inline DCF length per span</td>
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<td>Pre-compensation DCF length</td>
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<tr>
<td>DCF loss coefficient</td>
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<td>DCF CD coefficient</td>
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<td>SMF/DCF PMD coefficient</td>
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<td>SMF nonlinear coefficient</td>
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<td>DCF nonlinear coefficient</td>
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<td>ROADM maximum center frequency error</td>
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</tr>
<tr>
<td>Amplifier noise figure</td>
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<td>dB</td>
</tr>
</tbody>
</table>

Table 2.2 Parameters used in system model.

We have simulated a single channel in a wavelength-division-multiplexed system with nominal 50-GHz channel spacing. The modulation is single-carrier PM-QPSK using non-return-to-zero pulses at a symbol rate $R_s = 29.4152$ Gsymb/s. Each modulator is a quadrature Mach-Zehnder device. Each drive waveform is a train of rectangular pulses filtered by a five-pole Bessel LPF having a 3-dB bandwidth $1.4R_s = 41.2$ GHz. Percentages 90, 95 and 99% of the modulated signal energy are contained in bandwidths of 28.9, 33.2 and 40.4 GHz, respectively. We assume line coding at rate
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\[ r_L = 64/66, \] yielding an information bit rate \( R_b = 100 \text{ Gbit/s} \) for the highest overall code rate \( r_C R \).

The fiber network comprises multiple 80-km spans of standard single-mode fiber, using pre-compensation of chromatic dispersion (CD) by \(-510 \text{ ps/nm}\) and inline CD compensation to \(42.5 \text{ ps/nm (3.1\%)\}}\) residual dispersion per span. Each span uses a two-stage inline amplifier to compensate span loss, with the ratio between first- and second-stage gains optimized via simulation.

![Fig. 2.11 Effect of channel bandwidth narrowing due to multiple passes through ROADMs, which are present at every third span.](image)

In order to simulate transmission through a large-scale mesh network, a ROADM is inserted at every third span. Filtering by each wavelength-selective switch is simulated by a super-Gaussian bandpass filter of order 2.5 and 3-dB bandwidth of 40 GHz, consistent with typical devices [2]. The frequency response of the super-Gaussian filter is described by

\[ H(\omega) = \exp \left( -b \omega^{2n} \right), \quad (2.7) \]

where \( \omega \) is the angular frequency, \( n \) is the order of super-Gaussian function and the parameter \( b \) is given by
where $B$ is the 3-dB bandwidth. Over long distances, the ROADMs cause significant channel narrowing, as shown in Fig. 2.11. Typical networks would not have ROADMs at such precisely periodic intervals and at such high density throughout, but we choose a homogeneous, high-density distribution of ROADMs to be conservative and to obtain a simple trend of achievable bit rate vs. transmission distance.

The receiver employs a fifth-order Butterworth anti-aliasing filter of 3-dB bandwidth $R_s$, samples at a rate of $2R_s$ complex samples per polarization and performs digital compensation of CD and polarization-mode dispersion (PMD) using finite impulse response time-domain filtering, as described in [32]. At each transmission distance, the number of filter taps is optimized to minimize residual ISI. The number of taps employed is about twice that given by the formula in [32], which would result in a 2-dB penalty from residual ISI, where the penalty was measured as the ratio between the unbiased SNR at the MMSE equalizer output and the SNR considering only AWGN. The equalizer is adapted using the least mean squares algorithm with the parameters of step size, shift interval, and number of shifts optimized.

Signal propagation is simulated by numerical integration of the vector nonlinear Schrödinger equation by the split-step Fourier method [33].

At each transmission distance, and for each repetition factor $f_R$, the launched power is optimized to maximize $\text{SNR}_{\text{rep}}$, the SNR at the de-repeater output (RS-RS decoder input), which we show in Section 2.4 is equivalent to minimizing $P_{b,\text{in}}$, the corresponding BER. After simulation of a sufficient number of symbols to obtain at least 100 bit errors at the de-repeater output (at least for values of $P_{b,\text{in}}$ down to those corresponding to $P_{b,\text{out}} = 10^{-15}$), we record $P_{b,\text{in}}$, $\text{SNR}_{\text{rep}}$ and $F_{Q,\text{rep}}$ (at the de-repeater output) and $\text{SNR}$ and $F_Q$ (at the de-repeater input). We note that, in principle, all of this information could be determined by a real receiver.
2.4 Simulation Results

Fig. 2.12 shows $SNR_{rep}$, the SNR at the de-repeater output (RS-RS decoder input) vs. transmitted power $P_t$ for various repetition factors $f_R = 1, 2, 3, 4$, for a representative transmission distance, $L = 2400$ km. If the total “noise” were AWGN, we would expect improvements in $SNR_{rep}$ of 3.0, 4.8 and 6.0 dB for $f_R = 2, 3, 4$ as compared to $f_R = 1$. Employing repetition coding without permutation (dashed curves in Fig. 2.12), we observe improvements in $SNR_{rep}$ of 2.3, 3.5 and 4.4 dB for $f_R = 2, 3, 4$. We also notice that the optimized transmitted power $P_t$ is smaller for $f_R = 2, 3, 4$ than for $f_R = 1$. The observed improvement in $SNR_{rep}$ is less than expected for AWGN because about half of the total “noise” variance arises from nonlinear effects, including self-phase modulation (SPM) [3] and nonlinear phase noise (NLPN) [10], which are correlated between repeated symbols. Employing repetition coding with the permutation method described in Section 2.2.4 (solid curves in Fig. 2.12), we observe improvements in $SNR_{rep}$ of 3.0, 4.8 and 6.0 dB, which are the full improvements expected for AWGN. We note that the optimized transmitted power $P_t$ is the same for $f_R = 1, 2, 3, 4$. We also verified that repetition coding with permutation is equally effective in systems employing standard SMF without inline CD compensation, i.e., the specific permutation matrices shown in Fig. 2.9 are effective even when CD leads to a memory length as long as 35 symbols. Repetition coding with permutation is used in all studies described hereafter.

Fig. 2.13 compares the BER observed at the de-repeater output (RS-RS decoder input) to the BER predicted from $SNR_{rep}$, using (2.4) and (2.5) (only BERs obtained at optimized values of transmit power $P_t$ are shown). Good agreement is obtained, showing that maximizing $SNR_{rep}$ does minimize $P_{b,in}$, and suggesting that approximation of the total “noise” as Gaussian-distributed is valid. Under these circumstances, a rate adaptation algorithm using SNR as CSI could employ penalty parameters $\Delta_{f_R} = 0$ for all $f_R$. 
Fig. 2.12 SNR at de-repeater output (RS-RS decoder input) as a function of transmitted power, for various repetition factors, with and without permutation of repeated symbols, for $L = 2400$ km.

We have employed the rate adaptation algorithm of Section 2.2.3, using $P_{b,in}$ as CSI, and using all 20 transmission modes $(r_C, f_R)$ shown in Fig. 2.7. We assume a required FEC decoder output BER $P_{b,out,req} = 10^{-15}$, SNR margins $\mu_{up} = \mu_{down} = 0, 1,$
... 5 dB and counter parameters $N_{up} = N_{down} = 1$ (because we assume static channel conditions). Fig. 2.14 shows achievable information bit rates vs. transmission distance for the six different values of SNR margin. With zero margin, an information bit rate $R_b = 100$ Gbit/s can be realized up to 2000 km, with the achievable rate decreasing by approximately 40% for every additional 1000 km. Also, a repetition factor $f_R = 1$ (no repetition) is found to be optimal up to about 3500 km, beyond which the optimal $f_R$ increases by 1 approximately every 700 km.

![Graph showing achievable information bit rate vs. transmission distance for margins between 0 and 5 dB.](image)

Fig. 2.14 Achievable information bit rate vs. transmission distance for margins between 0 and 5 dB. The pair $(f_R, code)$ denotes the repetition factor and type of RS-RS code.

### 2.5 Discussion

In this section, we examine how the SNR in the model system using a fixed PM-QPSK constellation scales with transmission distance, and we provide an approximate estimate of the performance gap between the proposed rate-adaptive coding scheme and an ideal coding scheme achieving information-theoretic limits. We consider several different ways to quantify SNR and compare these to an equivalent SNR corresponding to the information bit rate achieved by the proposed rate-adaptive scheme. This discussion makes reference to Fig. 2.15.
The uppermost solid curve in Fig. 2.15 shows $\text{SNR}_{\text{AWGN}}$, which is an empirical estimate of the SNR per symbol (in two polarizations) as limited only by accumulated optical amplifier noise:

$$\text{SNR}_{\text{AWGN}} = \frac{P_t}{P_n}. \quad (2.9)$$

$P_t$ is the transmitted signal power, optimized at each transmission distance without repetition encoding ($f_R = 1$), which equals the received signal power at the demultiplexer input, since the network is designed to have unit signal gain. The noise power is:

$$P_n = S_n W, \quad (2.10)$$

where $S_n$ is the power spectral density (PSD) of the accumulated amplifier noise at the demultiplexer input, and $W$ is the noise bandwidth of the receiver filter. Since the receiver employs near-Nyquist rate sampling at rate $2R_c$ followed by a linear equalizer, we assume that the receiver noise bandwidth effectively equals that of an ideal matched filter, $W = R_c$. We compute the accumulated amplifier noise PSD (in two polarizations) analytically using:
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\[ S_n = 2 \sum_{k=1}^{N_{\text{sp}}} (S_{sp,1,k} e^{-\alpha_d L_d} T_k G_{2,k} + S_{sp,2,k}) \]  (2.11)

In the \( k \)th span, \( S_{sp,j,k} \) and \( G_{j,k} \) are the noise PSD and gain of the \( j \)th amplifier (\( j = 1,2 \)), \( T_k \leq 1 \) is the transmission factor of the ROADM (if present), and \( \alpha_d \) and \( L_d \) are the loss coefficient and length per span of the DCF. Eq. (2.11) makes use of the fact that the noises from the spans accumulate with equal weights, since each span has unit gain. The noise PSD (per polarization) \( S_{sp,j,k} \) is given by 

\[ S_{sp,j,k} = n_{sp,j} (G_{j,k} - 1) h \nu, \]

where \( n_{sp,j} \) is the population inversion factor, \( h \) is Planck's constant, and \( \nu \) is the optical frequency.

The dashed curve in Fig. 2.15 shows \( \text{SNR}_{\text{AWGN, best-fit}} \), which is a power-law fit to the observed \( \text{SNR}_{\text{AWGN}} \) as a function of transmission distance \( L \):

\[ \text{SNR}_{\text{AWGN, best-fit}} = \frac{A}{L^{0.55}}. \]  (2.12)

The constant \( A \) and the exponent of \( L \) have been found by curve fitting. The formula (2.12) fits \( \text{SNR}_{\text{AWGN}} \) well over nearly a 5:1 range in \( L \), corresponding to about a 10-dB variation in \( \text{SNR}_{\text{AWGN}} \).

It may be difficult to explain analytically the formula (2.12), because of CD, PMD and other factors in the model system. Nevertheless, we may gain some insight by comparing our results to simpler systems that have been analyzed. In phase-shift-keyed systems either without CD [10] or with CD [34],[35], it has been established that NLPN is the primary intrachannel nonlinear impairment, and that there exists a mean nonlinear phase shift that minimizes the sum of linear phase noise (corresponding simply to amplifier noise) and NLPN. The analysis is simplest for a dispersion-free system using one polarization [10]. Linear phase noise has a variance (at high SNR) \( \sigma_L^2 \approx 1/2 \text{SNR} \), while NLPN has a variance \( \sigma_{\text{NL}}^2 \approx 2 \langle \phi_{\text{NL}} \rangle^2 / 3 \text{SNR} \). Here, \( \langle \phi_{\text{NL}} \rangle \) denotes the mean nonlinear phase shift, where the brackets denote a time average over a modulated signal [3]. By differentiating the sum of these variances with respect to \( \langle \phi_{\text{NL}} \rangle \), one finds that the sum is minimized when the nonlinear phase
shift assumes the value \( \langle \phi_{NL} \rangle_{opt} = \sqrt{3}/2 \), independent of SNR, and thus independent of transmission distance \( L \). When \( \langle \phi_{NL} \rangle \) assumes this optimized value, one finds that the two phase variances are equal: \( \sigma^2_L = \sigma^2_{NL} \).

