Effects of supersonic relative velocity between baryons and dark matter in the early Universe

Greg Peairs
Advisor: Risa Wechsler

Evolution of the Universe following cosmic recombination is typically modeled only in linear approximation. Tseliakhovich and Hirata (2010) showed that supersonic bulk relative velocity between cold dark matter and baryonic matter, coherent over scales of tens to hundreds of comoving Mpc, produces important nonlinear effects during this era. We present a new pseudo-spectral simulation code, which is uniquely able to model the nonlinear evolution of the matter density fields beginning at recombination. Using this code, we investigate the relative-velocity effect and quantify the suppression of structure formation on small scales. We demonstrate that the effect is present even by redshift $z \sim 400$. By $z \sim 100$, baryonic power has been suppressed by around 40% at scales of $k = 80$ h/Mpc, as compared with the standard linear theory.

1 Introduction

Structure formation in the Universe involves the gravitational interactions between the two types of matter in the universe: ordinary or baryonic matter, and dark matter. The dark matter can be further divided into relativistic and non-relativistic or “hot” and “cold” components, although nearly all of it is thought to be cold. A small fraction of the energy density is contributed by the “hot” neutrinos of finite mass. It is cold dark matter (CDM) which provides large gravitational potentials that bind baryons into galaxies and other structures we observe in the Universe today. The better we understand the process of structure formation, the more we can learn about the early Universe that eventually gave rise to the observed galaxies and galaxy clusters.

Because the density and associated velocity fluctuations are very small initially, the evolution of the matter density fields after cosmic recombination is typically treated with linear perturbation theory, assuming that higher-order effects are negligible. However, Tseliakhovich and Hirata (2010) recently demonstrated the importance of large-scale motion of the CDM fluid with respect to baryons or small-scale structure at mass scales of about $10^8 M_\odot$. This relative velocity, around five times the speed of sound in the baryon fluid at recombination, significantly suppresses the earliest formation of structure at small scales. Intuitively, one can imagine the gravitational potential wells of the dark matter moving past baryons before they can “fall in” and form bound structures, overcoming internal pressure. (Current models say relativistic dark matter is not very common because it would have
caused a related, stronger effect not observed.) The linear approximation does not capture this nonlinear effect. Further, current simulations of nonlinear cosmic dynamics rely on N-body methods, which fail to sufficiently accurately model the coupled dynamics of the two fluids at such early times [9, 11].

Our new code, developed with Jeff Oishi and Oliver Hahn at SLAC and Keaton Burns at UC Berkeley, models the two matter fluids together using highly parallel pseudo-spectral methods. These are accurate in the mildly non-linear regime, allowing us to simulate the evolution of the Universe from recombination up through the early stages of structure formation. This is immediately useful in quantifying the relative velocity effect described above. Additionally, it opens up a number of promising approaches to problems in numerical cosmology, including the realistic generation of initial conditions for N-body simulations in the highly nonlinear regime, as well as the effect of primordial non-Gaussianity on astrophysical observables.

I was responsible for the bulk of the cosmology-specific code, along with parts of the general time-evolution, parallelization, and analysis frameworks. I also wrote and carried out the tests and full-resolution runs presented below.

1.1 Primordial fluctuations and the linear power spectrum

Following the Big Bang, the Universe was very hot, very dense, and very smooth. A period of rapid expansion known as inflation created small fluctuations in matter density, seeding the process of structure formation. For about 300,000 years, the Universe remained an ionized plasma. Photons could only travel short distances before scattering off of one of numerous free protons and electrons, and the tightly-coupled baryons and photons evolved as a single fluid. As the Universe expanded, it cooled, eventually becoming a neutral gas. The epoch during which this happened is known as recombination, referring to protons and electrons combining into neutral hydrogen atoms.

The period following recombination, up until around the formation of the first stars, is our main epoch of interest. This is when nonlinear evolution of fluctuations in the dark matter and baryon fluids becomes relevant, as discussed below. In general, these fluctuations are described in terms of the fractional overdensities of each fluid. For baryons, we write

\[ \delta_b(x) \equiv \frac{\rho_b(x) - \bar{\rho}_b}{\bar{\rho}_b}, \tag{1} \]

where \( \rho_b \) is the total baryon density field and \( \bar{\rho}_b \) its mean. We similarly have \( \delta_c \) for cold dark matter.

The particular pattern of fluctuations is of less interest than its statistical properties. These are mostly summarized by the two-point correlation function, which describes the average correlation between points at a given separation in space. For a Gaussian random field, this is a complete description; inflation under the simplest models produces such a primordial perturbation. Linear evolution preserves this property, while non-linear coupling adds non-trivial higher moments and phase correlations.

