Episodic degassing from unsteady lava lake convection in Ray Lava Lake, Mount Erebus, Antarctica

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Lava lakes are surface features that represent the upper portion of volcanic systems, and their behavior places limits on processes occurring in the interior of volcanoes. Persistently active lava lakes show continuous degassing and open convection over years to decades, which implies near steady-state conditions. The persistently active lava lake on Mount Erebus, Ross Island, Antarctica, known as Ray Lake, shows periodic activity in the form of small pulses of gas and hot magma at the surface every 5-18 minutes and occasional Strombolian eruptions while maintaining long-term near steady-state behavior in temperature, heat flux, gas flux, and composition when time-averaged over multiple cycles. This episodicity has been most commonly attributed to gas slugs: large gas bubbles rising through the conduit that burst at the surface. Alternate hypotheses, however, include unstable bidirectional flow in which episodicity is driven from the upper portion of the conduit. These hypotheses invoke a conduit source for episodicity. We present numerical simulations of Ray Lake with a constant inflow rate of gas-rich magma from the conduit and consider periodicity present in the convective pattern of the lava lake, consistent with a near-surface cause of episodicity. Our simulations of convection on Mount Erebus show drip instabilities with a periodicity of 5-20 minutes. Our results match the observed behavior well, showing a similar period as field observations. An increase in ascent velocity when magma enters the openly convecting lake can result in periodic behavior without invoking conduit dynamics, challenging existing ideas about near-surface volcanic conduit processes in persistently degassing volcanoes.
1 Introduction

Lava lakes provide a rare opportunity for direct observation of the near-surface portions of volcanic systems and act as natural laboratories to further our understanding of eruption mechanics of low-viscosity, open-conduit volcanoes that persistently release gas and heat. Persistently degassing volcanoes emit orders of magnitude more heat and gas than would be expected based on the volume of lava erupted alone (Beckett et al., 2014; Francis et al., 1993; Huppert & Hallworth, 2007; Palma et al., 2011; Kazahaya et al., 1994; Stevenson & Blake, 1998). This indicates continual recycling of lava through circulation in the conduit. Lava lake eruptions are frequent and of low explosivity, typically Strombolian to Hawaiian, making them readily observable and placing constraints on the dominant physical processes.

Episodic cycles punctuated by occasional eruptions are widely observed in low-viscosity, persistently-degassing systems, but the driving processes are not fully understood. Exsolution of volatiles is thought to be the primary driver of eruptive activity. As magma approaches the surface, the decrease in magmastatic pressure reduces the solubility of volatiles in the melt phase. As a result, bubbles form in the magma. Due to the high density contrast between bubbles and melt ($\Delta \rho \sim -2400 - 3000$ kg/m$^3$), vapor is a significant driver of convection. If bubbles are large and viscosity is low, bubbles may rise with respect to the melt and crystal phases. If bubble segregation is slow with respect to the melt, the magma moves as a bubbly suspension.

Field observations, laboratory experiments, and numerical models suggest a variety of potential eruption mechanisms in low-viscosity systems. The primary proposed mechanism for eruptions in open-conduit volcanoes is the ascent of gas slugs: large bubbles that occupy nearly the full width of the conduit and rise through an otherwise stagnant magma column to burst at the surface (Blackburn et al., 1976; Wilson & Head, 1981). This mechanism has been suggested on the basis of analog laboratory experiments (Jaupart & Vergniolle, 1988, 1989; Vergniolle & Jaupart, 1990). The formation of gas slugs depends upon high rates of
bubble coalescence (Wilson & Head, 1981) or upon bubble accumulation and foam collapse (Vergniolle & Jaupart, 1990).

When bubble coalescence is inefficient and bubble nucleation controls the formation of many small bubbles, exsolved bubbles segregate upwards relative to the carrier melt. At high concentrations, bubbles are thought to experience particle-particle interactions that increase the effective viscosity of the mixture, which may result in a local decrease in velocity (Manga, 1996). The formation of bubble waves due to instabilities in bubble ascent may result in planar to lenticular concentrations of gas bubbles (Manga, 1996; Michault et al., 2013). Gas is expected to be sourced from deep in the magma column and bubble waves form upon ascent. Bubble waves could be responsible for periodic increases in gas flow from the conduit.

If the vapor phase does not segregate from the melt the mixture may flow as a bubbly suspension when bubbles remain separate, or as a foam, when bubbles touch (generally when vesicularity exceeds 40-50% of the total volume). Surface activity may be driven by changes in convection. Instabilities in near-surface conduit flow could be responsible for generating periodic outgassing. Bidirectional, also known as countercurrent, flow in the conduit can arise where magma simultaneously flows both up and down in the conduit (Beckett et al., 2014; Huppert & Hallworth, 2007; Kazahaya et al., 1994; Stevenson & Blake, 1998). Bidirectional flow is unstable over a wide range of conditions and has been suggested as a driving mechanism for eruptions on Mount Erebus (Oppenheimer et al., 2009), Stromboli (Beckett et al., 2014), and Villarrica (Palma et al., 2011).

Observations of lava lakes are primarily restricted to the surface. As a result, modelling may help us understand how surface observations are related to internal dynamics. Molina et al. (2012) work towards this goal by investigating lava lake behavior through a thermally-driven convective model of the lava lake on Mount Erebus. Their results show convective overturn on time scales of 0.1-0.5 years with velocities of order $10^{-5} \text{ m/s}$, 4 orders magnitude
slower than the observed surface velocities. They resolve melt and crystal phases but omit
the gaseous phase. We seek to complement the work of Molina et al. (2012) by adding the
effects of a gaseous phase to simulations of convection in the lava lake.

In order to interpret surface observations from lava lakes that may be derived from conduit
flow, we need an improved understanding of the ability of near-surface convection within lava
lakes to generate episodicity that may modify or overprint conduit signals. Here, we test
whether lava-lake dynamics may result in episodic activity without requiring episodic influx
from the conduit through the use of a two-dimensional, phase-averaging transport model
of gas-buoyancy driven convection. The finite element, Stokes flow model includes bubble
segregation and is coupled to a thermal evolution model which determines a crystallinity-
dependent rheology. We simulate only the lava lake with controlled inflow from the conduit.
We run simulations over a range of rheological properties, inflow conditions, and rates of
surface degassing and cooling which allow us to investigate pertinent convective regimes.
Our model imposes a constant inflow of bubble-rich magma from the conduit to investigate
how convection within the lava lake can produce episodic records of gas and heat flux, and
convective velocities observed at the surface. The model output track surface temperature,
velocity, strain rate, gas flux and heat flux that we compare to observational data.