In general, \( \langle \phi_{NL} \rangle \) scales in proportion to the transmit power \( P_t \), and in proportion to the total number of fiber spans traversed, i.e., in proportion to \( L \). Since \( \langle \phi_{NL} \rangle \propto P_t L \), requiring \( \langle \phi_{NL} \rangle = \langle \phi_{NL} \rangle_{opt} \) requires the transmit power, which appears in the numerator of (2.9), to scale inversely with transmission distance: \( P_t \propto 1/L \). In general, the accumulated amplifier noise power \( P_n \), which appears in the denominator of (2.9), is proportional to the number of amplifiers traversed [36], and is thus proportional to transmission distance \( L \): \( P_n \propto L \). Thus, in a dispersion-free, single-polarization system [10], the SNR (2.9) is expected to scale approximately as \( 1/L^2 \).

By contrast, our model system includes CD and PMD, and the analysis of [10] is not precisely applicable. Fig. 2.16 shows the mean nonlinear phase shift \( \langle \phi_{NL} \rangle \) vs. transmission distance for our model system, where the transmit power \( P_t \) has been optimized at each transmission distance. This is computed using [37]:

\[
\langle \phi_{NL} \rangle = P_t \cdot \frac{8}{9} \sum_{k=1}^{N_{sp}} \left( \gamma_s L_{eff,s} + e^{-\alpha_s L_s} G_{1,k} \gamma_d L_{eff,d} \right).
\] (2.13)

The factor \( 8/9 \) accounts for the effect of polarization averaging on SPM, which is appropriate when the polarization scattering length is much shorter than the nonlinear interaction length. \( \gamma_s \) and \( \gamma_d \) are the nonlinear coefficients for SMF and DCF, respectively, \( \alpha_s \) and \( L_s \) are the loss coefficient and length per span of the SMF, and \( L_{eff,s} \) and \( L_{eff,d} \) are the effective interaction lengths [3] for SMF and DCF, respectively, computed using \( L_{eff,i} = \left( 1 - e^{-\alpha_i L_i} \right)/\alpha_i \). Unlike the dispersion-free, single-polarization system [10], the optimized \( \langle \phi_{NL} \rangle \) is not independent of transmission distance. In Fig. 2.16, the trend of mean nonlinear phase shift vs. transmission distance is seen to approximately follow the power law.
2. ADAPTIVE HARD-DECISION CODING AND FIXED MODULATION

\[ \langle \phi_{NL} \rangle_{best-fit} = B \cdot L^{0.45}, \]  

where the constant \( B \) and the exponent of \( L \) have been found by curve fitting. Given (2.14) and following the reasoning used for the idealized dispersion-free system, we would expect the optimized transmit power to scale as \( P_t \propto 1/L^{0.55} \), and the SNR to scale as \( \text{SNR} \propto 1/L^{1.55} \), which is consistent with the empirical curve fit (2.12).

Returning to Fig. 2.16, averaging \( \langle \phi_{NL} \rangle \) over \( L \) yields an average value \( \langle \phi_{NL} \rangle_{av} = 1.01 \) rad, fairly close to the value \( \langle \phi_{NL} \rangle_{opt} = \sqrt[3]{3}/2 \) for the idealized, dispersion-free system [10]. We should emphasize that the observed scaling of SNR and \( \langle \phi_{NL} \rangle \) with \( L \), given by (2.12) and (2.14), is not universal, but is dependent on the specific modulation format and dispersion map in the model system.

![Fig. 2.16 Mean nonlinear phase shift values computed using (2.13) and compared to the power law (2.14). The value averaged over distance is \( \langle \phi_{NL} \rangle_{av} = 1.01 \) rad.](image)

The middle solid curve in Fig. 2.15 shows \( \text{SNR}_{AWGN+NL, equivalent} \), which is the SNR per symbol observed empirically at the de-repeater input using a transmit power \( P_t \) optimized at each transmission distance. \( \text{SNR}_{AWGN+NL, equivalent} \) includes all effects of accumulated amplifier noise, fiber nonlinearity, residual ISI, and noise enhancement from the equalizer when the channel bandwidth decreases significantly. Here, “equivalent” refers to the fact that the sum of these “noises” is not necessarily Gaussian-distributed. At small \( L \), \( \text{SNR}_{AWGN+NL, equivalent} \) is about 3.2 dB lower than \( \text{SNR}_{AWGN} \), indicating that at the optimum \( P_t \), amplifier noise and nonlinear noise
powers are approximately equal, consistent with the analysis of [10], while at large $L$, the difference increases to about 4.4 dB, presumably because ROADM channel narrowing causes equalizer noise enhancement.

We would like to estimate the performance gap between the proposed rate-adaptive coding scheme and an ideal coding scheme achieving information-theoretic limits. Assuming an ideal discrete-time system transmitting $2R_s$ complex-valued symbols per second\(^3\) in the presence of AWGN, the capacity is [29]:

$$C = 2R_s \log_2(1 + \text{SNR}) \quad (2.15)$$

Inverting (2.15), we find the SNR required for an ideal, capacity-achieving coding scheme to achieve error-free transmission at the information bit rate $R_b$ achieved by the proposed coding scheme:

$$\text{SNR}_{\text{required,ideal}} = 2^{\frac{R_b}{2R_s}} - 1 \quad (2.16)$$

In Fig. 2.15, the bottom solid curve represents $\text{SNR}_{\text{required,ideal}}$ as a function of transmission distance. The vertical separation between $\text{SNR}_{\text{AWGN+NL,equivalent}}$ and $\text{SNR}_{\text{required,ideal}}$ is an estimate of the performance gap between an ideal rate-adaptive coding scheme and the scheme proposed here. The gap ranges from about 5.9 dB to about 7.5 dB as $L$ varies from 2000 to 5000 km. Much of the increase in the gap over this range of $L$ can be attributed to inefficiency of the repetition coding used for $L \geq 3280$ km. Over the entire range of $L$, we expect that some, but not all, of the performance gap can be closed by using more powerful FEC codes with iterative soft-decision decoding, but at the expense of greater implementation complexity. For $L < 2000$ km, the gap increases with decreasing $L$, reflecting our use of a fixed PM-QPSK constellation. We note that a small part of the observed gap (about 0.2 dB) arises from line coding.

In this study, we have considered only a single-channel system with intrachannel nonlinearities, in order to keep simulation run time reasonable (less than one week on a cluster of multi-core computers). We have not included multichannel effects,\(^3\) The factor of 2 corresponds to polarization multiplexing.
especially interchannel nonlinearities. If interchannel nonlinearities were included, we would expect our results to change quantitatively, via a slight reduction in the maximum distance at which a given rate can be achieved. We would not expect our results to change qualitatively, however, because interchannel and intrachannel nonlinearities scale similarly with transmission distance.
3. Adaptive Hard-Decision Coding and Modulation

3.1 Introduction

Variable-rate transmission using variable-rate codes with a fixed constellation was proposed in [38], where its performance was evaluated in systems using inline dispersion compensation. In this chapter, we propose variable-rate codes with variable-size constellations. In systems with or without inline CD compensation, we quantify the achievable information bit rate vs. distance and estimate the performance gap between the proposed scheme and an ideal coding scheme that achieves information-theoretic limits.

This chapter is organized as follows. In Section 3.2, we describe the proposed scheme using variable-rate codes and variable-size constellations, and a rate adaptation algorithm that uses measured SNR or FEC decoder input BER to determine the maximum information bit rate that can be supported. In Section 3.3, we describe simulations of the rate-adaptive scheme in a model terrestrial network. In Section 3.4, we present simulation results, including achievable information bit rates as a function of distance, with or without CD compensation. In Section 3.5, we discuss the observed trend of SNR vs. distance and compare the performance of the proposed scheme to information-theoretic limits. We also discuss laser linewidth requirements. In Appendix, we discuss error-probability calculations.

3.2 Rate-Adaptive Modulation and Coding Scheme

3.2.1 Variable-Size Constellations

We use PM-\(M\)-QAM with modulation orders \(M = 4, 8, 16\). The constellations for the various \(M\) are shown in Fig. 3.1, where the signal amplitudes have been scaled so the average energy is the same for all \(M\). Gray mapping is used for 4- and 16-QAM
square constellations. The bit mapping in [39] is used for the 8-QAM cross constellation, because we verified it performs best in the presence of nonlinearity (although it is not optimal on an AWGN channel [40]).

![QAM constellations with bit-to-symbol mappings. (a) Square 4-QAM. (b) Cross 8-QAM. (c) Square 16-QAM.](image)

3.2.2 Variable-Rate FEC Coding Scheme

The FEC scheme uses serially concatenated Reed-Solomon (RS) codes, referred to as RS-RS codes, in GF(2^8). A family of variable-rate RS-RS codes is constructed by puncturing and shortening a mother code that has a rate in the middle of the desired range, as described in [38], trading off hardware complexity and performance. Higher-rate and lower-rate codes are generated by puncturing and shortening the inner RS code of the mother code, respectively. Hard-decision decoding of the RS-RS code is employed. The rates and performance of the constructed codes are shown in Fig. 2.3, where \( n \) and \( k \) indicate the length in bytes of the code word and message word, respectively, and \( P_{b,in} \) and \( P_{b,out} \) are input and output BERs of the RS-RS decoder. The curves in Fig. 2.3 have been calculated analytically assuming independent errors and no decoding failures [28]. The highest-rate code, with code rate 0.8765, offers a net coding gain of about 7.6 dB at a decoded BER of \( 10^{-13} \), comparable to second-generation FEC codes [24].

To compensate for the diminishing gain obtained by hard-decision decoding of RS-RS codes at low rates, as in [38], an inner repetition code is added, with repetition factor \( f_R \) ranging from 1 to 4. While suboptimal, repetition coding allows operation at
very low SNR with a minimal increase in complexity. Repetitions are separated by 66 symbols, to make the scheme compatible with the 64b/66b line coding used in 10 Gbit Ethernet and proposed for 100 Gbit Ethernet. In order to minimize noise correlation between repeated symbols caused by fiber nonlinearity and CD, permutation of repeated symbols is performed, as in [38]. Soft-decision decoding of the repetition code is employed.

We assume that line encoding, RS-RS encoding, and repetition encoding are performed on a single serial bit stream, after which, blocks of $66f_R$ bits are permuted and mapped in round-robin fashion to the $2\log_2 M$ tributaries of the PM-$M$-QAM signal. In order to minimize noise correlation between repeated symbols caused by fiber nonlinearity, when $f_R > 1$, the second, third and fourth repetition of a block are permuted using different matrices as in [38]. The symbol mapping of FEC-encoded and permuted bits is illustrated in Fig. 3.2, where $x$ and $y$ are two polarizations and the bit index $b_i$ is defined in Fig. 3.1.