As will become evident, it is convenient to think about the Fourier transform of the correlation function, called the power spectrum. This is defined as the average over wave-
vectors \( \mathbf{k} \) on a sphere of radius \( k \)

\[
P(k) = \langle \hat{\delta}(\mathbf{k})\hat{\delta}^*(\mathbf{k}) \rangle, \tag{2}
\]

where \( \hat{\delta}(\mathbf{k}) \) is the Fourier transform of the overdensity field and \( ^* \) denotes complex conjugation. The peaks of \( P(k) \) thus represent ‘typical’ clustering scales of matter.

Current models of inflation generally produce initial fluctuations with a power spectrum of the simple form \( P(k) \propto k^{n_s} \), with the spectral index \( n_s \approx 1 \). Linear evolution (up to around recombination) can be described in terms of the growth of that initial power spectrum:

\[
P_{\text{linear}}(k) \propto k^{n_s}T^2(k), \tag{3}
\]

where \( T(k) \) is the “transfer function” that describes the growth. Once matter overtakes radiation as the dominant component of the Universe around \( z = 3200 \), perturbations at all scales will grow at the same rate. Even following recombination, the power spectrum should grow nearly according to linear evolution, especially at large scales. We are interested in an important nonlinear correction which produces a suppression of power at small scales, or high wavenumber \( k \). This becomes a suppression of structure formation at those scales; thus its effects can be seen not only in the 21-cm signal from the intergalactic medium at redshifts prior to star formation, but also in the distribution of galaxies and smaller objects.

1.2 Recombination

Cosmic recombination caused a decoupling of photons from matter, as photons could then travel great distances without scattering off of a charged particle. In fact, photons which have been traveling freely since decoupling can be observed today at microwave frequencies, coming from all points in the sky. This is the Cosmic Microwave Background (CMB), an important source of information about the Universe during this era. The spectrum of the CMB is that of a blackbody at a nearly uniform temperature, directly related to the cosmic temperature at photon decoupling according to photon redshift from the expansion of the Universe.

Of more immediate interest to structure formation are the small temperature variations in the CMB from different regions of the sky, fluctuations of about a part in \( 10^5 \) of the overall temperature. Fluctuations spanning more than around 1° in the sky provide a map of the dark matter density at the time of last scattering. A photon scattered from a region of high density must have climbed out of a large gravitational potential, losing some of its energy; we observe a further redshifted spectrum from that region of the sky. Photons from density minima will have correspondingly higher energies.

There are also temperature fluctuations at some characteristic smaller angular scales, but a different mechanism is mainly responsible for these [3]. These can be described statistically by the two-point correlation function of the CMB measurements, or again equivalently by the power spectrum. When photons and baryons were still tightly coupled, they effectively formed a single fluid. This fluid undergoes gravitational collapse in the potential wells created by dark matter, and rebound outward from internal pressure. These competing forces produce oscillations in the fluid, or standing sound waves that collapse and expand at different rates depending on the scale of the oscillating overdensity. Thus we see a series of
peaks in the power spectrum, corresponding to additional temperature fluctuations at scales which are at the fully collapsed or expanded phase of an oscillation at that time. The largest of these fluctuations are around angular size $1^\circ$ in the sky, above which scale overdensities did not have enough time to fully collapse by photon decoupling. This corresponds to a spatial scale of around 100 Mpc/h. At sufficiently small scales, photon diffusion damped the oscillations, since the photons and baryons were not perfectly coupled.

The clustering of baryonic matter due to this effect is known as baryon acoustic oscillations (BAO). This has been detected with high significance in the distribution of galaxies, most recently by the Baryon Oscillation Spectroscopic Survey (BOSS, [1]).

1.3 The relative velocity effect

Objects below a certain scale, known as the Jeans length, will not collapse under their own gravity. This is roughly the scale at which the time for gravitational collapse is the same as the time for a sound wave to travel from the boundary to the center of the overdensity. In this case, somewhat simplistically, gravity creates a pressure front which rebounds before collapse can occur, causing an acoustic standing wave. This happens at all relevant scales before decoupling: the sound speed in the photon-baryon fluid is around $c/\sqrt{3}$, giving a very large Jeans length.

After recombination and subsequent decoupling, the sound speed of the baryon fluid drops dramatically to about 6 km/s. Thus the Jeans length also drops dramatically, to about $\lambda_J \approx 30$ kpc. Baryons overdensities larger than that begin to collapse into gravitationally bound structures. The smallest structures tend to form earliest.