2 Observational constraints on the Mount Erebus lava
lake

We examine the case study of Mount Erebus, which is part of the volcanic group in Ross
Island, Antarctica. The volcano supports a persistently active lava lake, known as Ray Lake
located at the bottom of the Inner Crater of Mount Erebus (fig. 3). The lava lake has
been degassing and openly convecting for at least several decades since its discovery in 1972
and appears to be in near steady state (Calkins et al., 2008; Dibble et al., 2008; Peters,
oppenheimer, kyle, et al., 2014). when surface observables are time-averaged over several hours, the system has approximately constant surface temperature, heat flux, and gas flux. about these mean values, ray lake shows periods of increased outgassing, heat loss, and surface velocities every 5 to 18 minutes (calkins et al., 2008; oppenheimer & kyle, 2008; oppenheimer et al., 2009; peters, oppenheimer, kyle, et al., 2014; peters, oppenheimer, killingsworth et al., 2014; sweeney et al., 2008). steady-state behavior is accompanied by occasional strombolian eruptions in which mildly explosive activity disrupts the surface and partially evacuates the lake (dibble et al., 2008).

persistent activity of the lava lake is thought to be maintained by continual supply of low-viscosity (~10^4–10^6 Pa.s) magma (giordano et al., 2008). magma viscosity is highly dependent upon temperature, crystallinity, composition, and volatile content. petrologic studies of ejecta reveal a phonolitic composition that has remained largely unchanged at the volcano since 17 ka (kelly et al., 2008). the silica undersaturated magma is thought to be the result of small degrees of partial melting in the mantle that produces magma along the basanite-phonolite lineage (eschenbacher, 1998; kelly et al., 2008; moore & thirlwall, 1992; moussallam et al., 2013).

observational studies of the ray lava lake utilize visual monitoring to characterize the convective and explosive activity (dibble, kyle, & rowe, 2008). following larger strombolian eruptions, the lake has been evacuated to reveal a simple cone-like geometry that is ~40 m across and ~30 m deep, although the level of the lake fluctuates by ~10 m. a 4-10 m wide conduit supplies fresh magma to the base of the lake (oppenheimer et al., 2009).

during steady-state activity, a 5-18 min periodic behavior is evident in the mean and peak surface velocities (peters, oppenheimer, killingsworth et al., 2014). mean surface velocities are typically less than ~0.1 m/s and reach up to ~0.15 m/s during periods of high activity (oppenheimer et al., 2009; peters, oppenheimer, killingsworth et al., 2014). the peak velocity varies between ~0.1-0.5 m/s with a period of 5-10 minutes (oppenheimer
et al., 2009), but may reach speeds above 0.8 m/s when the lake is most active (Calkins et al., 2008; E. Lev, personal communication). Strombolian eruptions are associated with high mean and maximum velocities, but the explosive eruptions are not correlated with the periodic background activity and do not regulate surface velocities beyond the ~ 5 min in which the lake refills (Peters, Oppenheimer, Killingsworth et al., 2014).

Thermal imaging of the lava lake is used to measure the surface temperature and estimate the heat flux from the lava lake to the atmosphere (Calkins et al., 2008; Oppenheimer et al., 2009). Surface temperatures vary from ~ 275 – 900°C with mean temperatures ~ 525 – 750°C (Calkins et al., 2008). Calkins and others (2008) found periodic activity every ~ 5 min increases the maximum and mean by ~ 120°C and ~ 20°C, respectively, with thermal maxima lasting ~ 1-2 min. Ground-based thermal flux estimates when the lake level is high (surface area of ~ 1400 m²) are 30 ± 10 MW and 15 ± 5 MW when the lake level is low (surface area of ~ 770 m²) (Calkins et al., 2008; Oppenheimer et al., 2009). Satellite observations show higher maximum radiant fluxes up to 100 MW (Wright & Pilger, 2008).

Studies of gas emissions from the lake show periodic activity in total flux and composition that is phase-locked with the surface activity (Peters, Oppenheimer, Killingsworth et al., 2014). Plume measurements of total SO₂ flux show a dominant 10 min periodic behavior with a flux of ~ 0.7 ± 0.3 kg/s SO₂ (Sweeney et al., 2008). The vapor composition shows the same periodicity where increased surface activity is associated with the emission of gas enriched in SO₂, H₂O, HCl, and HF with respect to CO₂ and an increase in CO₂/OCS (Oppenheimer & Kyle, 2008; Oppenheimer et al., 2009; Peters, Oppenheimer, Killingsworth et al., 2014). Calculations based on the composition and SO₂ flux measurements yields an estimate for the total gas flux of ~27.3 kg/s from the volcano summit.

Periodic behavior in the lake is observed in a variety of metrics which show phase-locked episodicity. An increase in temperature, heat, gas flux, and volatile gas species suggest that the episodic behavior is the result of fresh magma reaching the surface. This has widely been
attributed to unstable bidirectional flow from the conduit (Oppenheimer et al., 2009).

We evaluate the possibility of convection in the lava lake to generate episodicity present in the steady-state behavior of the lake. The simple geometry of the lake and abundant observations make it well suited to a modelling approach to understand possible sources of episodicity in the uppermost portion of the plumbing system. In the following, we simulate convection in the lava lake and compare model outputs to surface observations. An understanding of how convective processes create surface activity or modify signals derived from the conduit below is essential for connecting surface observations to predictions about conduit processes.

3 Methods & Techniques

3.1 Physical Model Description

Simulations of Ray Lake are performed using the Matlab code FEM3PHASE (Keller et al., 2013). The model takes a phase-averaging approach in which the phases are described as interpenetrating continuum fields at the system scale representing the spatially averaged interactions of a large number of phase constituents (bubbles, crystals, melt films) at the local scale. Each point on the continuum field corresponds to a unit volume of the multiphase aggregate that is small with respect to the system size but large with respect to the local-scale phase constituents (e.g. bubble size). The magma includes liquid melt, solid crystals, and vapor bubbles which advect according to the local aggregate velocity field. The vapor phase has a segregation velocity with respect to the overall fluid motion according to a hindered Stokes-segregation law. The bubble fraction also diffuses as a result of local-scale fluctuations of bubble motion with respect to the systems scale segregation velocity. We do not allow for phase separation of the crystals which is valid for small particles that experience negligible relative motion with respect to the magma. This condition is a simplification in our
model that may not best capture the effects of crystal settling on the 5-10 cm anorthoclase megacrysts present in the natural system (Molina et al., 2012; Moussallam et al., 2013).

3.1.1 Conservation of Mass and Momentum

The total mass of the aggregate is conserved for incompressible flow:

$$\nabla \cdot \mathbf{v} = 0 .$$  \hspace{1cm} (1)

Where $\mathbf{v}$ is the velocity of the aggregate magma. We find the Reynolds number:

$$\text{Re} = \frac{\rho u L}{\eta} ,$$  \hspace{1cm} (2)

where $\rho$ is the fluid density ($\approx 2600 \text{ kg/m}^3$), $u$ is the maximum velocity ($\approx 1 \text{ m/s}$), a characteristic linear dimension we take as the conduit diameter ($\approx 10 \text{ m}$), and $\eta$ is the dynamic viscosity ($\approx 10^4 \text{ Pa.s}$), yielding a $\text{Re} = 2.6 << 10^2$. The low value for $\text{Re}$ justifies the use of the Stokes flow equation.

Magma flow is density-driven where the density $\bar{\rho}$ is the aggregate density in the Stokes flow linear momentum balance:

$$-\nabla \cdot 2\eta \bar{\mathbf{D}}(\mathbf{v}) + \nabla P = \bar{\rho} \mathbf{g} ,$$  \hspace{1cm} (3)

where $\bar{\eta}$ is the aggregate viscosity, $P$ is pressure on the aggregate fluid volume, $\mathbf{g}$ is the gravitational acceleration and $\bar{\mathbf{D}}(\mathbf{v})$ is the deviatoric strain tensor of velocity:

$$\bar{\mathbf{D}}(\mathbf{v}) = \frac{1}{2}(\nabla \mathbf{v} + [\nabla \mathbf{v}]^T) - \frac{1}{3} \nabla \cdot \mathbf{v} .$$  \hspace{1cm} (4)

Where the density $\bar{\rho}$ is multiplied by gravity, as in the linear momentum balance, we use a
<table>
<thead>
<tr>
<th>Mineral</th>
<th>Volume Fraction</th>
<th>Density (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anorthoclase feldspar</td>
<td>0.91</td>
<td>2580</td>
</tr>
<tr>
<td>Titanomagnetite</td>
<td>0.03</td>
<td>5180</td>
</tr>
<tr>
<td>Olivine</td>
<td>0.02</td>
<td>3800</td>
</tr>
<tr>
<td>Clinopyroxene</td>
<td>0.02</td>
<td>3200</td>
</tr>
<tr>
<td>Fluorapatite</td>
<td>0.02</td>
<td>3200</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td></td>
<td><strong>2720</strong></td>
</tr>
</tbody>
</table>

Table 1: Mineral densities for the calculation of weighted average crystal density. The crystal volume fractions are an average of measurements from Moussallam et al. (2013), and density of mineral phases from Klein (2002).