Fig. 3.2 Mapping of FEC-encoded and permuted bits to PM-$M$-QAM symbols assumed in model system, where $x$ and $y$ are two polarizations and the bit index $b_i$ follows the definition in Fig. 3.1.
3. ADAPTIVE HARD-DECISION CODING AND MODULATION

3.2.3 Rate Adaptation Algorithm

Given a symbol rate $R_s$, an RS-RS code rate $r_C = k/n$, a repetition code rate $r_R = 1/f_R$, a line code rate $r_L$, and a modulation order $M$, the information bit rate $R_b$ can be calculated as

$$R_b = 2r_Lr_Cr_R R_s \log_2 M,$$

assuming PM transmission.

The achievable bit rate $R_b$ is quantized into multiple segments $R_{b,i}$, $i = 1, 2, \ldots, N_s$, where $R_{b,i} < R_{b,i+1}$. Each $R_{b,i}$ can be represented by a set of transmission parameters $(M, f_R, r_C)_i$, commonly referred to as a mode. For each transmission mode, we define a threshold value for channel state information (CSI), at which $(M, f_R, r_C)_i$ can achieve a target decoder output BER, $P_{b,\text{out},\text{req}}$, on an AWGN channel, which is commonly assumed for modeling optical channels [30]. The controller runs a rate adaptation algorithm based on CSI that is estimated at the receiver and reported to the transmitter. The transmitter monitors variations of reported CSI, and adjusts the RS-RS and repetition code rates and constellation size to maximize $R_b$, while achieving $P_{b,\text{out},\text{req}}$. We consider two possible choices for CSI: (a) SNR, which is the SNR per symbol estimated at the de-repeater soft input, or (b) $P_{b,\text{in}}$, which is the BER of hard decisions measured at the de-repeater output (RS-RS decoder input). We design the rate adaptation algorithm such that it can provide an arbitrary specified SNR margin.

A rate adaptation algorithm using SNR as CSI may not reliably yield the highest feasible information bit rate when SNR is not an accurate predictor of $P_{b,\text{out}}$, but it is straightforward in terms of mapping a CSI value to an expected transmission rate as described in Section 2.2.3, so it is described first. Fig. 3.3 shows information bit rates and threshold values of SNR for different combinations of $(M, f_R, r_C)$ on an AWGN channel. The plots for $f_R = 1$ are obtained assuming $P_{b,\text{out},\text{req}} = 10^{-15}$ and using Fig. 2.3, (3.1), and the following relationships between SNR and $P_{b,\text{in}}$ for a QAM constellation on an AWGN channel [29]:

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3. ADAPTIVE HARD-DECISION CODING AND MODULATION

\[
P_{s,in} \leq \begin{cases} 
4 \cdot \left(1 - \frac{1}{M} \right) Q \left( \frac{3}{M-1} \right), & M = 4, 16 \\
3 \cdot Q \left( \sqrt{0.423 \cdot SNR} \right), & M = 8 
\end{cases},
\]

(3.2)

\[
P_{b,in} \approx \begin{cases} 
\frac{1}{\log_2 M} P_{s,in}, & M = 4, 16 \\
\frac{1.375}{3} P_{s,in}, & M = 8 
\end{cases},
\]

(3.3)

where \( P_{s,in} \) is the upper bound on the symbol-error ratio (SER) at FEC decoder input considering only nearest neighbors, and \( P_{b,in} \) is the decoder input BER using the bit mappings shown in Fig. 3.1. The plots for \( f_R \geq 2 \) in Fig. 3.3 are computed from those for \( f_R = 1 \) by dividing \( R_b \) by \( f_R \) and subtracting \( 10 \log_{10}(f_R) \) from the threshold value of \( SNR \). The filled \((M,f_R,r_C)\) combinations represent a possible choice of transmission modes.

A pseudocode for the algorithm using \( SNR \) as CSI is given as follows:

1. Initialize parameters to the highest-rate mode (assuming that short-distance transmission is the dominant regime).
   - Initialize mode: \((M,f_R,r_C)_i \rightarrow (M,f_R,r_C)_{N_i} \)
   - Initialize up/down counters: \( C_{up} = C_{down} = 0 \)

2. Check if rate change is necessary.
   - if \( SNR < (SNR_{th,i} + \Delta_i + \mu_{down}) \)
     - \( C_{down} = C_{down} + 1 \)
     - if \( C_{down} \geq N_{down} \), \((M,f_R,r_C)_i \rightarrow (M,f_R,r_C)_{i-1} \)
   - elseif \( SNR \geq (SNR_{th,i+1} + \Delta_i + \mu_{up}) \)
     - \( C_{up} = C_{up} + 1 \)
     - if \( C_{up} \geq N_{up} \), \((M,f_R,r_C)_i \rightarrow (M,f_R,r_C)_{i+1} \)
   - else
     - \( C_{up} = C_{down} = 0 \)
     - \((M,f_R,r_C)_i \rightarrow (M,f_R,r_C)_i \)

3. Go to step 2.

\( \mu_{up} \) and \( \mu_{down} \) are SNR margins when changing rate up and down, respectively, and \( N_{up} \) and \( N_{down} \) are counters when changing rate up and down, respectively. Since \( SNR \) may
not be a sufficiently accurate predictor of $P_{b,in}$, as in [38], we introduce the SNR penalty parameters $\Delta_i$, which quantify the increase in SNR required to achieve a specified $P_{b,in}$ when using $(M_{fR},rC)_i$, as compared to the AWGN case. Finding $\Delta_i$ would require searching for the SNR that achieves the target $R_{b,i}$ (satisfying $P_{b,out,req}$) that is predicted by the threshold SNR under the AWGN assumption.

An algorithm using $P_{b,in}$ as CSI requires a more complicated CSI-to-rate mapping compared to using SNR as CSI, but can reliably yield the highest feasible information bit rate, since $P_{b,in}$ is an accurate predictor of $P_{b,out}$ for a hard-decision RS-RS decoder as described in Section 2.2.3. $P_{b,in}$ can be estimated by the RS decoder from the number of bit errors corrected in decoding [27].

Fig. 3.3 Information bit rate vs. threshold SNR per symbol for PM-M-QAM on AWGN channel. Information bit rates are computed using (3.1), while SNR values are computed by combining (3.2), (3.3) and Fig. 2.3. The set of filled $(M_{fR},code)$ combinations represent a possible choice of modes for rate-adaptive transmission.

Fig. 3.4 shows information bit rates and threshold values of $P_{b,in}$ for different combinations of $(M_{fR},rC)$, computed using Fig. 2.3 and (3.1)-(3.3). A threshold value of $P_{b,in}$ for mode $i$ with margin $\mu$ dB, $P_{b,in,th,i,\mu}$, is defined as the value of $P_{b,in}$ for $(M_{fR},rC)_i$ at $SNR_{th,i} + \mu$ dB on an AWGN channel. A pseudocode for the rate-adaptation algorithm using $P_{b,in}$ as CSI is as follows:

1. Initialize parameters to the highest-rate mode.
   - Initialize mode: $(M_{fR},rC)_i \rightarrow (M_{fR},rC)_N$.
2. Check if rate change is necessary.

- if \( P_{b,in} > P_{b,in,th,i,\mu_{\text{down}}} \)
  - \( C_{\text{down}} = C_{\text{down}} + 1 \)
  - if \( C_{\text{down}} \geq N_{\text{down}} \), \( (M_{fR},r_C)_i \rightarrow (M_{fR},r_C)_{i-1} \)
- elseif \( P_{b,in} \leq P_{b,in,th,i+1,\mu_{\text{up}}} \)
  - \( C_{\text{up}} = C_{\text{up}} + 1 \)
  - if \( C_{\text{up}} \geq N_{\text{up}} \), \( (M_{fR},r_C)_i \rightarrow (M_{fR},r_C)_{i+1} \)
- else
  - \( C_{\text{up}} = C_{\text{down}} = 0 \)
  - \( (M_{fR},r_C)_i \rightarrow (M_{fR},r_C)_i \)

3. Go to step 2.

We note that SNR penalty parameters are not required when using \( P_{b,in} \) as CSI [38].

Fig. 3.4 Information bit rate vs. threshold \( P_{b,in} \) for PM-M-QAM on AWGN channel. Information bit rates are computed using (3.1), while \( P_{b,in} \) values are computed by combining (3.2), (3.3) and Fig. 2.3. The set of filled \((M_{fR},\text{code})\) combinations represent a possible choice of modes for rate-adaptive transmission.

### 3.3 System Simulations

We have evaluated the rate-adaptive transmission scheme in the model long-haul system shown in Fig. 3.5. We simulated a single channel in a wavelength-division-multiplexed (WDM) system with center wavelength of 1550 nm and nominal
50-GHz channel spacing. The modulation is single-carrier PM-M-QAM using non-return-to-zero pulses at a symbol rate $R_s = 29.4152$ Gsymb/s. Each modulator is a quadrature Mach-Zehnder device. Each drive waveform is a train of rectangular pulses filtered by a five-pole Bessel LPF having a 3-dB bandwidth $1.4R_s = 41.2$ GHz. Percentages 90, 95 and 99% of the modulated signal energy are contained in bandwidths of 28.9, 33.2 and 40.4 GHz, respectively. We assume line coding at rate $r_L = 64/66$, yielding an information bit rate $R_b = 200$ Gbit/s for the highest overall code rate $r_C r_R$ and $M = 16$.

The fiber network comprises multiple 80-km spans of standard single-mode fiber (SMF). In dispersion-compensated systems, dispersion-compensating fiber (DCF) is used to pre-compensate CD by $-510$ ps/nm, and to compensate CD in each span to a residual CD of 42.5 ps/nm, corresponding to 3.1% residual dispersion per span (RDPS). The parameters for SMF and DCF are summarized in Table 3.1. Each span uses a two-stage inline amplifier to compensate total span loss, with the ratio between first- and second-stage gains optimized via simulation, as in [38]. Amplifier noise figures of 5 dB are assumed. In dispersion-uncompensated systems, the pre-compensation and inline DCFs are eliminated.

In order to simulate transmission through a large-scale mesh network, a ROADM is inserted at every third span. We choose a homogeneous, high-density distribution of ROADM s to be conservative and to obtain a simple trend of achievable
bit rate vs. transmission distance. Filtering by each wavelength-selective switch is simulated by a super-Gaussian bandpass filter of order 2.5, 3-dB bandwidth of 40 GHz and insertion loss of 12 dB, consistent with typical devices [2]. We assume 3-dB bandwidth of 40 GHz and insertion losses of 6 dB each for the multiplexer and the de-multiplexer.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>SMF length per span</td>
<td>80</td>
<td>km</td>
</tr>
<tr>
<td>Inline DCF length per span</td>
<td>15.5</td>
<td>km</td>
</tr>
<tr>
<td>Pre-compensation DCF length</td>
<td>6</td>
<td>km</td>
</tr>
<tr>
<td>SMF loss coefficient</td>
<td>0.25</td>
<td>dBm/km</td>
</tr>
<tr>
<td>DCF loss coefficient</td>
<td>0.6</td>
<td>dBm/km</td>
</tr>
<tr>
<td>SMF CD coefficient</td>
<td>17</td>
<td>ps/(nm·km)</td>
</tr>
<tr>
<td>DCF CD coefficient</td>
<td>−85</td>
<td>ps/(nm·km)</td>
</tr>
<tr>
<td>SMF/DCF PMD coefficient</td>
<td>0.1</td>
<td>ps/km^{1/2}</td>
</tr>
<tr>
<td>SMF nonlinear coefficient</td>
<td>0.0012</td>
<td>W m^{-1}</td>
</tr>
<tr>
<td>DCF nonlinear coefficient</td>
<td>0.0053</td>
<td>W m^{-1}</td>
</tr>
</tbody>
</table>

Table 3.1 Parameters for SMF and DCF. DCF is omitted from dispersion-uncompensated systems.