Tseliakhovich and Hirata have noted another related effect that comes in to play at this time. The acoustic oscillations before photon decoupling also produce a relative velocity between baryons and the background dark matter. While the effect is isotropic on large scales, the root mean square relative velocity between dark matter and baryons at the time of recombination is around 30 km/s, five times the speed of sound in the baryon fluid.

The relative velocity mostly comes from the BAO regime, indicating that the relative flow is coherent over scales smaller than that (a few Mpc and below). Bulk flows will thus pull baryons out of the gravitational potential wells of dark matter haloes, suppressing the growth of structure at scales below about five times the Jeans length.

Tseliakhovich and Hirata use the fact that the coherence length is large compared to the size of the typical first structures to perturbatively approximate the magnitude of this effect. They found the largest suppression of power near the Jeans scale $k_J = aH/c_s \sim 200$ Mpc$^{-1}$, with a difference of $\sim 15\%$ compared to the standard linear theory.

1.4 Related work

A number of numerical studies have been conducted, mostly confirming the presence of the effect, e.g. [4,7–9]. Many of these suffer from initialization procedures which are do not include self-consistent evolution up to the start time, and further start at late times, missing much of the effect. Numerical problems with N-body codes also make these results less reliable. O’Leary and McQuinn (2012) performed the most recent investigation of the streaming
velocity effect, which is also the most convincing. Using a more realistic procedure for initializing nonlinear simulations, they found that the relative velocities significantly reduces the accumulation of baryons in dark matter halos and delays the formation of the first stars. Previous studies had used existing initial-conditions codes and simply increased the baryon velocity at the initialization time, which is not necessarily physically consistent with the assumptions those codes use. O’Leary and McQuinn demonstrated that such initialization missed a significant amount of the suppression of growth, and that it missed more for later initialization times.

The better procedure uses linear evolution which accounts for the relative velocities along with a self-consistent realization. A simulation initialized in this way at $z = 200$ was consistent with one initialized at $z = 400$, indicating convergence of the linear evolution and initialization procedure with respect to initialization time. They then compare nonlinear evolution according to two different N-body codes, GADGET and Enzo. These are generally in agreement, but they suffer from some effects that produce spurious nonlinear growth. With GADGET (which uses smooth particle hydrodynamics), coupling between nearby dark matter and baryon particles is a significant problem. Adaptive mesh refinement in Enzo also introduces extra nonlinear growth. We attempt to improve on these with a more faithful simulation method which follows nonlinear growth self-consistently all the way from recombination.

### 2 Numerical methods

#### 2.1 The equations

We model both CDM and baryons with fully nonlinear fluid equations. These are expressed in terms of the fractional overdensity $\delta_i$ of each fluid component $i$ (1). The evolution of the CDM is governed by the pressureless fluid equations (in coordinates comoving with an expanding Universe with scale factor $a(t)$):

\begin{equation}
\partial_t \delta_c = -a^{-1}(1 + \delta_c) \nabla \cdot \mathbf{v}_c - a^{-1} \mathbf{v}_c \cdot \nabla \delta_c
\end{equation}

\begin{equation}
\partial_t \mathbf{v}_c = -a^{-1} \nabla \phi - \frac{\dot{a}}{a} \mathbf{v}_c - a^{-1} (\mathbf{v}_c \cdot \nabla) \mathbf{v}_c
\end{equation}

The baryons evolve according the the same equations with pressure:

\begin{equation}
\partial_t \delta_b = -a^{-1}(1 + \delta_b) \nabla \cdot \mathbf{v}_b - a^{-1} \mathbf{v}_b \cdot \nabla \delta_b
\end{equation}

\begin{equation}
\partial_t \mathbf{v}_b = -a^{-1} \nabla \phi - \frac{\dot{a}}{a} \mathbf{v}_b - a^{-1} (\mathbf{v}_b \cdot \nabla) \mathbf{v}_b - a^{-1} c_s^2 \nabla \delta_b,
\end{equation}

where the sound speed $c_s(t)$ is assumed to be spatially uniform, and calculated at each time step as in [6].