The volume-fraction weighted average of the components:

$$\bar{\rho} = (1 - \phi)[(1 - \chi)\rho^m + \chi\rho^x] + \phi\rho^v,$$

where \(\rho^m\) is the density of the melt, \(\rho^x\) is a weighted average density of the crystals shown in table 1, and \(\rho^v\) is the vapor density, shown in table 2 with other reference parameter values.

We define \(\Delta\rho\) as the density contrast between the gas bubbles and crystal-melt suspension:

$$\Delta\rho = ((1 - \chi)\rho^m + \chi\rho^x) = \rho^v.$$

The effects of thermal expansion and compressibility of each phase for \(\Delta T \approx 50^\circ C\), and \(\Delta P \approx 50\) kPa is small compared to the density contrasts between phases so temperature and pressure expansivity effects are neglected. In other conservation equations, we use the Boussinesq approximation by employing a constant reference density \(\rho_0 = \rho^m = 2545\) kg/m³.
3.1.2 Conservation of Energy

The Stokes flow equations for mass and linear momentum balance are coupled with the conservation of energy given by:

\[
\frac{\partial T}{\partial t} = -\mathbf{v} \cdot \nabla T + \kappa_T \nabla^2 T + \frac{L}{c_p} \Gamma_\chi + \Gamma_T, \tag{7}
\]

where \(c_p\) is the specific heat capacity and \(\kappa_T\) is the thermal diffusivity of the melt. \(L_\chi\) is the latent heat of crystallization and \(\Gamma_\chi\) is a volumetric crystallization rate given by:

\[
\Gamma_\chi = \frac{\chi - \chi^{eq}}{\tau_\chi}, \tag{8}
\]

where \(\chi\) is the volume fraction of crystals in the melt:

\[
\chi = \frac{\phi^\chi}{\phi^\chi + \phi^m}, \tag{9}
\]

\(\phi^\chi\) and \(\phi^m\) are the volume fraction per unit volume of crystals and melt respectively. \(\chi^{eq}\) is the equilibrium crystallinity and \(\tau_\chi\) is a characteristic time for crystallization taken to be small enough to ensure a well equilibrated system on the time scale of magma flow.

\(\Gamma_T\) is a parameterized surface cooling rate that incorporates the effects of diffusive, advective, and radiative heat loss from the top of the lake. This condition is more appropriate than applying an isothermal boundary because the convective motion of the lake maintains a sharp thermal boundary at the surface which is not well resolved on our numerical grid:

\[
\Gamma_T = -\frac{T - T_{atm}}{\tau_T} \exp \left( -\frac{z}{\delta} \right), \tag{10}
\]

where \(T_{atm}\) is the air temperature above the lake, buffered at the boiling point of water (100 °C), \(\tau_T\) is the characteristic time scale over which cooling occurs, calibrated to match field observations of thermal flux, \(z\) is depth, and \(\delta\) is the characteristic length scale of surface
cooling. δ is limited to be greater than or equal to the height of four grid cells to be well resolved.

3.1.3 Vesicularity Evolution Model

Vesicularity, where \( \phi \) is the volume fraction of vapor, is conserved. Vesicularity evolution is modelled by:

\[
\frac{\partial \phi}{\partial t} = -\nabla \cdot (\phi \mathbf{v}^v) + \kappa_\phi \nabla^2 \phi + \Gamma_\phi, \tag{11}
\]

Where \( \kappa_\phi \) is the diffusivity due to local-scale fluctuations in bubble motion (Segre et al., 2001), \( \mathbf{v}^v \) is the vapor velocity given by the magma velocity \( \mathbf{v} \) and an additional segregation velocity using a hindered Stokes rise speed:

\[
\mathbf{v}^v = \mathbf{v} - \frac{2a_0^2}{9\eta} (1 - \phi)^\mu \Delta \rho g, \tag{12}
\]

where \( a_0 \) is the radius of the bubbles or bubble clusters, \( \mu \) controls the bubble hindering and is between 1 and 5 (Manga, 1996 after Russell et al., 1989; Segre et al., 2001 after Richardson & Zaki, 1954).

\( \Gamma_\phi \) is an imposed surface degassing rate of a form similar to the surface cooling in equation 8 above:

\[
\Gamma_\phi = -\frac{\phi}{\tau_\phi} \exp \left( -\frac{z}{\delta} \right), \tag{13}
\]

in which \( \tau_\phi \) is the characteristic time scale of degassing. The form of the degassing rate assumes that the final state of degassing results in no remaining fraction of gas which we assume for simplicity of our conceptual model. The surface degassing is imposed to increase the degassing rate due to bubble advection, which alone is insufficient for complete degassing, leading to gas accumulation in a foam layer near the surface.

Over the height of the domain (\( \approx 30m \)) with a gas compressibility of \( \beta \sim 10^{-6} \) 1/Pa, decompression expansion is negligible. We also neglect the divergence of bubble segregation.
velocity in the total mass conservation equation as it would have minor effect on flow but would require an evolving total volume in the domain as the lava lake degasses in time.

### 3.1.4 Crystallinity

We solve for crystallinity using the advection-reaction equation

\[
\frac{\partial \chi}{\partial t} = -\mathbf{v} \cdot \nabla \chi + \Gamma_x .
\]  

(14)

The crystallinity is assumed to be near equilibrium. We assume the magma composition of the lake is constant because convection in the lake and conduit causes the system to be well mixed and there is no significant segregation of the melt and crystal phases on the time and spatial scales considered here. Equilibrium crystallinity is a function of temperature and pressure. Because the pressure variations over the depth of the domain are small, the crystallinity is idealized as a function of only temperature using a power-law parameterization of the form:

\[
\chi^{eq} = \left( \frac{T - T_{liq}}{T_{sol} - T_{liq}} \right)^q ,
\]  

(15)

where \( T_{sol} \) is the solidus temperature, \( T_{liq} \) is the liquidus temperature and \( q \) is a fitting parameter near unity. The fitting parameters are determined to approximate the equilibrium crystal volume fraction measured by Moussallam et al. (2013). If the theoretical solidus and liquidus temperatures of the phonolite (Giordano et al., 2008) are used, the power-law parameterization yields a poor fit to the data. A fitted solidus temperature of \( T_{sol} = 884.5 \, ^\circ\text{C} \) and \( q = 1.25 \) give the best fit, shown in figure 1. The discrepancy between the fitted and theoretical solidus temperatures may be due to the presence of incompatible components in the melt that preferentially remain in a residual melt of less than 5% by volume. Because the region of interest in the temperature field is 100 \, ^\circ\text{C} \) higher, the fitted solidus temperature gives the best relation using the power-law parameterization.
Figure 1: Power-law fitting in temperature to equilibrium crystallinity data from Moussallam et al., 2013). The power-law fit matches the data poorly when using the theoretical values for the solidus temperature (red). A fitted solidus temperature of 884.5°C yields the best fit with the power-law crystallinity parameterization (blue).