The receiver employs a fifth-order Butterworth anti-aliasing filter of 3-dB bandwidth $R_s$, samples at a rate of $2R_s$ complex samples per polarization and performs digital compensation of CD and polarization-mode dispersion (PMD) using finite impulse response time-domain filtering, as described in [32]. At each transmission distance, the number of filter taps is optimized to make residual ISI negligible. The equalizer is adapted using the least mean squares algorithm with the parameters of step size, shift interval, and number of shifts optimized. The signal at the equalizer output is scaled to remove the bias introduced by the MMSE equalizer [41].

We empirically found that in order to render residual ISI minimized, the number of equalizer taps required is:

$$N = 2 \cdot 2\pi (|\beta_2|L)_{\text{eff}} R_s^2 (M/K) + 7$$
$$\approx 12.6 (|\beta_2|L)_{\text{eff}} R_s^2 (M/K) + 7. \quad (3.4)$$

This is about twice the number given in [32], which would result in a 2-dB penalty from residual ISI. In (3.4), $\beta_2$ is the fiber group-velocity dispersion (GVD) parameter,
3. ADAPTIVE HARD-DECISION CODING AND MODULATION

$L$ is the fiber length, and $M/K$ is the oversampling rate. The effective value of the product $|\beta_2|L$ can be computed as

$$
( |\beta_2|L )_{\text{eff}} = \sum_{k=1}^{N_{\text{span}}} \left( \beta_{2,\text{SMF}} L_{k,\text{SMF}} + \beta_{2,\text{DCF}} L_{k,\text{DCF}} \right) \quad (3.5)
$$

$$
= N_{\text{span}} \left| \beta_{2,\text{SMF}} L_{k,\text{SMF}} + \beta_{2,\text{DCF}} L_{k,\text{DCF}} \right|,
$$

where $N_{\text{span}}$ is the number of spans, $\beta_{2,\text{SMF}}$ and $\beta_{2,\text{DCF}}$ are GVD parameter of SMF and DCF respectively, and $L_{k,\text{SMF}}$ and $L_{k,\text{DCF}}$ are the length of SMF and DCF in the $k$th span respectively. Using (3.4), Fig. 3.6 shows that the dispersion-uncompensated system requires more filter taps than the compensated system, by more than an order of magnitude for $L > 1000$ km.

![Fig. 3.6 The number of filter taps employed in the equalizer based on the formula (3.4), for dispersion-compensated and -uncompensated systems. For $L > 1000$ km, an uncompensated system requires more than ten times as many filter taps as a compensated system.](image)

Signal propagation is simulated by numerical integration of the vector nonlinear Schrödinger equation by the split-step Fourier method [33].

We estimate $P_{b,\text{in}}$ at the de-repeater output using two different methods, depending on the confidence level of $P_{b,\text{in}}$ obtained by error counting. In the first regime, we estimate $P_{b,\text{in}}$ by error counting. We define $P_{b,\text{in},95,M}$, $M = 4, 8, 16$, to be the value of the estimate of $P_{b,\text{in}}$ such that the true value does not exceed 130% of the estimate with 95% confidence, i.e. the confidence interval is $[0.7 \cdot P_{b,\text{in}}, 1.3 \cdot P_{b,\text{in}}]$ with probability 0.95. We find that $P_{b,\text{in},95,4} = 9.1 \times 10^{-4}$, $P_{b,\text{in},95,8} = 6.2 \times 10^{-4}$ and $P_{b,\text{in},95,16} =$
4.5×10^{-4}. At least for estimated values of $P_{b,in}$ down to $P_{b,in,95,M}$, we simulate a sufficient number of symbols in the model optical system to obtain the specified confidence level. In this regime, we find that the uncertainty in $P_{b,in}$ corresponds to 0.2 dB uncertainty in SNR. In the second regime, when $P_{b,in}$ estimated by error counting is below $P_{b,in,95,M}$, we estimate noise standard deviations and compute $P_{b,in}$ as described in the Appendix, in order to reduce simulation time for very low values of $P_{b,in}$. We estimate that the BER computation method in the Appendix yields uncertainties in $P_{b,in}$ corresponding to SNR uncertainties of 0.18, 0.56, and 0.38 dB for $M = 4, 8,$ and 16, respectively.

At each transmission distance, and for each repetition factor $f_R$, the launched power is optimized with 1 dB resolution, to minimize the value of $P_{b,in}$ estimated using the two methods described in the previous paragraph. We have found that in cases when both methods have been used, they result in the same optimized power in about 80% of the cases, and only a 1-dB difference in the remaining cases.

### 3.4 Simulation Results

In order to evaluate the rate-adaptive scheme, at each transmission distance, for each modulation order, and for each repetition factor, after the transmit power is optimized, the received SNR per symbol and decoder input BER are recorded. We estimate the SNR per symbol by calculating the ratio between the average symbol power and the noise variance, which is empirically measured at the de-repeater input or output.
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Fig. 3.7 SNR observed at de-repeater output (RS-RS decoder input) as a function of transmitted power, for various repetition factors, with and without dispersion compensation. (a) 4-QAM: $L = 2400$ km and $4000$ km. (b) 8-QAM: $L = 1200$ km and $2400$ km. (c) 16-QAM: $L = 800$ km and $1200$ km.
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Fig. 3.7 shows the SNR per symbol observed at the de-repeater output vs. transmitted power $P_t$ for various repetition factors $f_R = 1, 2, 3, 4$, for a representative transmission distance for each modulation order, with and without dispersion compensation. In Fig. 3.7 (a), for $M = 4$, we observe improvements in SNR of 2.9-3.0, 4.5-4.8 and 5.7-6.0 dB for $f_R = 2, 3, 4$ as compared to $f_R = 1$, which are close to the full repetition gains expected for AWGN. Fig. 3.7 (b) and (c) show improvements in SNR of 2.6-2.8, 4.0-4.2, and 5.0-5.2 dB for $f_R = 2, 3, 4$ for $M = 8$, and 2.5-2.7, 4.0-4.2, and 5.1-5.3 dB for $M = 16$. For multi-level modulations, the observed improvement in SNR is less than expected for AWGN presumably because they are more sensitive to nonlinearity effects.

We have employed the rate adaptation algorithm of Section 3.2.3, using $P_{b,\text{in}}$ as CSI, and using all possible transmission modes in Fig. 3.4. We assume a required FEC decoder output BER $P_{b,\text{out,req}} = 10^{-15}$, SNR margins $\mu_{\text{up}} = \mu_{\text{down}} = 0, 1, \ldots, 5$ dB and counter parameters $N_{\text{up}} = N_{\text{down}} = 1$ (because we assume static channel conditions). Fig. 3.8 (a) and (b) present achievable information bit rates with and without inline dispersion compensation respectively, as a function of transmission distance. In dispersion-compensated systems, with zero margin, a bit rate $R_b = 200$ Gbit/s can be realized up to 640 km, with the achievable rate decreasing by approximately a factor of two for every additional 1000 km. In dispersion-uncompensated systems, with zero margin, a bit rate $R_b = 200$ Gbit/s can be realized up to 1120 km, with the achievable rate decreasing by approximately a factor of two for every additional 2000 km.

We used $P_{b,\text{in}}$ as CSI, and evaluated $P_{b,\text{in}}$ as described in Section 2.3. We find that the uncertainty of $P_{b,\text{in}}$ (either measured by error counting above $P_{b,\text{in,95,}M}$ or computed as described in the Appendix below $P_{b,\text{in,95,}M}$) may result in a reduced transmission distance by at most one span for dispersion-compensated system, but the penalty may increase to two spans for uncompensated system for distances above 5000 km.
Fig. 3.8 Achievable information bit rates vs. transmission distance for different SNR margins. The set \((M,fR,code)\) denotes the modulation order, repetition factor and type of RS-RS code. (a) Dispersion-compensated system. (b) Dispersion-uncompensated system.

### 3.5 Discussion

In this section, we examine how the optimized mean nonlinear phase shift and SNR in the model system scale with transmission distance, and estimate the performance gap between the proposed rate-adaptive scheme and an ideal coding scheme achieving information-theoretic limits. We also address laser linewidth requirements.
In general, the mean (i.e., time-averaged) nonlinear phase shift, $<\phi_{NL}>$, scales in proportion to the transmit power $P_t$ and the total number of fiber spans traversed [3], i.e., $<\phi_{NL}> \propto P_t L$. After optimizing the transmitted power $P_t$, we computed the resulting mean nonlinear phase shift $<\phi_{NL}>$ using equation (11) from [38]. Optimized values of $<\phi_{NL}>$ vs. transmission distance $L$ for unit repetition factor, $f_R = 1$ are shown in Fig. 3.9 (a) and (b) for dispersion-compensated and -uncompensated systems, respectively. We observed that the $<\phi_{NL}>$ approximately follow power laws:
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\[
⟨φ_{NL}⟩_{best-fit} = \begin{cases} 
B \cdot L^{0.7}, & 3.1\% \text{ RDPS} \\
B \cdot L^{0.85}, & 100\% \text{ RDPS}
\end{cases}
\]  

(3.6)

where the constant \(B\) and the exponent of \(L\) have been found by curve fitting. In the dispersion-compensated system, \(⟨φ_{NL}⟩\) scales as \(L^{0.7}\), a slightly stronger dependence than in systems using a fixed PM-QPSK constellation [38], where the dependence was found to be \(L^{0.45}\). In the dispersion-uncompensated system, the \(L^{0.85}\) dependence is consistent with [42], where the optimized power was found to be roughly independent of \(L\).

Fig. 3.10 Different measures of SNR compared to the SNR required for an ideal capacity-achieving coding scheme to achieve error-free transmission at the information bit rate \(R_b\) achieved by the proposed rate-adaptive scheme. (a) Dispersion-compensated system. (b) Dispersion-uncompensated system.
We consider several different measures of SNR and compare these to an equivalent SNR corresponding to the information bit rate achieved by the proposed rate-adaptive scheme. This discussion makes reference to Fig. 3.10 (a) and (b), which describe dispersion-compensated and -uncompensated systems, respectively.

The uppermost solid curves in Fig. 3.10 (a) and (b) show $SNR_{AWGN}$, which is an empirical estimate of the SNR per symbol (in two polarizations) as limited only by accumulated optical amplifier noise:

$$SNR_{AWGN} = \frac{P_t}{P_n}.$$  \hspace{1cm} (3.7)

$P_t$ is the transmitted signal power, optimized at each transmission distance without repetition encoding ($f_R = 1$), which equals the received signal power at the demultiplexer input, since the network is designed to have unit signal gain. $P_n$ is the noise power in two polarizations, and is computed using (8) in [38]. In general, the noise power $P_n$ is proportional to the number of amplifiers traversed [36], so it scales linearly with distance, $P_n \propto L$. The dashed curves in Fig. 3.10 (a) and (b) show $SNR_{AWGN,best-fit}$ for dispersion-compensated and -uncompensated systems respectively, which is a power-law fit to the observed $SNR_{AWGN}$ as a function of transmission distance $L$:

$$SNR_{AWGN,best-fit} = \begin{cases} A \left( \frac{L}{L_{RDPS}} \right)^{1.30}, & 3.1\% \text{ RDPS} \\ A \left( \frac{L}{L_{RDPS}} \right)^{1.05}, & 100\% \text{ RDPS} \end{cases}.$$  \hspace{1cm} (3.8)

The constant $A$ and the exponent of $L$ have been found by curve fitting. In the dispersion-compensated system, $SNR_{AWGN}$ scales as $L^{-1.30}$, a slightly weaker dependence than in systems using a fixed PM-QPSK constellation [38], where the dependence was found to be $L^{-1.55}$. In the dispersion-uncompensated system, the $L^{-1.05}$ dependence is consistent with [42], where the optimized SNR was found to scale as $L^{-1}$. Considering the dependencies of $\langle \phi_{NL} \rangle$ and $P_n$ on $L$, the power laws in (3.6) and (3.8) are consistent.
The middle solid curves in Fig. 3.10 (a) and (b) show $SNR_{AWGN+NL,\text{equivalent}}$ for dispersion-compensated and -uncompensated systems, respectively, which is the SNR per symbol observed empirically at the de-repeater input, and includes all effects of amplifier noise and fiber nonlinearity (“equivalent” refers to the fact that their sum may not be Gauss-distributed). At small $L$, $SNR_{AWGN+NL,\text{equivalent}}$ is about 2.6 dB and 2.7 dB lower than $SNR_{AWGN}$ for dispersion-compensated and -uncompensated systems, respectively, indicating that at the optimum $P_t$, amplifier noise and nonlinear noise powers are approximately equal, while at large $L$, the difference increases to about 4.3 dB and 7.0 dB for dispersion-compensated and -uncompensated systems, respectively, presumably because significant ROADM channel narrowing over long distances causes equalizer noise enhancement.