The gravitational potential $\phi$ is given by Poisson’s equation:

\begin{equation}
a^{-2} \Delta \phi = 4\pi G \overline{\rho}_m \delta_m.
\end{equation}
The total matter perturbation $\delta_m$ is the sum of the two component perturbations. In a matter-dominated flat universe, we can rewrite this in terms of the Hubble parameter $H$:

$$a^{-2} \Delta \phi = \frac{3}{2} H^2 \left( \frac{\Omega_c}{\Omega_m} \delta_c + \frac{\Omega_b}{\Omega_m} \delta_b \right)$$ (9)

The scale factor $a(t)$, which describes the expansion of the Universe in the Friedmann-Robertson-Walker metric, evolves according to the Friedmann equation:

$$\dot{a} = H_0 \left( \frac{\Omega_{r,0}}{a^2} + \Omega_{m,0}a + \Omega_{\Lambda,0}a^2 + (1 - \Omega_0) \right),$$ (10)

where $\Omega_{r,0}, \Omega_{m,0}, \Omega_{\Lambda,0}, \Omega_0$ are the present-day radiation, matter, dark energy, and total density parameters, respectively. In general, we use redshift $z = \frac{1}{a} - 1$ to identify times in the history of the Universe. Present day is $z = 1$, and our simulations start with $z = 1000$, shortly after recombination.

We also account for general relativistic effects and Thomson drag at linear order as in [6]. General relativity is important only at the largest scales of our simulations, and Thomson drag is important only at the highest redshifts when there are still free electrons. Both of these are well within the linear regime at all times.

2.2 The spectral method

The code uses a pseudo-spectral method to evolve these equations. The overdensity and velocity fields are generally represented as grids of coefficients in their discrete Fourier transform. That is, we write

$$\delta(x) = \sum_k c_k e^{i k \cdot x}$$ (11)

and likewise for $v(x)$, where the sum is over a three-dimensional grid of discrete wavenumbers $k = (k_x, k_y, k_z)$. The Fourier transformed fields $\tilde{\delta}(k)$ and $\tilde{v}(k)$ are stored as the coefficients $c_k$ (with appropriate normalization) and used for most computations. The real-space fields can be recovered with a 3D discrete Fourier transform. For the most part, however, evolution in Fourier space is simpler, and we are interested in statistical properties of the overdensity and velocity fields which are better described in terms of the Fourier components.

We thus consider the Fourier transforms of the evolution equations:

$$\partial_t \tilde{\delta}_c = -a^{-1} \left( i k \cdot \tilde{v}_c + \int e^{-i k \cdot x} \delta_c \nabla \cdot v_c d^3x + \int e^{-i k \cdot x} v_c \cdot \nabla \delta_c d^3x \right)$$ (12)

$$\partial_t \tilde{v}_c = -a^{-1} \left( i k \tilde{\phi} + \int e^{-i k \cdot x} (v_c \cdot \nabla) v_c d^3x \right) - \frac{\dot{a}}{a} \tilde{v}_c$$ (13)

$$\partial_t \tilde{\delta}_b = -a^{-1} \left( i k \cdot \tilde{v}_b + \int e^{-i k \cdot x} \delta_b \nabla \cdot v_b d^3x + \int e^{-i k \cdot x} v_b \cdot \nabla \delta_b d^3x \right)$$ (14)

$$\partial_t \tilde{v}_b = -a^{-1} \left( i k \tilde{\phi} + \int e^{-i k \cdot x} (v_b \cdot \nabla) v_b d^3x \right) - \frac{\dot{a}}{a} \tilde{v}_b - a^{-1} c_s^2 (i k \tilde{\delta}_b)$$ (15)

$$-a^{-2} |k|^2 \tilde{\phi} = \frac{3}{2} H^2 \left( \frac{\Omega_c}{\Omega_m} \tilde{\delta}_c + \frac{\Omega_b}{\Omega_m} \tilde{\delta}_b \right)$$ (16)
Spatial derivatives \( \partial/\partial x_j \) have become multiplications by \( ik_j \), and this takes care of linear terms immediately. Nonlinear terms—for example, the advective term \( v_b \cdot \nabla \delta_b \) in the continuity equation for baryons (6)—involve real-space multiplication which becomes a computationally expensive convolution in Fourier space. To evaluate these terms, we first compute the necessary derivatives in terms of the Fourier transformed fields

\[
\nabla \delta \to ik\delta(k),
\]

(17)

\[
\nabla \cdot v \to ik \cdot \tilde{v}(k).
\]

(18)

We then inverse Fourier transform the fields to real space, multiply them pointwise, and transform the result back.