3.1.5 Rheology

Laboratory experiments show a strong dependence of viscosity on crystal content (Krieger & Dougherty, 1959; Renner et al., 2000; Heymann et al., 2002; Costa et al., 2009; Pistone et al., 2012; Mader et al., 2013; Le Losq et al., 2015). We assume a viscosity model with a smooth step function of the form:

\[ X = \frac{1}{2} \left[ 1 + \tanh \left( \frac{\chi - \chi_{\text{crit}}}{w_\chi} \right) \right], \tag{16} \]

which is centered about a critical crystallinity \( \chi_{\text{crit}} \) and has a width of \( w_\chi \). The change in viscosity in addition to the step function is given by:

\[ \bar{\eta} = [\eta_s \exp(-\lambda(1 - \chi))]^X \times [\eta_t \exp(\lambda\chi)]^{(1-X)}, \tag{17} \]
Figure 2: Crystallinity and temperature dependence of viscosity at the reference strain rate of $10^{-3}$ s$^{-1}$. At low temperatures the viscosity approaches $10^{12}$ Pa.s and at high temperatures the viscosity approaches the melt viscosity of $10^4$ Pa.s.

where $\eta_l$ and $\eta_s$ are the reference viscosities for the melt and solid phases respectively (Giordano et al., 2008; Costa et al., 2009), and $\lambda_l$ and $\lambda_s$ are the slopes of viscosity with temperature away from the step function (Giordano et al., 2008; Costa et al., 2009; Moussallam et al., 2013).

Because the viscosity is highly dependent on the crystallinity, and the crystallinity is itself a function of only temperature in our model, the viscosity may be equivalently expressed as a function of temperature illustrated in figure 2. We constrain the maximum viscosity at $10^{12}$ Pa.s, which is below the viscosity of solid rock at these conditions, but presents a large enough contrast with the lake interior to prevent the lake walls from deforming on the time scale of simulation runs.

Crystal- and bubble-bearing magmas exhibit strain-rate dependent, non-Newtonian rheology above strain-rates of about $10^{-3}$ s$^{-1}$ (Saar et al., 2001; Caricchi et al., 2007; Renner et al., 2000; Heymann et al., 2002; Costa et al., 2009; Pistone et al., 2012; Mader et al.,
2013; Le Losq et al., 2015). Above this critical strain rate, the model imposes a power-law rheology for shear-thinning behavior. At high strain rates, above a yield stress, Binghamian behavior is introduced (Caricchi et al., 2007). The Bingham plastic flow may be important in maintaining mobility in the chilled surface layer. Binghamian yielding is not included in most model runs, but we test the effect on convective flow in one parameter test.

3.2 Numerical Model Description

3.2.1 Geometry

The geometry of the modelled lake bed is 38 m in diameter and 28 m in depth which tapers to a conduit of 8 m diameter, consistent with field observations of a cone-like lake narrowing to a 5-10 m conduit (Dibble et al., 2008; Oppenheimer et al., 2009). The model idealizes the geometry of the lake in two dimensions as a simple cone. We model from the top of the conduit to the surface of the lake and extend the domain laterally by 1 meter on each side at the surface of the lake, as shown in figure 3.

3.2.2 Model Discretization

The velocity-pressure solution is obtained by a Continuous Galerkin Finite-Element method, where variables are discretized on a regular rectangular mesh with linear shape functions for velocity and piece-wise constant for pressure (Q1P0 elements). The advection-diffusion-reaction equations are discretized with a staggered-grid finite-differences method on the same grid.

The model uses Fromm’s method for a grid based advection scheme in which the state of the advected function within a cell is a piecewise linear function of position with a centered slope. This method is second-order accurate and sufficiently maintains the sharp boundaries in the system without creating regions of spurious values in the vicinity of large contrasts.
Figure 3: A schematic cross section of the main crater of Mount Erebus shows the position of Ray Lake in fig. 3A. The box around Ray Lake defines the model domain. Fig. 3B places the boundary conditions in the domain. The top of the lake is flat and a cooling rate is imposed in the energy conservation equation that decays exponentially away from the surface (see eq. 6). The bottom boundary along the top of the conduit has a flux boundary with a prescribed inflow velocity, vesicularity, and temperature, and downwelling velocity. Grid-based advection is used to calculate the temperature and vesicularity fields. The gas vesicularity is advected with the vapor velocity, which is a combination of the magma and bubble segregation velocities. When gas bubbles are small we expect the advection of the fluid to be the dominant transport mechanism and bubble separation will be minimal.

3.2.3 Initial Conditions

The temperature in the walls is initially set to $T = 100^{\circ}\text{C}$, (an approximate surface temperature which is buffered by water vapor condensation) except for a thermal boundary layer of approximately 1 m, centered at the contact between the lake and the wall. The initial temperature of the lake is $T = 965^{\circ}\text{C}$ which best matches the observed 30% crystallinity with the temperature-crystallinity relationship of the model (Moussallam et al., 2013). The initial distribution of vesicularity is $\phi = 0$ everywhere.
3.2.4 Boundary Conditions

The cold walls pictured in fig. 3 above form the boundaries of the convective region of the lake and extend to the edges of the domain. The model initializes the walls of the lake as a material with the same thermodynamic properties as the magma in the lake interior. The walls and lake interior are separated by a thermal boundary where in the walls $T = 100^\circ$C, resulting in a much higher viscosity (fig. 2). The velocity field is not calculated in walls where the viscosity exceeds a critical limit for computational efficiency since the deformation in the walls is negligible ($w = u = 0$) resulting in internal no-slip boundaries at the walls. The left and right boundaries of the domain have a no-slip condition and are thermally insulating ($\frac{\partial T}{\partial z} = 0$ normal to boundary).

The top boundary is free-slip ($\frac{\partial u}{\partial z} = 0$, $w = 0$), such that melt cannot be advected vertically out of the domain. Gas can advect out the top of the boundary. Disruption of a surface layer of cooled material is inhibited by this technique which biases the simulations towards a stagnant-lid regime. The thermal boundary at the surface is insulating ($\frac{\partial T}{\partial z} = 0$) and the surface cooling is imposed in the conservation equations (eq. 4-6).

At the bottom of the model domain in regions away from the central conduit, no-slip ($w = u = 0$) conditions are imposed and the boundary is isothermal. In the conduit, there is no horizontal velocity ($u = 0$) and the vertical velocity is imposed by a sinusoidal function of the form $\sin(2\pi x)$ for asymmetrical flow, and $\frac{1}{2}(\cos(2\pi x) + \cos(4\pi x))$ for symmetrical flow. To avoid disrupting the walls along the mouth of the lava lake, the velocity profile is applied on the central 80% of the 10 m conduit with a thermal boundary along the edges resulting in flow across an 8 m conduit. The incoming magma has a defined gas fraction and temperature which are detailed in the parameter variations described below. For most parameter variations, the velocity profile is constant in time, however it can also vary temporally using a simple sinusoid.
4 Results

4.1 Dimensional Analysis

To find the characteristic physical scales of the problem, we perform dimensional analysis of the governing equations. Non-dimensional numbers that compare different characteristic scales of the problem guide our understanding of how parameter variations control system behavior. The structure of the governing equations suggests several characteristic velocity scales in terms of a characteristic density contrast \( \Delta \rho_0 \) between the crystal mush and vapor, gravitational acceleration \( g_0 \), characteristic length \( l_0 \), characteristic pressure scale which we take as the relative buoyancy difference of the system length scale \( P_0 = \Delta \rho_0 g_0 l_0 \), characteristic magma viscosity \( \eta_0 \), bubble size \( a_0 \), bubble diffusivity \( \kappa_\phi \), thermal diffusivity \( \kappa_T \), degassing timescale \( \tau_\phi \), cooling timescale \( \tau_T \), and crystallization reaction timescale \( \tau_\chi \):

\[
\begin{align*}
    u_{\text{conv}} &= \frac{\Delta \rho_0 g_0 l_0^2}{\eta_0}, \\
    u_{\text{segr}} &= \frac{\Delta \rho_0 g_0 a_0^2}{\eta_0}, \\
    u_{\text{dif},\phi} &= \frac{\kappa_\phi}{l_0}, \\
    u_{\text{dif},T} &= \frac{\kappa_T}{l_0}, \\
    u_{\text{degas}} &= \frac{l_0}{\tau_\phi}, \\
    u_{\text{cool}} &= \frac{l_0}{\tau_T}, \\
    u_{\text{react}} &= \frac{l_0}{\tau_\chi},
\end{align*}
\]

which define the characteristic velocities for magma convection, bubble segregation, bubble diffusion, thermal diffusion, volumetric degassing, volumetric cooling, and volumetric crystallization.