The bottom solid curves in Fig. 3.10 (a) and (b), $SNR_{\text{required,ideal}}$, are computed by inverting the formula for the capacity of an ideal discrete-time AWGN channel transmitting at symbol rate $R_s$ in two polarizations, i.e.,

$$C = 2R_s \log_2 (1 + SNR)$$

$$SNR_{\text{required,ideal}} = \frac{R_s}{22R_s} - 1.$$  \hspace{1cm} (3.9)

$SNR_{\text{required,ideal}}$ corresponds to the SNR required for an ideal, capacity-achieving coding scheme to achieve error-free transmission at the information bit rate $R_b$ achieved by the proposed scheme. The vertical separation between $SNR_{AWGN+NL,\text{equivalent}}$ and $SNR_{\text{required,ideal}}$ is an estimate of the performance gap between an ideal rate-adaptive scheme and the scheme proposed here. For dispersion-compensated systems, the gap ranges from about 6.4 dB to about 7.6 dB as $L$ varies from 640 to 5040 km. For dispersion-uncompensated systems, the gap ranges from about 6.6 dB to about 7.5 dB as $L$ varies from 1120 to 7600 km. Much of the increase in the gap arises from the inefficiency of the repetition coding used beyond 3280 km and 5760 km for dispersion-compensated and -uncompensated systems, respectively. We expect that some, but not all, of this gap can be closed by using more powerful FEC codes.

We now discuss requirements placed on laser linewidths by the proposed rate-adaptive scheme. In [43], transmitter and local oscillator (LO) linewidth requirements for 4-, 8- and 16-QAM were studied for the regime in which CD is negligible. In [44],
it was shown that significant uncompensated CD can lead to equalization-enhanced phase noise (EEPN), making linewidth requirements for the LO more stringent, while not affecting transmitter linewidth requirements.

Fig. 3.11 Requirements on transmitter (Tx) and local oscillator (LO) laser linewidths, $\Delta \nu_{\text{Tx}}$ and $\Delta \nu_{\text{LO}}$, when operating at a transmission mode $(M_{fR}, r_C)$ corresponding to 1-dB margin, as specified in Fig. 3.8. Dashed lines: $\Delta \nu_{\text{Tx}} = \Delta \nu_{\text{LO}}$ required for a total phase-noise penalty of 1 dB, neglecting equalization-enhanced phase noise (EEPN). Dotted lines: $\Delta \nu_{\text{LO}}$ required for an additional penalty of 1 dB from EEPN. Solid lines: $\Delta \nu_{\text{Tx}} = \Delta \nu_{\text{LO}}$ required for a total phase-noise penalty of 1 dB, including EEPN, where the total penalty was optimally distributed between the Tx and LO so as to minimize the linewidth requirement. (a) Dispersion-compensated system; the linewidth requirement is determined by EEPN for $L$ exceeding about 1920 km. (b) Dispersion-uncompensated system; the linewidth requirement is dominated by EEPN for all $L$.

We assume the system will be designed so that all phase noise effects cause a 1-dB penalty, and that to accommodate this penalty, the transmission mode $(M_{fR}, r_C)$ is chosen to provide a margin of 1 dB at each transmission distance, as specified in Fig.
3. ADAPTIVE HARD-DECISION CODING AND MODULATION

3.8. Linewidth requirements are shown in Fig. 3.11. Dashed lines present the transmitter and LO linewidths (assumed equal) for a penalty of 1 dB, neglecting EEPN, assuming decision-directed feedforward carrier recovery, as analyzed in [43]. Dotted lines present the LO linewidths for an additional penalty of 1 dB from EEPN. Solid lines present the transmitter and LO linewidths (assumed equal) for a total penalty of 1 dB, including EEPN.

Fig. 3.11 (a) shows linewidth requirements for dispersion-compensated systems. For distances below about 1920 km, EEPN is negligible, while for distances above 1920 km, the linewidth requirement is determined by EEPN. The narrowest linewidth of about 2.6 MHz is required at the shortest distances with 1 dB phase-noise penalty (solid line). Fig. 3.11 (b) shows linewidth requirements for dispersion-uncompensated systems. For all distances, linewidth requirements are dictated by EEPN, with the narrowest linewidth of about 800 kHz required at the shortest distances with 1 dB phase-noise penalty (solid line). All of the above linewidth requirements can be met using commercially available lasers, such as distributed feedback devices with linewidths less than 200 kHz [45].

Timing recovery should be designed carefully to yield acceptably small jitter at the low operating SNRs enabled by the proposed rate-adaptive scheme.
4. Adaptive Soft-Decision Coding and Modulation

4.1 Introduction

Variable-rate transmission using variable-rate codes with fixed constellations was proposed in [38], and was extended to variable-size constellations in [46]. Both [38] and [46] employed hard-decision FEC decoding. FEC schemes in optical communications have evolved over several generations, starting from a single block code with hard-decision decoding (HDD), followed by concatenated block codes and then soft-decision codes. Soft-decision FEC schemes have been widely used in wireless and wireline communication systems for more than decades. By contrast, soft-decision FEC schemes have been adopted in the optical communication systems quite recently as demands for higher data rates increased and advanced high-speed ASIC technologies became available [11]. In recent years especially, intensive research has addressed iterative soft-decision decoding (SDD) of various codes, e.g., block turbo codes [12] and low-density parity-check codes (LDPC) [13], seeking the highest possible coding gain. Variable-rate transmission using SDD FEC was studied in [22], although a limited range of rate variation was provided. In this chapter, we propose variable-size constellations and variable-rate FEC codes with SDD, providing a wide range of rate variation. In systems with or without inline chromatic dispersion (CD) compensation, we quantify the achievable information bit rate vs. distance and estimate the performance gap between the proposed scheme and an ideal coding scheme that achieves information-theoretic limits. We also compare the results of this chapter to those in [46] in order to quantify performance improvements between hard- and soft-decision FEC schemes.

This chapter is organized as follows. In Section 4.2, we describe the proposed scheme using variable-rate codes and variable-size constellations, and a rate adaptation algorithm that uses the measured signal-to-noise ratio (SNR) or bit-error
ratio (BER) to determine the maximum information bit rate that can be supported. In Section 4.3, we describe simulations of the rate-adaptive scheme in a model terrestrial network. In Section 4.4, we present simulation results, including achievable information bit rates as a function of distance, with or without CD compensation. In Section 4.5, we discuss observed trends of SNR vs. distance and compare the performance of the proposed scheme to information-theoretic limits.

### 4.2 Rate-Adaptive Modulation and Coding Scheme

#### 4.2.1 Variable-Size Constellations

We use PM-$M$-QAM with modulation orders $M = 4, 8, 16$. The constellations for the various $M$ are shown in Fig. 4.1, where the signal amplitudes have been scaled so the average energy is the same for all $M$. We use Gray mapping for 4- and 16-QAM square constellations and an optimal bit mapping [40] (different from the bit mapping in Fig. 3.1 (b)) for the 8-QAM cross constellation.

![Fig. 4.1 QAM constellations with bit-to-symbol mappings. (a) Square 4-QAM. (b) Cross 8-QAM. (c) Square 16-QAM.](image)

#### 4.2.2 Variable-Rate FEC Coding Scheme

Our FEC scheme uses concatenated Reed-Solomon (RS) codes and LDPC codes, referred to as RS-LDPC codes, throughout the remainder of this chapter. An LDPC code is a linear block code defined by a parity-check matrix that consists of a low density of one bits, and which can achieve performance approaching information-theoretic limits [47]. While similar coding gains can be achieved by Turbo codes, it is
known that LDPC codes offer advantages over Turbo codes in terms of coding gain at high code rates and decoder complexity [48], making LDPC codes an attractive option for systems requiring high data rates without the possibility of retransmission. A RS code is a linear block code with strong capability to correct both random and burst errors [27]. In a RS-LDPC code, the inner LDPC code provides most of the coding gain. Since LDPC codes exhibit error floors at very low error rates [49], we add a high-rate outer RS code to reduce error floor effects, as in [13].

We employ a family of variable-rate LDPC codes that are used in second-generation digital video broadcasting (DVB-S2), which broadcasts high-speed data from satellites to mobile devices [50]. The DVB-S2 LDPC codes are designed using an extended irregular repeat-accumulate method that is suitable for producing high-rate codes offering good performance and reduced encoding complexity (which can be high for general LDPC codes) [51]. Note that there are other methods for constructing encoding-efficient LDPC codes [52],[53],[54]. The DVB-S2 standard provides two sets of codes with different codeword lengths, 16200 bits and 64800 bits, with code rate ranges from about 0.25 to 0.9 in each set. We choose six codes from the set with shorter codeword length, in order to reduce decoding complexity. The highest-rate code, with code rate 0.89, offers a net coding gain of about 10.5 dB at a decoded BER of $10^{-13}$ with SDD, comparable to third-generation FEC codes [11]. Here, we perform iterative SDD using the belief propagation (BP) algorithm [55]. To reduce implementation complexity of the BP algorithm, many studies have proposed suboptimal but simplified decoding algorithms [56],[57],[58]. We have chosen to use the standard BP algorithm, in order to enable a performance comparison between HDD and SDD schemes that are optimal or near-optimal. The rates of our constructed codes and their performance for $M = 4, 8, 16$ on an additive white Gaussian noise (AWGN) channel are shown in Fig. 4.2, where $n_{LDPC}$ and $k_{LDPC}$ indicate the length in bits of the codeword and message word, respectively.
4. ADAPTIVE SOFT-DECISION CODING AND MODULATION

Fig. 4.2 Performance of inner LDPC codes on AWGN channel with six different rates, in terms of LDPC decoder output BER vs. SNR per symbol. The BP algorithm uses 50 iterations. The SNR per symbol is measured at the equalizer output (i.e., the LDPC decoder input) and $P_{b,\text{out,LDPC}}$ is measured at the LDPC decoder output (i.e., the RS encoder input). (a) $M = 4$. (b) $M = 8$. (c) $M = 16$. 