Ratios of characteristic velocities define non-dimensional numbers that compare the rela-
tive importance of physical processes in determining system behavior. We define the non-dimensional number $R_{\text{segr}}$ as the ratio between the characteristic velocity scales of bubble segregation and convection:

$$R_{\text{segr}} = \frac{u_{\text{segr}}}{u_{\text{cnvt}}} = \frac{a_0^2}{l_0^2},$$

(19)

$R_{\text{segr}}$ compares the relative importance of buoyant magma and bubble ascent on the system. We choose $l_0$ to be the conduit radius ($5 \text{ m}$) as a proxy for the radius of a rising magma diapir which is always much larger than the bubble radius ($a_0 = 2.5-10 \text{ cm}$), suggesting that convection should be more efficient than the ascent of individual bubbles or bubble clusters.

We consider another non-dimensional parameter $R_{\text{in}}$ which compares the characteristic velocities of the imposed inflow rate at the bottom boundary and the convective velocity:

$$R_{\text{in}} = \frac{u_{\text{in}}}{u_{\text{cnvt}}} = \frac{u_{\text{in}} l_0}{\Delta \rho g_0 l_0^2},$$

(20)

For high values of $R_{\text{in}}$, the inflow rate exceeds the convective transport, should result in a pile-up of buoyant material near the boundary. When $R_{\text{in}}$ is small, material convects away from the boundary more quickly than it is fed in, which may result in a drip instability.

We can compare the volumetric transport rates $u_{\text{degas}}$, $u_{\text{cool}}$, and $u_{\text{react}}$ with the convective velocity to define three Damköhler numbers, which describe the rates of reaction compared to the rate of transport. We use the concept here to describe volumetric rates, of which only the crystallization rate is a true reaction:

$$D_{a\phi} = \frac{u_{\text{degas}}}{u_{\text{cnvt}}} = \frac{\eta_0 \delta}{\tau_\phi \Delta \rho g_0 l_0^2},$$

(21a)

$$D_{aT} = \frac{u_{\text{cool}}}{u_{\text{cnvt}}} = \frac{\eta_0 \delta}{\tau_T \Delta \rho g_0 l_0^2},$$

(21b)

$$D_{a\chi} = \frac{u_{\text{react}}}{u_{\text{cnvt}}} = \frac{\eta_0 \delta}{\tau_\chi \Delta \rho g_0 l_0^2}.$$

(21c)

Where $l_0$ is chosen to be the conduit radius and $\delta$ is the characteristic depth for surface cooling.
and crystallization. $\text{Da}_\phi$ characterizes the relative rates of surface degassing and convection. For small values of $\text{Da}_\phi$, we would expect degassing to be inefficient and gas may accumulate in the domain, while large values of $\text{Da}_\phi$ would be expected to result in efficient degassing. $\text{Da}_T$ compares the relative rates of surface cooling to convective transport of heat. For small $\text{Da}_T$ we expect surface cooling to dominate behavior, and for large $\text{Da}_T$ we expect convective transport to exceed surface cooling. $\text{Da}_\chi$ compares the relative rates of crystallization and convective transport. Large $\text{Da}_\chi$ due to the imposed small characteristic crystallization time suggests near-equilibrium crystallinity.

Ratios relating the characteristic velocities of convective and diffusive transport of bubbles and heat yield two Peclet numbers:

$$\text{Pe}_\phi = \frac{u_{\text{convt}}}{u_{\text{diff},\phi}} = \frac{\Delta \rho_0 g_0 l_0^3}{\kappa_\phi \eta_0},$$

(22a)

$$\text{Pe}_T = \frac{u_{\text{convt}}}{u_{\text{diff},T}} = \frac{\Delta \rho_0 g_0 l_0^3}{\kappa_T \eta_0}.$$  

(22b)

Both Peclet numbers are very large ($\text{Pe}_\phi > 10^6$ and $\text{Pe}_T > 10^7$) in our simulations, suggesting that advection is expected to be more efficient than diffusion in transporting gas and heat. We take the large values for $\text{Pe}_\phi$ and $\text{Pe}_T$ as justification for neglecting variations in the bubble and thermal diffusivities because changes due to variability with temperatures and phase proportions are likely minor and do not greatly increase the relative efficiency of bubble and thermal diffusion.

We compare the ratio of sensible heat to latent heat given by the Stefan number:

$$\text{St} = \frac{c_p \Delta T_0}{L_\chi},$$

(23)

where $\Delta T_0$ is the difference in temperature between the upwelling and downwelling magma. With the model parameter $\frac{L_\chi}{c_p} = 300$, and $\Delta T \approx 10^5\circ C$, we find $\text{St} \approx 3 \times 10^{-2}$, such that we expect the latent heat may be important with respect to sensible heat. We only consider

the effect of latent heat in one run to reduce the complexity of the reference model and to isolate the effect of latent heat from other parameter variations.

4.2 Reference Parameters

The reference parameters are chosen to match the expected conditions at Mount Erebus. System behavior is described by the model outputs of velocity, temperature, crystallinity, vesicularity, density, viscosity, and stress and strain rate fields that are calculated at each grid point in the model. Additionally, the model calculates a linear gas and heat flux which are scaled to estimate the heat flux from a two-dimensional lake surface for comparison with observational data:

\[ F_{3D} = \frac{A}{W} \sum_{i=1}^{n} F_{2D}(i) \Delta x, \]  

where \( F_{3D} \) is the volumetric flux, \( F_{2D} \) is the calculated linear flux for transport across the boundaries as well as volumetric contributions of degassing and cooling rates, \( \Delta x \) is the grid cell width, \( n \) is the number of grid cells, \( A \) is the area of the lava lake taken to be 1400 m\(^2\) (Calkins et al., 2008), and \( W \) is the model lava lake width taken to be 40 m.
For the reference simulations and all other parameter variations, the flow field is allowed to develop over two hours of model time. The first portion of the time history shows a spin-up period in which the flow field disrupts the initial conditions, generally found to be about 30-40 min. After the lake is thoroughly mixed, the system reaches a dynamic equilibrium with a characteristic convective regime. Simulation output show coupling of the crystallinity, temperature, vesicularity, and density fields shown in figure 4. Flow is defined by a dripping instability, in which roughly equant diapirs rise from the base of the lake or from upwelling magma along the lake wall. Figure 5 shows the time evolution of the dripping plume. We calculate the value of the non-dimensional numbers using known values for the gravitational acceleration, conduit radius, bubble radius, bubble and thermal diffusivities, characteristic time scales for surface degassing and cooling, characteristic length scale for surface degassing and cooling, and inflow velocity which are proscribed in the system setup. We calculate the value for $\Delta \rho$ using the difference between the average density of the lake interior and the minimum density found in the plume. The representative viscosity is calculated using a geometric average of the lake interior. We find a calculated value of $R_{in} = \frac{u_{in}}{u_{cv}} = \frac{0.2 \text{ m/s}}{3.4 \text{ m/s}} = 0.06$, predicting a convective velocity that exceeds the inflow velocity from the conduit, consistent with the dripping behavior we observe.

The net gas flux into and out of the lake approach the same magnitude following the initial spin-up period, $\approx 0.7 \text{ kg/s}$, as shown in figure 6, suggesting a near-equilibrium state for vesicularity in the lake. Temporal variations in the surface degassing result from the arrival of higher vesicularity magma near the surface of the lake. The net inflow of gas is determined by the flow field at the base of the conduit. The boundary has a constant inflow of material with $\phi = 0.2$, but the sinusoidal velocity field removes downwelling material with variable vapor fraction, resulting in temporal variations in the net gas flux at the base of the lake.

The simulation time for the reference case, and for other parameter variations with high
Figure 4: Density, temperature, vesicularity, and crystallinity fields of the reference simulations after 80 min. The characteristic shape of the dripping plume shows magma moving away from the conduit along the high-shear region at the wall of the lake before plume heads rise and detach. The variable fields are coupled and show the same regions of fresh magma with variations at the surface where degassing and cooling are most rapid.
Figure 5: The density fields from the reference simulations from 79-85 min show the time evolution of the dripping plume, including plume heads rising from the left wall of the lake at 79 min and rising directly from the conduit at 83 min.

surface heat flux, is insufficient to reach thermal equilibrium. The reference case shows more heat loss from the surface, \( \approx 150 \text{ MW} \), than flux in from the conduit, \( \approx 70 \text{ MW} \) (fig. 6). The characteristic timescale of surface cooling is large (\( \tau_T = 8 \text{ hr} \)), which smooths the temporal variations in surface heat flux that are small relative to the magnitude of flux. Similarly to the influx of gas, the heat flux in at the bottom boundary is variable in time as a result of evolving temperature of downwellling magma.

Convective behavior is recorded in the time series of surface velocity, shown in figure 7, which has maximum values between 0.05 and 0.25 m/s and average values between 0.02 and 0.05 m/s. The large positive excursions in maximum and average velocity correlate with magma drips reaching the lake surface.
Figure 6: Time history of gas and heat flux from the reference simulations. The time spans one hour from 40 to 100 minutes of model time. Net flux in the bottom of the lake from the conduit is shown in blue and flux from the top of the lake is shown in red.

Figure 7: Time history of surface velocities from the reference simulations. The time spans one hour from 40 to 100 minutes of model time. The mean velocity is shown in blue and the maximum velocity is shown in red.
4.3 Parameter Variations

We perform a series of simulations to investigate the possible behaviors of the system in response to small changes in parameters. We change a single parameter in each simulation so that it may be readily compared to the reference simulation. A summary of all simulations is presented in table 3. We explore the effects of varying the inflow conditions, surface flux rates, rheology, bubble segregation velocity, latent heat and inflow asymmetry to understand possible controls on the natural system which has uncertain parameters.

First, we test the sensitivity of the model to changes in the gas flux from the conduit. We vary the vesicularity of the upwelling magma from $\phi_0 = 0.1$ to 0.3 about the reference value of 0.2. Model outputs are shown in figure 8, selected at times when the simulation has reached steady-state flow. When $\phi_0 = 0.1$, the lava lake surface completely crystallizes within the two hour model time. The decreased vapor fraction of the upwelling magma decreases the density contrast with the partially degassed magma in the lake. A decrease in $\Delta \rho$ by $\approx 50\%$ correlates with a proportional decrease in the expected convective velocity $u_{cnvt}$ and an increase in $\text{Da}_T = \frac{u_{cnvt}}{u_{cnvt}}$. For $\phi_0 = 0.1$, we calculate $\text{Da}_T = 5.8 \times 10^{-5}$, which exceeds the value of $\text{Da}_T = 1.0 \times 10^{-5}$ for the reference case, suggesting a relative increase
in the importance of surface cooling. Greater surface cooling effects with respect to the convective transport of heat here leads to a stagnant lid regime.

When the inflow vesicularity is increased to $\phi_0 = 0.3$, we expect a higher density contrast $\Delta\rho$ between the upwelling magma and magma within the lake. This increase in $\Delta\rho$ drives a higher convective velocity, which decreases the $R_{in}$ from 0.06 in the reference case to 0.04 for large $\phi_0$. The increased convective vigor is sufficient to keep the surface open with $Da_T = 6.7 \times 10^{-6}$. The simulations show a dripping regime similar to that described for the reference simulations (fig. 8), which is consistent with a convective velocity that greatly exceeds the velocity of upwelling magma from the conduit as predicted by the low value for $R_{in}$.

The absolute amount of vapor coming from the conduit can also be varied by changing the upwelling magma velocity. We test the effect of decreasing the upwelling magma velocity from 0.2 m/s to 0.1 m/s while holding the inflow vesicularity constant at $\phi = 0.2$, resulting in $R_{in} = 0.01$ and $Da_T = 1.5 \times 10^{-5}$. Under these conditions, the lava lake enters a stagnant lid regime because the decrease in upwelling magma velocity decreases the heat flux into the lake and decreases the buoyancy of the rising plume such that there is not enough heat to keep the lake open.

We increase the velocity of the upwelling magma from the conduit to increase the amount of gas and heat flux into the lake. By increasing $u_{in}$, we increase the relative importance of the inflow velocity to convective velocity: $R_{in}$ increases to 0.10 when $u_{in} = 0.3$ m/s. The greater thermal flux is sufficient to maintain an open lake with $Da_T = 1.1 \times 10^{-5}$. As shown in figure 8, the simulation shows a free plume in the middle of the lava lake, rather than clinging to the lake wall, with pulses of gas-rich magma that move along the plume. In this regime, we do not observe magma dripping directly from the conduit, but occasionally pulses will rise from the edge of the arcuate plume to ascend through the center of the lake.

We consider the importance of the surface degassing and cooling through variations in
Influx Parameter Variations

Figure 8: Vesicularity fields for simulations with upwelling magma vapor fraction $\phi_0 = 0.1$ and 0.3 and upwelling magma velocity $u_{in} = 0.1$ and 0.3 m/s. Frames are chosen at times that best show the characteristic flow patterns. These simulations change the absolute rates of gas flux into the lake from the conduit.
their characteristic time and length scales. The characteristic timescale for degassing $\tau_\phi$ is tested between 30 and 120 s. The characteristic surface degassing accounts for a portion of the gas loss from the lake surface in addition to the advective transport provided by bubble segregation. A decrease in the degassing time is shown in figure 9 which shows a dripping instability. The higher rate of surface degassing increases the relative importance of surface degassing with respect to convective transport, an increase in Da$_\phi$ from $5 \times 10^{-3}$ to $8 \times 10^{-3}$. Efficient degassing results in a low vapor fraction in the lake interior, which increases the density contrast between the plume and the lava lake. The high density contrast increases the convective velocity and reduces $R_{\text{in}}$ to 0.05, favoring formation of a dripping instability.

Conversely, if the characteristic timescale of degassing is increased to 120 s, degassing becomes inefficient as Da$_\phi$ decreases to $4 \times 10^{-3}$. A pulsing magma plume occurs in the lake (fig. 9). The high vapor fraction in the lake decreases the density contrast between the lake and the upwelling magma from the plume, resulting in a lower value for $u_{\text{conv}}$ of 1.2 m/s and a higher value of $R_{\text{in}} = 0.09$.

We select the timescales of surface cooling to capture open convection and stagnant lid regimes as well as to match surface observations of heat flux. The lower limits on the characteristic timescale for degassing is $\tau_T = 6$ hr. The short timescale forces rapid cooling of the surface that results in a stagnant lid (fig. 9). This result is consistent with a large rate of cooling and a large Da$_T$ of $2.7 \times 10^{-5}$. The presence of the stagnant lid causes an increase in the overall temperature of the lake interior by insulating the magma from the surface cooling effects, which is a well-known property of stagnant-lid convective regimes.