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We run the BP algorithm in the log domain, which requires knowledge of the log-likelihood-ratio (LLR) of received bits. We use LLR values computed assuming an AWGN channel. Let the BP decoder input signal be

\[ z = x + w, \]  

(4.1)

where \( x \) is a complex QAM symbol and \( w \) is complex Gaussian noise. We define an LLR for the \( k \)-th bit in a QAM constellation point as

\[ LLR_k = \log \left( \frac{\Pr \{ b_k = 0 \mid z \}}{\Pr \{ b_k = 1 \mid z \}} \right), \]  

(4.2)

where \( b_k \) follows the notation in Fig. 4.1. Eq. (4.2) can be rewritten as

\[
LLR_k = \log \frac{\sum_{\alpha \in S_k^{(0)}} \Pr \{ x = \alpha \mid z \}}{\sum_{\alpha \in S_k^{(1)}} \Pr \{ x = \alpha \mid z \}} \\
= \log \frac{\sum_{\alpha \in S_k^{(0)}} \Pr \{ z \mid x = \alpha \}}{\sum_{\alpha \in S_k^{(1)}} \Pr \{ z \mid x = \alpha \}} \\
= \log \left( \sum_{\alpha \in S_k^{(0)}} e^{-\frac{|z-\alpha|^2}{2\sigma^2}} \right) - \log \left( \sum_{\alpha \in S_k^{(1)}} e^{-\frac{|z-\alpha|^2}{2\sigma^2}} \right),
\]

(4.3)

where \( S_k^{(i)} \) denotes a set of QAM symbols with a binary value \( i \) in the \( k \)th location, and \( \sigma^2 \) is the variance of a complex Gaussian noise. In (4.3), the first equality holds by the law of total probability, the second equality follows using Bayes’ rule and the assumption of equal priori probability of \( x \), and the third equality holds for AWGN. Fig. 4.3-Fig. 4.5 show computed LLRs for \( M = 4, 8, 16 \) assuming \( \sigma^2 = 0.1 \).
There are several options for generating variable-rate LDPC codes. First, one can use different codes for different code rates. This approach increases encoder/decoder hardware complexity. Second, one can puncture a low-rate mother code to generate higher-rate codes [59]. Puncturing is effective typically over only a small range of code rates, and puncturing patterns must be carefully chosen to prevent performance degradation. Third, one can shorten a high-rate mother code to generate lower-rate codes [60]. Shortening results in a code with a shorter message word length as compared to codes having the same code rate but having codeword length equal to that of the mother code, which prevents construction of very high-rate outer RS codes that have a fixed error-correction capability. Shortening also reduces codeword length (as compared to the mother code), which mandates higher decoder throughput requirements at lower information bit rates. Fourth, one can employ various puncturing-and-extending method, as proposed in [61],[62],[63], but these exhibit some limitations, such as limited supported code rate ranges and increased decoder complexity. Based on these considerations, we have chosen to use different LDPC codes for different rates, utilizing verified good codes at each different rate, avoiding a long search time to find good puncturing patterns, minimizing overhead when concatenating with outer RS codes, and minimizing decoder throughput requirements.
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Fig. 4.4 Calculated LLR values for $b_0$, $b_1$ and $b_2$ for 8-QAM, using (4.3) and assuming the bit mapping of Fig. 4.1 (b) and $\sigma^2 = 0.1$, where $z_i$ and $z_Q$ denote the in-phase and quadrature components of the equalizer output signal.
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Fig. 4.5 Calculated LLR values for $b_0$ and $b_1$ for 16-QAM, using (4.3) and assuming the bit mapping of Fig. 4.1 (c) and $\sigma^2 = 0.1$, where $z_Q$ denotes the quadrature component of the equalizer output signal. The LLR values for $b_2$ and $b_3$ are, respectively, reflections of the curves for $b_0$ and $b_1$ about the line $z_Q = 0$, replacing $z_Q$ by $z_I$ (the in-phase component).

In DVB-S2, Bose/Ray-Chaudhuri/Hocquenghem codes are used as an outer code to cope with error floors, with an error correction capability of up to 12 bits. We employ a stronger RS block code offering an error correction capability of 25 bits. A family of variable-rate RS codes in $\text{GF}(2^{11})$ is constructed by shortening a mother code that has a highest rate of 0.98. Note that the RS shortening does not induce any additional decoder throughput requirements, which is determined by the length of
inner LDPC codeword. We employ HDD of the RS code [27]. The rates and performance of our constructed codes are shown in Fig. 4.6, where $n_{RS}$ and $k_{RS}$ indicate the length of the codeword and message word in units of 11 bits, respectively, and $P_{b,\text{in},RS}$ and $P_{b,\text{out},RS}$ are input and output BERs of the RS decoder. The curves in Fig. 4.6 have been calculated analytically assuming independent errors and no decoding failures [28].

![Fig. 4.6 Performance of outer RS codes with six different rates, in terms of input and output BERs of the RS decoder. The $P_{b,\text{in},RS}$ and $P_{b,\text{out},RS}$ are measured at the input and output of the RS decoder, respectively.](image)

<table>
<thead>
<tr>
<th>Code</th>
<th>$k_{RS}(k_{RS,m})$</th>
<th>$n_{RS}$</th>
<th>$k_{LDPC}(k_{LDPC,m})$</th>
<th>$n_{LDPC}$</th>
<th>$r_C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1259 (1997)</td>
<td>1309</td>
<td>14399 (14400)</td>
<td>16199</td>
<td>0.855</td>
</tr>
<tr>
<td>2</td>
<td>1095 (1997)</td>
<td>1145</td>
<td>12595 (12600)</td>
<td>16195</td>
<td>0.744</td>
</tr>
<tr>
<td>3</td>
<td>833 (1997)</td>
<td>883</td>
<td>9713 (9720)</td>
<td>16193</td>
<td>0.566</td>
</tr>
<tr>
<td>4</td>
<td>604 (1997)</td>
<td>654</td>
<td>7194 (7200)</td>
<td>16194</td>
<td>0.410</td>
</tr>
<tr>
<td>5</td>
<td>440 (1997)</td>
<td>490</td>
<td>5390 (5400)</td>
<td>16190</td>
<td>0.299</td>
</tr>
<tr>
<td>6</td>
<td>244 (1997)</td>
<td>294</td>
<td>3234 (3240)</td>
<td>16194</td>
<td>0.166</td>
</tr>
</tbody>
</table>

Table 4.1 RS and LDPC code parameters: lengths of message word and codeword. The notation is defined in the text.

We linearly interleave RS-encoded bits prior to LDPC encoding. To align the number of RS-encoder output bits ($n_{RS}$) to the number of LDPC-encoder input bits ($k_{LDPC}$), we perform a shortening of DVB-S2 LDPC codes (by less than 11 bits). The
code parameters for RS and LDPC codes are listed in Table 4.1, where \( k_m \) denotes the length of message word for the mother code (the codeword length for the mother code, \( n_m \), can be calculated as \( n_m = k_m + n - k \)). The encoding procedure of RS-LDPC codes is illustrated in Fig. 4.7.

![Fig. 4.7 Constructing variable-rate inner RS and outer LDPC codes by shortening mother codes. The overall RS-LDPC encoding process is also illustrated.](image)

We assume that line encoding, RS-LDPC encoding are performed on a single serial bit stream, after which bits are mapped in round-robin fashion to the \( 2\log_2 M \) tributaries of the PM-\( M \)-QAM signal as in [46], following the bit index \( b_i \) defined in Fig. 4.1.

### 4.2.3 Rate Adaptation Algorithm

Given a symbol rate \( R_s \), an RS-LDPC code rate \( r_C \), a line code rate \( r_L \), and a modulation order \( M \), the information bit rate \( R_b \) can be calculated as

\[
R_b = 2r_L r_C R_s \log_2 M ,
\]

assuming PM transmission.

We quantize the achievable bit rate \( R_b \) into multiple segments \( R_{b,i} \), \( i = 1, 2, \ldots, N_s \), where \( R_{b,i} < R_{b,i+1} \). Each \( R_{b,i} \) can be represented by a set of transmission parameters \((M, r_C)\), commonly referred to as a mode. For each transmission mode, we define a
threshold value for channel state information (CSI), $CSI_{th,i}$, at which $(M,r_C)_i$ can achieve a target decoder output BER, $P_{b,\text{out,req}}$, on an AWGN channel, which is commonly assumed for modeling optical channels [30]. In addition, decoders for SDD algorithms, e.g., Turbo decoding and BP algorithms, are typically designed based on the AWGN assumption in advanced optical fiber systems, and [12],[13] show that the AWGN assumption works well, through experiments in practical setups. The controller runs a rate adaptation algorithm based on CSI that is estimated at the receiver and reported to the transmitter. The transmitter monitors variations of reported CSI, and adjusts the RS-LDPC code rate and constellation size to maximize $R_b$, while achieving the target BER $P_{b,\text{out,req}}$ at the RS decoder output. We consider two possible choices for CSI: (a) $\text{SNR}$, which is the SNR per symbol estimated at the equalizer output, or (b) $P_{b,\text{in,RS}}$, which is the BER measured at the RS decoder input (LDPC decoder output). We design the rate adaptation algorithm such that it can provide an arbitrary specified SNR margin $\mu$ in dB.

Fig. 4.8 Information bit rate vs. threshold SNR per symbol for PM-M-QAM on AWGN channel.

Fig. 4.8 shows information bit rates and threshold values of SNR for different combinations of $(M,r_C)$ on an AWGN channel. Information bit rates are computed using (4.4), and required SNR values are obtained by combining Fig. 4.2 and Fig. 4.6. The set of filled $(M,\text{code})$ combinations represent a possible choice of modes for rate-adaptive transmission.
transmission modes using SNR as CSI. The transmission modes using $P_{b, in, RS}$ as CSI can be defined as described in [38].

A rate adaptation algorithm using SNR as CSI may not reliably yield the highest feasible information bit rate when SNR is not an accurate predictor of $P_{b, out, RS}$, but it is straightforward in terms of mapping a CSI value to an expected transmission rate as described in Section 2.2.3. Since SNR may not be a sufficiently accurate predictor of $P_{b, in, RS}$, as in [38], we introduce SNR penalty parameters $\Delta_i$, which quantify the increase in SNR required to achieve a specified $P_{b, in, RS}$ when using $(M, r_C)_i$ as compared to the AWGN case. Finding $\Delta_i$ would require searching for the SNR that achieves the target $R_{b, i}$ (satisfying $P_{b, out, req}$) that is predicted by the threshold SNR with an AWGN assumption. An algorithm using $P_{b, in, RS}$ as CSI requires a more complicated CSI-to-rate mapping compared to using SNR as CSI, but can reliably yield the highest feasible information bit rate, since $P_{b, in, RS}$ is an accurate predictor of $P_{b, out, RS}$ for a hard-decision RS decoder as described in Section 2.2.3. $P_{b, in, RS}$ can be estimated by the decoder from the number of bit errors corrected in decoding. We note that SNR penalty parameters are not required when using $P_{b, in, RS}$ as CSI [38].

A pseudocode for the algorithm is given as follows, where $CSI_{th, i} \{\Delta_i, \mu\}$ denotes a threshold CSI value for mode $i$ considering SNR penalty $\Delta_i$ and SNR margin $\mu$:

1. Initialize parameters to the highest-rate mode.
   - Initialize mode: $(M, r_C)_i \rightarrow (M, r_C)_N$.
   - Initialize up/down counters: $C_{up} = C_{down} = 0$
2. Check if rate change is necessary.
   - if $CSI \leq (or \geq) CSI_{th, i} \{\Delta_i, \mu\downarrow\}$ when using SNR (or $P_{b, in, RS}$) as CSI
     - $C_{down} = C_{down} + 1$
     - if $C_{down} \geq N_{down}$, $(M, r_C)_i \rightarrow (M, r_C)_{i-1}$
   - elseif $CSI \geq (or \leq) CSI_{th, i} \{\Delta_i, \mu\downarrow\}$ when using SNR (or $P_{b, in, RS}$) as CSI
     - $C_{up} = C_{up} + 1$
     - if $C_{up} \geq N_{up}$, $(M, r_C)_i \rightarrow (M, r_C)_{i+1}$
   - else
     - $C_{up} = C_{down} = 0$
4. ADAPTIVE SOFT-DECISION CODING AND MODULATION

- \((M, rC)_i \rightarrow (M, rC)_i\)

3. Go to step 2.

The parameters \(\mu_{up}\) and \(\mu_{down}\) are SNR margins when changing rate up and down, respectively, and \(N_{up}\) and \(N_{down}\) are counters when changing rate up and down, respectively.