Larger values for the characteristic timescale of cooling maintain an openly convecting lava lake. We test values of $\tau_T = 12$ and 16 hr to keep the lake open which have Da$_T$ = $6.2 \times 10^{-5}$ and $4.2 \times 10^{-5}$ respectively. For the simulations with the highest value of $\tau_T$ we find a time-averaged heat flux of $\approx 70$ MW, which is within the range of estimates of heat flux from Ray lake based upon field observations.
We also vary the characteristic depth $\delta$ over which degassing and cooling occur. When this length scale is small ($\delta = 0.5$ m), we observe a stagnant lid regime. The small thickness over which degassing and cooling occur result in low values for $Da_\phi$ and $Da_T$ of $6 \times 10^{-3}$ and $1.3 \times 10^{-5}$, respectively. The accumulation of gas and heat in the lake decreases the density contrast between the rising plume and lake interior leading to a high value of $R_{in}$ of 0.01. The buoyancy contrast is low, such that the surface is not sufficiently disturbed before crystallizing completely.

Increasing the characteristic length scale for degassing and cooling to $\delta = 2$ results in a stagnant lid. Degassing and cooling are more important compared to convective transport in the reference simulations, with $Da_\phi = 0.016$ and $Da_T = 2.3 \times 10^{-5}$. The thermal insulation provided by the stagnant lid increases the internal temperature of the lake which reduces the viscosity such that $R_{in} = 0.01$.

We explore the sensitivity of the model to changes in the temperature of upwelling magma. When the temperature $T_0 = 970$ °C, the lava lake crystallizes over to form a stagnant lid as depicted in figure 10. The low temperature correlates with a higher crystallinity, that increases the viscosity of the magma. The viscosity enters the $Da_T$ term through the convective velocity, and results in $Da_T = 3.3 \times 10^{-5}$ suggesting that surface cooling is more important with respect to convective transport than in the reference simulations.

For high temperatures of upwelling magma, a dripping instability characterizes the flow field (fig. 10). Low crystallinity in the magma results in a lower average viscosity, which inversely relates to the convective velocity. For $T_0 = 980$°C, the low average viscosity results in $R_{in} = 0.04$, and as a result of the high convective speed, the lava lake shows dripping behavior.

To evaluate how $R_{in}$ controls the regime, we consider the idealized scenario in which the lake interior has a constant viscosity everywhere. By reducing the rheological complexity such that the convective speed is set exactly by one viscosity. When the constant viscosity
Figure 9: Vesicularity fields for simulations showing variations in the characteristic degassing time $\tau_{\phi} = 30$ and 120 s. Temperature fields for simulations showing variations in the characteristic cooling time $\tau_T = 6$, 12, and 16 hr. Density fields for simulations showing variations in the characteristic depth of degassing and cooling $\delta = 0.5$ and 2 m.
is $10^4$ Pa.s, a dripping instability emerges in which the magma rises along the side of the lake walls, as observed in the reference simulations (fig. 10). We would predict that the convective velocity is important in comparison to the upwelling magma velocity because $R_{in} = 0.02$, which is much lower than $R_{in}$ for the reference simulations.

When the viscosity is constant and very high ($\eta = 10^6$), a stable plume is observed in the center of the lake (fig. 10). At downwelling regions, high-crystallinity magma is reincorporated into the lake which is not observed in simulations with variable viscosity because the high crystallinity regions have high viscosity and are difficult to disrupt. $Da_T = 3.3 \times 10^{-4}$, which, according to results from other simulations, we would expect to force a stagnant lid regime. However, the constant viscosity condition prevents the formation of a crust. Instead a single wide plume forms where $R_{in} = 1.9$, for which we would expect the inflowing magma velocity to be somewhat higher than the convective velocity.

The rheological power-law for the reference simulations is set to $n = 1.5$. We investigate $n = 1$ and 3, where $n = 1$ is a Newtonian fluid. The simulations for $n = 1$ show a dripping instability. The viscosity for the majority of the domain is similar to that of the reference case, resulting in $R_{in} = 0.07$, slightly higher than the value for the reference case. However, we expect that the absence of shear-thinning would result in a higher effective viscosity in regions of high shear with respect to the reference case, including along the sides of the domain and at the interface between the buoyant plume and surrounding magma.

For $n = 3$, the simulation falls into the pulsing plume regime (fig. 10). The average viscosity in the lava lake remains largely unchanged, but the shear-thinning behavior of the magma may result in a localization of strain that leads to a higher effective viscosity in regions of the domain away from the plume and lake walls. The value for $R_{in}$ is 0.06, approximately the same as the reference solution. But local effective velocity changes likely result in the change in convective style.

The effect of introducing Binghamian yielding results in a pulsing plume in the center
Figure 10: Density field from simulations showing variations in upwelling magma temperature $T_0 = 970$ and 980 $\degree$C which control magma viscosity, constant viscosity simulations with $\eta = 10^4$, $10^5$ and $10^6$ Pa.s, simulations with rheology power-law exponents of $n = 1$ and 3, and simulation with Binghamian plastic yielding.

of the lava lake (fig. 10). The finite yield strength is most relevant in disrupting the high-crystallinity boundary layer that can form at the surface which allows more crystalline magma to be re-incorporated into the center of the lava lake. The resulting slight increase in the crystallinity of the lake increases the viscosity such that $R_{in} = 0.07$.

We assume that the bubble radius is constant across the domain and between bubbles, but we vary the size constant in a further two parameter sets to investigate the importance of bubble segregation. When the bubble size is small, $a_0 = 2.5$ cm, bubble segregation compares
Figure 11: Vesiculality field from simulations with variations in bubble size $a_0 = 2.5$ and 10 cm. Volume fraction of vapor is held constant.

to convective velocity as $R_{segr} = \frac{a_0^2}{\rho_0} = 2.5 \times 10^{-5}$. We expect that bubble segregation is less important for smaller bubbles, which results in inefficient degassing. When $a_0 = 2.5$ cm, the flow pattern is characterized by a pulsing plume, shown in figure 11. The residual vapor slightly decreases the density contrast between the rising plume and the lake interior, which results in $R_{in} = 0.05$.

The flow regime for large bubbles or bubble clusters of radius $a_0 = 10$ cm is characterized by a free plume in the center of the lake with occasional pulses of magma through the plume. For a 10 cm radius bubble, $R_{segr} = 4 \times 10^{-4}$. An increase in the bubble segregation velocity allows the bubbles to advect ahead of and vertically out of the plume where the plume flows laterally. The loss of bubbles from the plume into the lake increases the density of the plume by approximately 30 kg/m$^3$ which decreases the density contrast with the surrounding magma. Changing $\Delta \rho$ decreases the convective speed such that $R_{in} = 0.05$.

Latent heat has a buffering effect on cooling lava lakes and helps to maintain a high temperature near the surface. The flow regime when latent heat is considered is that of the pulsing plume as shown in figure 12. There is a slight decrease in the average viscosity of the lake which decreases the value of $R_{in}$ to 0.06.
Figure 12: Temperature field from simulations with latent heat. Crystallization occurs rapidly such that the crystals and melt are in near-equilibrium.