4.3 System Simulations

We have evaluated the rate-adaptive transmission scheme in the model long-haul system shown in Fig. 4.9. We simulated a single channel in a wavelength-division-multiplexed (WDM) system with center wavelength of 1550 nm and nominal 50-GHz channel spacing. The modulation is single-carrier PM-M-QAM using non-return-to-zero pulses at a symbol rate \(R_s = 30.156\) Gsymb/s. Each modulator is a quadrature Mach-Zehnder device. Each drive waveform is a train of rectangular pulses filtered by a five-pole Bessel LPF having a 3-dB bandwidth \(1.4R_s = 42.2\) GHz. Percentages 90, 95 and 99% of the modulated signal energy are contained in bandwidths of 29.3, 33.7 and 41 GHz, respectively. We employ line coding at rate \(r_L = 64/66\), yielding an information bit rate \(R_b = 200\) Gbit/s for the highest overall code rate \(r_C\) and \(M = 16\).
4. ADAPTIVE SOFT-DECISION CODING AND MODULATION

The fiber network comprises multiple 80-km spans of standard single-mode fiber (SMF). In dispersion-compensated systems, dispersion-compensating fiber (DCF) is used to pre-compensate CD by \(-510\) ps/nm, and to compensate CD in each span to a residual CD of 42.5 ps/nm, corresponding to 3.1% residual dispersion per span (RDPS). The parameters for SMF and DCF are summarized in Table 3.1. Each span uses a two-stage inline amplifier to compensate total span loss, with the ratio between first- and second-stage gains optimized via simulation, as in [38]. Amplifier noise figures of 5 dB are assumed. In dispersion-uncompensated systems, the pre-compensation and inline DCFs are omitted.

In order to simulate transmission through a large-scale mesh network, a ROADM is inserted at every third span. We choose a homogeneous, high-density distribution of ROADMs to be conservative and to obtain a simple trend of achievable bit rate vs. transmission distance. Filtering by each wavelength-selective switch is simulated by a super-Gaussian bandpass filter of order 2.5, 3-dB bandwidth of 40 GHz and insertion loss of 12 dB, consistent with typical devices [2]. We assume 3-dB bandwidth of 40 GHz and insertion losses of 6 dB each for the multiplexer and the de-multiplexer.

The receiver employs a fifth-order Butterworth anti-aliasing filter of 3-dB bandwidth $R_s$, samples at a rate of $2R_s$ complex samples per polarization and performs digital compensation of CD and polarization-mode dispersion (PMD) using finite impulse response time-domain filtering, as described in [32]. At each transmission distance, the number of filter taps is optimized to make residual ISI negligible. The equalizer is adapted using the least mean squares algorithm with the parameters of step size, shift interval, and number of shifts optimized. The signal at the equalizer output is scaled to remove the bias introduced by the MMSE equalizer [41].

Signal propagation is simulated by numerical integration of the vector nonlinear Schrödinger equation by the split-step Fourier method [33].

We run the BP algorithm for LDPC decoding with 50 iterations. Then we measure $P_{b,in,RS}$ at the RS decoder input (or LDPC decoder output). We simulate a
4. ADAPTIVE SOFT-DECISION CODING AND MODULATION

sufficient number of symbols such that the true $P_{b,in,RS}$ value does not exceed 130% of the estimate with 95% confidence down to measured BERs of $P_{b,in,RS,95,M} = 1.9 \times 10^{-4}$, $1.3 \times 10^{-4}$, and $9.5 \times 10^{-5}$ for $M = 4, 8, 16$. We estimate that the uncertainty in measured $P_{b,in,RS}$ corresponds to 0.1 dB uncertainty in SNR.

At each transmission distance, the launched power is optimized with 1 dB resolution, to minimize the value of $P_{b,in,RS}$. When the measured $P_{b,in,RS}$ is below $P_{b,in,RS,95,M}$, the launched power is optimized to minimize the value of $P_{b,in,LDPC}$ (or BER at the equalizer output) that is computed as described in [46]. We have found that in cases when measured $P_{b,in,RS}$ is above the threshold values, minimizing $P_{b,in,RS}$ and minimizing $P_{b,in,LDPC}$ result in the same optimized power in about 85% of the cases, and only a 1-dB difference in the remaining cases.

4.4 Simulation Results

In order to evaluate the rate-adaptive scheme, at each transmission distance, for each modulation order, and for each RS-LDPC code, after the transmit power is optimized, the received SNR per symbol and RS decoder input BER are recorded. We estimate the SNR per symbol by calculating the ratio between the average symbol power and the noise variance, which is empirically measured at the equalizer output.

We have employed the rate adaptation algorithm of Section 2.2.3, using $P_{b,in,RS}$ as CSI, and using all possible transmission modes. We assume a required RS-LDPC decoder output BER $P_{b,\text{out,req}} = 10^{-15}$, SNR margins $\mu_{\text{up}} = \mu_{\text{down}} = 0, 1, \ldots, 5$ dB and counter parameters $N_{\text{up}} = N_{\text{down}} = 1$ (because we assume static channel conditions). Fig. 4.10 (a) and (b) present achievable information bit rates with and without inline dispersion compensation, respectively, as a function of transmission distance. In dispersion-compensated systems, with zero margin, a bit rate $R_b = 200$ Gbit/s can be realized up to 960 km, with the achievable rate decreasing by approximately a factor of two for every additional 2000 km. In dispersion-uncompensated systems, with zero margin, a bit rate $R_b = 200$ Gbit/s can be realized up to 1920 km, with the achievable rate decreasing by approximately a factor of two.
for every additional 3000 km. Compared to the hard-decision FEC scheme of [46], the transmission distances have increased by about 50%.

Fig. 4.10 Achievable information bit rates vs. transmission distance for different SNR margins. The set $(M,\text{code})$ denotes the modulation order and type of RS-LDPC code. (a) Dispersion-compensated system. (b) Dispersion-uncompensated system.

We used $P_{b,\text{in,RS}}$ as CSI, and find that the uncertainty of measured $P_{b,\text{in,RS}}$ may result in a reduced transmission distance by at most two spans for dispersion-compensated systems for distances beyond 5000 km, but the penalty may increase to three spans for uncompensated systems for distances beyond 10000 km.
4. ADAPTIVE SOFT-DECISION CODING AND MODULATION

4.5 Discussion

In this section, we examine how the SNR in the model system scales with transmission distance, and estimate the performance gap between the proposed rate-adaptive scheme and an ideal coding scheme achieving information-theoretic limits.

Fig. 4.11 Two different measures of SNR compared to the SNR required for an ideal capacity-achieving coding scheme to achieve error-free transmission at the information bit rate $R_b$ achieved by the proposed rate-adaptive scheme. (a) Dispersion-compensated system. (b) Dispersion-uncompensated system.

We consider several different measures of SNR and compare these to an equivalent SNR corresponding to the information bit rate achieved by the proposed rate-adaptive scheme. This discussion makes reference to Fig. 4.11 (a) and (b), which describe dispersion-compensated and -uncompensated systems, respectively.
**The uppermost solid curves in Fig. 4.11 (a) and (b) show \( SNR_{AWGN} \), which is an empirical estimate of the SNR per symbol (in two polarizations) as limited only by accumulated optical amplifier noise:**

\[
SNR_{AWGN} = \frac{P_L}{P_n}. \tag{4.5}
\]

Here, \( P_t \) is the transmitted signal power, optimized at each transmission distance, which equals the received signal power at the demultiplexer input, since the network is designed to have unit signal gain. \( P_n \) is the noise power in two polarizations, and is computed using (8) in [38]. The dashed curves in Fig. 4.11 (a) and (b) show \( SNR_{AWGN, best-fit} \) for dispersion-compensated and -uncompensated systems, respectively, which is a power-law fit to the observed \( SNR_{AWGN} \) as a function of transmission distance \( L \):

\[
SNR_{AWGN, best-fit} = \begin{cases} 
\frac{A}{L^{1.40}}, & 3.1\% ~ RDPS \\
\frac{A}{L^{1.15}}, & 100\% ~ RDPS 
\end{cases} \tag{4.6}
\]

The constant \( A \) and the exponent of \( L \) have been found by curve fitting. We obtained trends of \( SNR_{AWGN} \) similar to those in [46]. In the dispersion-compensated system, \( SNR_{AWGN} \) scales as \( L^{-1.40} \), a slightly weaker dependence than in systems using a fixed PM-QPSK constellation [38], where the dependence was found to be \( L^{-1.55} \). In the dispersion-uncompensated system, the \( L^{-1.15} \) dependence is consistent with [42], where the optimized SNR was found to scale as \( L^{-1} \).

**The middle solid curves in Fig. 4.11 (a) and (b) show \( SNR_{AWGN+NL, equivalent} \) for dispersion-compensated and -uncompensated systems, respectively, which is the SNR per symbol observed empirically at the equalizer output, and includes all linear noise (amplifier noise and equalizer noise enhancement) and nonlinear noise (arising from fiber nonlinearity). “Equivalent” refers to the fact that their sum may not be white or Gaussian-distributed. At small \( L \), \( SNR_{AWGN+NL, equivalent} \) is about 3.1 dB and 4.2 dB lower than \( SNR_{AWGN} \) for dispersion-compensated and -uncompensated systems, respectively, indicating that at the optimum \( P_t \), amplifier noise and nonlinear noise**
powers are roughly equal. At large $L$, the difference increases to about 5.9 dB and 8.6 dB for dispersion-compensated and -uncompensated systems, respectively, presumably because significant ROADM channel narrowing over long distances causes equalizer noise enhancement. The noise enhancement at large $L$ is more pronounced here as compared to [46] because the SDD FEC scheme here allows the signal to propagate over longer distances than the HDD FEC scheme used in [46].

The bottom solid curves in Fig. 4.11 (a) and (b), $SNR_{\text{required,ideal}}$, are computed by inverting the formula for the capacity of a ideal discrete-time AWGN channel transmitting at symbol rate $R_s$ in two polarizations, i.e., $C = 2R_s \log_2(1 + SNR)$:

$$SNR_{\text{required,ideal}} = 2^{\frac{R_b}{2R_s}} - 1.$$ (4.7)

$SNR_{\text{required,ideal}}$ corresponds to the SNR required for an ideal, capacity-achieving coding scheme to achieve error-free transmission on an AWGN channel at the information bit rate $R_b$ achieved by the proposed scheme. The vertical separation between $SNR_{AWGN+NL,\text{equivalent}}$ and $SNR_{\text{required,ideal}}$ is an estimate of the performance gap between an ideal coding scheme and the scheme proposed here. For dispersion-compensated systems, the gap ranges from about 4.0 dB to about 2.7 dB as $L$ varies from 960 to 9680 km. For dispersion-uncompensated systems, the gap ranges from about 3.9 dB to about 2.9 dB as $L$ varies from 1920 to 19360 km.

Several observations about these performance gaps can be made.

First, we observe that the gaps at small $L$, where higher modulation orders are used, are larger by about 1 dB than the gaps at large $L$, where lower modulation orders are used. We expect that at small $L$, further improvement can be achieved by constellation shaping [64], using either non-uniformly spaced constellations or uniformly spaced constellations with non-uniform probability distributions. By contrast, we do not expect shaping to be beneficial at large $L$, where binary signaling per dimension (such as PM-4-QAM) should be optimal.