The majority of our simulations consider time-invariant bidirectional flow from the conduit in a core-annular regime in which the upwelling magma rises from the center of the conduit with a thinner region closer to the conduit walls that allows for downwelling. But, we test a single variation in which magma flows up on one side of the conduit and down on the other, shown in figure 13. In our model, when the buoyant material rises up from the conduit, it moves towards the middle of the lake where it feels the effect of the imposed velocity boundary condition such that a portion of the upwelling magma is advected back down the conduit out of the domain. This occurs to a small extent in the core-annular flow, but causes noticeable dripping for the side-by-side conduit flow. Due to the immediate downwelling of a large potion of the fresh magma, the effective inflow rate of bubbly magma is less than $u_{in}$, and the pulsing behavior of the source flow dominates the lake behavior and outputs.
5 Discussion

5.1 Convective regimes

Over the range of parameters explored in our simulations, we find several thermo-mechanical convective regimes that we correlate with relevant non-dimensional numbers. We find that when $\text{Da}_T > 1.5 \times 10^{-5}$ surface cooling becomes an important control on system behavior, leading to a stagnant lid regime. During open convection, we identify two different flow regimes. The first is defined by a dripping instability in which the buoyant material rises along a portion of the lake wall, or magma rises as buoyant plume diapirs that disconnect from the flow along the conduit wall or directly from the conduit. The second regime is characterized by an arcuate plume in the center of the lake through which periodic pulses of larger volumes of gas rich magma travel. The plume migrates with time although it usually impinges on the surface near the center of the lava lake. When the curvature becomes large, magma may pinch off and rise independently. The openly convecting regimes are distinguished by the non-dimensional numbers $\text{Da}_T$ and $R_{in}$ such that the dripping regime

![Side-by-side Flow Parameter Variation](image)

Figure 13: Density field output from simulations with asymmetrical bi-directional flow from the conduit. Magma is upwelling on the left and is being advected out of the domain on the right side of the conduit.
Figure 14: Modelled behavioral regimes shown in the non-dimensional space of $R_{in}$ and $Da_T$ which compare the relative rates of inflow velocity and convective velocity that arises from natural scales of the problem, and the volumetric surface cooling rate and convective velocity. Non-dimensional numbers are calculated for simulations and color shows the regime. Open circles represent simulations with constant viscosity that fall within the stagnant lid regime space, but cannot form a solid crust.
occurs for generally higher values of \( \text{Da}_T \) below the transition to stagnant lid, and \( R_{\text{in}} < 0.06 \), illustrated in figure 14. While other model numbers control slight variations by modifying the effective density contrast and average lake rheology, it is these two numbers that mainly control the convective regime.

### 5.2 Comparison to field data

Simulations show several possible regimes over a range of parameters that are consistent with measurements or estimates from field data collected at Mount Erebus. We compare the model outputs to observational data of the lava lake. Ray lake is persistently open, so the stagnant lid regime does not represent what we expect from the natural system. Therefore, we focus our discussion on the two openly-convecting regimes of dripping diapirs and pulsing plumes. We consider the gas and heat flux, surface velocity, and the locations of maximum strain at the surface.

Observational studies from Erebus show periodic fluctuations in gas flux that correspond with changes in gas composition every 5-18 min (Oppenheimer et al., 2008, 2009; Peters, Oppenheimer, Killingsworth et al., 2014; Sweeney et al., 2008). Total gas flux estimates are on average 27.3 kg/s which we take as the upper bound on gas flux from the summit because it incorporates the combined effects of the lake along with diffuse degassing from the surrounding crater walls. Model outputs scaled to a lake size of 1400 m\(^2\) (Calkins et al., 2008) have an average gas flux of 0.7 kg/s for the reference case and up to 1.2 kg/s when the inflow vesicularity \( \phi = 0.3 \) and show periodicities of 5-20 min. Gas fluxes from the model outputs account for \( \sim 5\% \) of the observational value, but we show volumetric flux rates consistent with estimates from field data (Oppenheimer & Kyle, 2008).

Thermal flux estimates range from 30-100 MW with fluctuations in output phase locked with gas flux (Calkins et al., 2008; Oppenheimer et al., 2009; Wright & Pilger, 2008). Simulation outputs show a thermal flux of 140 MW for the reference case and as low as 70
MW when the characteristic cooling time is 16 hr. Simulations in the stagnant lid regime show higher heat fluxes, up to 200 MW when the characteristic cooling time is 6 hr, but are unable to reach thermal equilibrium in the simulation time. Periodic fluctuations in the heat flux in model outputs are very small with respect to the overall heat flux because we do not resolve a quenched skin at the surface that can cause large differences in surface temperature when disturbed. Thermal flux measurements are similar for both openly convecting regimes.

Measurements of surface velocity show periodicity phase locked to the gas and heat flux with maximum speeds between 0.1-0.3 m/s with occasional peaks above 0.8 m/s, and mean speeds between 0.1 and 0.3 m/s (Calkins et al., 2008; E. Lev, personal communication; Oppenheimer et al., 2009; Peters, Oppenheimer, Killingsworth et al., 2014). Two one-hour time series of maximum and mean velocity are shown in figure 15 (E. Lev, personal communication). Our simulations show surface velocity fluctuations every 5-20 min with mean speeds below 0.05 m/s and maximum speeds between 0.05 and 0.25 m/s consistent with field observations. Surface velocity time series are similar for the dripping and pulsing plume regimes, but the dripping regimes shows larger high-frequency oscillations shown in figure 16. Neither regime captures the complexity of rupturing a chilled skin on the surface of the lava lake. A Fourier analysis of the maximum surface velocity in figure 17 shows high frequency oscillations and amplitude peaks at ~ 8, 12, and 20 min periodicities.

Observations of the locations of maximum divergence or burst locations at the surface of the lake show temporal variability and cover nearly the entire surface of the lake as shown in figure 18. Model outputs include the location of maximum strain that corresponds to the burst location. The dripping regime simulations show significant variations in the location of maximum strain that cover the entire surface of the lake as displayed in figure 19. Outputs from simulations with a pulsing regime show focusing of the maximum strain location to the center of the lake, with slow wandering as the plume evolves and sharp excursions when small amounts of magma pinch off from the plume to rise buoyantly.
Figure 15: Surface velocity observations from Dec 6 and Dec 26, 2012. Each time series lasts one hour (E. Lev, personal communication). Mean velocity is shown in blue, maximum velocity is shown in red. Peak velocities reach 2-3 m/s.
Figure 16: Surface velocity time series for the dripping, pulsing, and steady plume regimes taken from simulations with $\phi_0 = 0.3$, $v_{in} = 0.3$ m/s, and constant viscosity $\eta = 10^6$ Pa.s respectively. Mean velocities are shown in blue, maximum velocities are shown in red.

Figure 17: Single-sided amplitude of Fourier analysis of the reference simulations maximum surface velocity time series showing amplitude peaks at 8, 12, and 20 minutes.
Figure 18: Time evolution of the location of maximum divergence of Ray lake. The variability in locations shows a moving source of upwelling magma (E. Lev, personal communication).

6 Summary & Conclusions

Both of the openly convecting regimes we describe are consistent with heat and gas flux and surface velocity measurements from Mount Erebus. Simulations in which magma drips from the conduit and plumes along the walls of the lava lake best replicate variability in the locations of maximum divergence, suggesting that detached magma diapirs may be important in replicating the variability in burst location. Our simulations replicate field measurements with a constant inflow velocity of hot, gas-rich magma, suggests that lava lake dynamics may be able to drive episodic activity at the surface. These results complicate the interpretation of surface measurements from lava lakes in placing constraints on conduit dynamics.

Future directions from this work include investigating through modelling or laboratory experiments the impact of lava lake dynamics on time-varying inflow conditions from the conduit which would offer better insights into how surface observations can be used to make predictions about processes occurring at depth. This study focuses on the steady-state
Figure 19: Time series of x-location of maximum strain for the dripping, pulsing, and steady plume regimes taken from simulations with $\phi_0 = 0.3$, $v_{in} = 0.3$ m/s, and constant viscosity $\eta = 10^6$ Pa.s respectively.
behavior of the lake, but another avenue of investigation would be to investigate causes of the Strombolian eruptions that disrupt steady-state behavior, which may include gas slugs. Additionally, this model could be expanded to include the upper portion of the conduit along with a model for the evolution of volatile content in the magma and vapor phases. The approach detailed here could be expanded to investigate the processes governing the behavior of other lava lakes.
7 References

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