Second, we observe that the performance gaps at large $L$ are greater than for the DVB-S2 BCH-LDPC codes on AWGN channels, where the lowest-rate codes have
gaps to capacity of about 0.9 dB [50]. We have replaced the outer BCH codes of [50] with RS codes in order to provide stronger error correction capability, further reducing the error floor. Compared to the DVB-S2 BCH codes, the RS codes provide slightly higher coding gains (about 0.1 dB), but their lower rate corresponds to an increase in the gap of about 0.8 dB, computed using (4.7). The gap to capacity for our lowest-rate RS-LDPC codes on AWGN channels is estimated to be $0.9 \text{ dB} - 0.1 \text{ dB} + 0.8 \text{ dB} = 1.6 \text{ dB}$. The performance gaps observed in dispersion-compensated and uncompensated systems are, respectively, about 1.1 dB and 1.3 dB higher than expected on AWGN channels. These increases in the performance gap can be attributed, at least in part, to residual channel memory (for both signal and noise) caused by the combined effects of CD and nonlinearity. We expect that these gaps can be reduced by using digital backward propagation, as was used in [42].

Third, we observe that using our SDD scheme, the performance gaps at small $L$ are about 2.5 dB smaller than for the HDD scheme in [46], close to the expected advantage of SDD over HDD [65]. It is less relevant to compare the schemes at large $L$, because the HDD scheme in [46] used repetition coding, which is known to be suboptimal.

We have evaluated laser linewidth requirements of the proposed rate-adaptive scheme, applying the same analysis as in [46]. We find worst-case linewidth requirements of about 2.6 MHz and 800 kHz for dispersion-compensated and uncompensated systems, respectively. These linewidth requirements are similar to those in [46], and can be met using commercially available lasers [45].
5. Summary and Future Work

5.1 Summary

In this thesis, we studied rate-adaptive optical transmission schemes using variable-rate FEC codes and variable signal constellations with a fixed symbol rate.

In the first scheme, we studied rate-adaptive transmission using variable-rate FEC codes with a fixed signal constellation and a fixed symbol rate. The FEC scheme employs serially concatenated RS codes with hard-decision decoding, using shortening and puncturing to vary the code rate. An inner repetition code with soft combining provides further rate variation. We have combined the FEC scheme with single-carrier polarization-multiplexed QPSK and digital coherent detection, evaluating performance in a model long-haul system with inline dispersion compensation. With zero SNR margin, the achievable information bit rate varies from 100 Gbit/s at 2000 km, to about 60 Gbit/s at 3000 km, to about 35 Gbit/s at 4000 km. Compared to an ideal coding scheme achieving information-theoretic limits on an AWGN channel, the proposed coding scheme exhibits a performance gap ranging from about 5.9 dB at 2000 km to about 7.5 dB at 5000 km. Much of the increase in the gap can be attributed to sub-optimality of the repetition coding used beyond 3280 km. We found that the SNR as limited by amplifier noise scales with distance $L$ as $L^{-1.55}$, as compared to the $L^{-2}$ expected in a dispersion-free system.

In the second scheme, we studied rate-adaptive transmission using variable-rate FEC codes, variable signal constellations, and a fixed symbol rate. The FEC scheme employs serially concatenated RS codes with hard-decision decoding, using shortening and puncturing to vary the code rate. An inner repetition code with soft combining provides further rate variation. We have combined the FEC scheme with single-carrier PM-$M$-QAM with varying $M$ and digital coherent detection, evaluating performance in a model long-haul system with and without inline dispersion compensation. With zero SNR margin, the information bit rates of 200/100/50 Gbit/s
are achieved over distances of 640/2080/3040 km and 1120/3760/5440 km for
dispersion-compensated and -uncompensated systems, respectively. Compared to an
ideal coding scheme achieving information-theoretic limits on an AWGN channel, the
proposed scheme exhibits a performance gap ranging from about 6.4 dB at 640 km to
7.6 dB at 5040 km for dispersion-compensated system, and from about 6.6 dB at 1120
km to 7.5 dB at 7600 km for dispersion-uncompensated system. Much of the increase
in the gap can be attributed to sub-optimality of the repetition coding used beyond
3280 km and 5760 km for dispersion-compensated and -uncompensated systems
respectively. We found that the SNR as limited by amplifier noise scales with distance
$L$ as $L^{-1.30}$ and $L^{-1.05}$ for dispersion-compensated and -uncompensated systems
respectively.

In the last scheme, we studied rate-adaptive transmission using variable-rate
FEC codes, variable signal constellations, and a fixed symbol rate. The FEC scheme
employs a family of concatenated RS codes (with HDD) and LDPC codes (with
iterative SDD). We have combined the FEC scheme with single-carrier PM-$M$-QAM
with varying $M$ and digital coherent detection, evaluating performance in a model
long-haul system with and without inline dispersion compensation. With zero SNR
margin, the information bit rates of 200/100/50 Gbit/s are achieved over distances of
960/2800/4400 km and 1920/4960/8160 km for dispersion-compensated and -
uncompensated systems, respectively. Compared to an ideal coding scheme achieving
information-theoretic limits on an AWGN channel, the proposed scheme exhibits a
performance gap ranging from about 4.0 dB at 960 km to 2.7 dB at 9680 km for
dispersion-compensated systems, and from about 3.9 dB at 1920 km to 2.9 dB at
19360 km for dispersion-uncompensated systems. The larger gap at short distances
can be attributed to sub-optimality of the uniform QAM constellations, as compared to
optimal Gaussian-distributed constellations. Some of the gap at large distances may be
explained by residual channel memory (for both signal and noise), caused by the
combined effect of CD and nonlinearity. Compared to a reference system using an
HDD FEC scheme, we observed approximately 50% increases in transmission
distance, and approximately 2.5-dB reductions in the performance gap from an ideal
coding scheme. We have simulated SDD in floating-point arithmetic, and would expect a slight performance loss (less than 0.2 dB on an AWGN channel) when implementing a fixed-point decoder [66].

In this work, we have considered only a single-channel system with intrachannel nonlinearities, in order to keep simulation run time reasonable. Simulating both dispersion-compensated and -uncompensated systems takes about 1 month and 2 months for the HDD and SDD schemes, respectively. Based on [67], we estimate that to simulate a dense WDM system with $k$ channels, the required simulation time would increase as $k^{3.5}$. Assuming $k = 3$, the required simulation time would increase by a factor of 46.8, which is impractical. If we included the effects of interchannel nonlinearities, we would expect a slight reduction in the maximum distance at which a given bit rate can be achieved. However, we would not expect our results to change qualitatively, because interchannel and intrachannel nonlinearities scale similarly with transmission distance.

5.2 Future work

In this thesis, we have studied the performance of our rate-adaptive coding and modulation schemes only in single-channel systems with intrachannel nonlinearities, because of limited available computing resources (a cluster of ten four-core processors). In the future, as computing resources improve, we may perform multi-channel simulations to evaluate the effects of interchannel nonlinearities.

At shorter transmission distances, where higher-order constellations are used to achieve high spectral efficiency, we can further improve spectral efficiency or transmission distance by constellation shaping [64]. The maximal theoretical shaping gain is 1.53dB. Shaping, which is well-known in non-optical communications, makes the probability density of the transmitted signal better approximate a two-dimensional joint Gaussian distribution, which is known to be the capacity-achieving distribution on an additive Gaussian noise channel. Shaping can be achieved using either non-uniformly spaced constellations or uniformly spaced constellations with non-uniform
5. SUMMARY AND FUTURE WORK

probability distributions. In fact, a member of my research group is now pursuing the latter approach, building on the LDPC ACM scheme described above.

In order to increase coding gains in the AC and ACM schemes using concatenated RS-RS codes, we can perform soft-decision iterative decoding between the inner and outer RS codes. Also, we can obtain modestly higher coding gains by constructing separate RS-RS codes for different code rates, instead of shortening or puncturing a single mother code.
In this Appendix, we discuss methods to compute error probabilities. All quantities considered here are measured at the de-repeater output (in the case $f_R = 1$, all quantities are the same at the de-repeater input). As described in Section 3.2.1, we assume Gray mapping for $M = 4, 16$ or the mapping of [39] for $M = 8$, so the BER $P_{b,in}$ and SER $P_{s,in}$ are related by (3.3). A constellation with $M$ points can be divided into $J$ equivalence classes of points, denoted by $j = 1, \ldots, J$, where class $j$ includes $M_j$ points. The constellation can also be described as including $K$ equivalence classes of pairs of points, denoted by $k = 1, \ldots, K$. This notation for points and pairs of points is illustrated in Fig. A.1. Values of the parameters for constellations with $M = 4, 8, 16$ are given in Table A.1.

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Table A.1 Classes of constellation points and Q factors. The notation is defined in the text.
Suppose the minimum distance between points in a constellation is $d$. Let $N_{jk}$ denote the number of nearest neighbors at distance $d$ with which a point in class $j$ forms pairs in class $k$. Fig. A.2 shows the conditional probability densities of the received signal (at the equalizer output) along the line joining a point with its neighbor. Let $\sigma_{jk}$ be the ensemble-average standard deviation of the total “noise” (including residual ISI and nonlinearity) on the point in class $j$, and let $\sigma'_{jk}$ denote the corresponding quantity on the neighboring point. Assuming the decision threshold is set midway between the points (as in our implementation), assuming the total “noise” is Gaussian, the ensemble-average SER for a point in class $j$ can be approximated as:

$$P_{s,jm,j} \approx \sum_{k=1}^{K} N_{jk} Q \left( \frac{d}{2\sigma_{jk}} \right).$$

(A.1)

If $\sigma_{jk}$ and $\sigma'_{jk}$ are unequal (as indicated in Fig. A.2), the error probability can be minimized by setting the threshold at a distance $\frac{\sigma_{j} \sigma'_{jk}}{\sigma_j + \sigma'_{jk}} d$ from the point in class $j$.

Defining a $Q$ factor for the pair of class $k$:

$$F_{Q,k} = \frac{d}{\sigma_{jk} + \sigma'_{jk}},$$

(A.2)

the SER for the point in class $j$ is approximated as:

$$P_{s,im,j} \approx \sum_{k=1}^{K} N_{jk} Q(F_{Q,k}).$$

(A.3)
APPENDIX

In any case, the average SER is given by:

$$P_{s,in} = \frac{1}{M} \sum_{j=1}^{M} P_{s,in,j}.$$  \hfill (A.4)

Note that in the AWGN case, for a given constellation, all $\sigma_{jk}$ and $\sigma'_{jk}$ become equal, all $F_{Q,k}$ become equal, and (A.1) and (A.4) (or (A.3) and (A.4)) become equivalent to (3.2).

![Conditional probability densities of received signal along a line joining a point in class $j$ with a neighbor with which it forms a pair in class $k$.](image)

To test the accuracy of our error-probability analysis, after propagating signals through our nonlinear fiber network, we estimated the standard deviation of the total “noise” on each constellation point. As a first test, we computed $P_{s,in,j}$ using (A.1), and employed (A.4) and (3.3) to compute $P_{b,in}$. We then compared the results to $P_{b,in}$ measured by error counting. For $M = 4$, predicted and measured values of $P_{b,in}$ agreed within 0.18 dB, as in [38]. For $M = 8, 16$, measured values exceeded computed values by factors corresponding to 0.56 and 0.38 dB, respectively. The prediction error is larger for higher modulation orders presumably because they are more sensitive to nonlinearity effects. As a second test, we computed the $F_{Q,k}$ using (A.2), and employed (A.3), (A.4) and (3.3) to predict $P_{b,in}$. Compared to values of $P_{b,in}$ measured by error counting, we observed the same errors (within 0.01 dB) as with the first method. The $P_{b,in}$ computation method using (A.1), (A.4) and (3.3) was employed in optimizing the transmit power and estimating $P_{b,in}$ when the expected $P_{b,in}$ is very low, as described in Section 2.3.
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