This is still relatively computationally expensive, but it allows us to take advantage of the parallelism of the fast Fourier transform (FFT) algorithm. (We use the open-source FFTW library to perform individual transforms.) Consider a simulation on an \( N \times N \times N \) grid, parallelized among \( n \) processes. The fields are divided into \( n \) slices along one dimension, each of size \( N/n \times N \times N \), and each process handles its own slice. Linear terms can be calculated in parallel, since the linear evolution of a component at a given wavenumber is independent of all other components. For nonlinear terms, because each process holds the entire range of \( k_y \) and \( k_z \) values for some given \( k_x \), each process can independently perform a \( y-z \) 2D inverse FFT. The processes must then communicate to perform a global transpose of the field, so that each holds (say) an \( N \times N \times N/n \) slice in the \( x-y \) plane. Then 1D inverse FFTs can be performed in the \( x \) direction in parallel, and the data can be transposed back to its original orientation, now fully inverse Fourier transformed. Multiplications in real space are then pointwise operations, and the fields are transformed back to Fourier space. Since most operations are performed entirely in parallel, the running time scales very well with number of concurrent processes.

When we compute nonlinear terms in this way, we must take care to avoid numerical error from aliasing. Multiplying sinusoidal components creates a sum of sinusoids at different frequencies; for example

\[
\sin k_a x \sin k_b x = \frac{1}{2}(\cos(k_a - k_b)x - \cos(k_a + k_b)x).
\]

But \( k_a + k_b \) may lie outside the range of wavenumbers represented by our simulation. Such a component will be aliased to another frequency where the discrete sampling in space produces the same values. For \( N' \) evenly spaced grid points in the discrete Fourier transform, this will be \( k_a + k_b - N' \). We use the '3/2' de-aliasing rule, in which all fields are padded with an extra \( N/4 \) zeros in each direction, where \( N \) is the number of grid points actually used for the simulation. Then \( N' = \frac{3}{2}N \), and even the product of the highest-frequency components will be aliased to the zero-padded region, set to zero and ignored.

### 2.3 Time evolution

Time evolution is computed with the second-order Runge-Kutta method. The time derivative of a field \( f \) at time \( t \) is used to estimate \( f \) at a time \( t + \Delta t \) to first order: 

\[
f(t) + \frac{\Delta t}{2} f'(t).
\]
Then the time evolution of $f$ is computed using the derivative at that “midpoint”:

$$f(t + \Delta t) = f(t) + \Delta t f'(t + \frac{\Delta t}{2}).$$  \hspace{1cm} (19)

Thus we compute $\partial_t \tilde{\delta}(\mathbf{k})$ and $\partial_t \tilde{v}(\mathbf{k})$ simultaneously for each component, according to Equations (12) - (18), to obtain field values for the midpoint of the time step $\Delta t$. Then we use those midpoint values to obtain new time derivatives which we use to perform the time step. This is slightly more accurate than the straightforward Euler’s method, but still has advantages over higher-order methods in terms of memory requirements.

We face the usual constraint on the size of time steps that $|\mathbf{v}|_{\text{max}} \Delta t$ must be small compared to the grid point spacing (the Courant-Friedrichs-Lewy or CFL condition). To enforce this, the maximum over $|\mathbf{v}_c|$ and $|\mathbf{v}_b| + c_s$ is determined at each time step, and $\Delta t$ reduced if necessary. The maximum baryon velocity must also consider the sound speed, as a pressure wave propagating through a fluid with some bulk velocity $\mathbf{v}_b$ suffers the same numerical problems if it propagates through multiple grid points in one time step.

### 2.4 Initial conditions

Initial conditions are generated with \textsc{Linger++}, a linear spectral code which models dark matter, baryons, radiation and neutrinos, according to the synchronous gauge equations in Ma & Bertschinger (1995). This produces for an arbitrary time $t$ the primordial density perturbations $\tilde{\delta}_c$ and velocity divergence $\theta_c = i k \tilde{v}_c'$ for CDM. These actually represent the linear transfer from the primordial perturbations (3), so we multiply by $k^{n_s/2}$ to obtain

$$\tilde{\delta}_c(k) = k^{n_s/2} \tilde{\delta}_c'$$

$$\tilde{v}_c(k) = -i k^{n_s/2-1} \theta_c.$$ \hspace{1cm} (20)

Baryons are handled identically, although those transfer functions are significantly different due to BAO.

Inflationary cosmology predicts that initial density perturbations are very close to a Gaussian random field. We create a three-dimensional realization by drawing random complex values

$$\tilde{\delta}_c(k) = \tilde{\delta}_c(k) [G(0, 1) + iG(0, 1)]$$  \hspace{1cm} (22)

where $G(0,1)$ represents a Gaussian distributed random number of zero mean and unit variance. Since the inverse transformed fields have to be real fields, care has to be taken to ensure the constraint $f(\mathbf{k}) = f^*(-\mathbf{k})$ for all fields $f$. The next step is to generate a velocity field consistent with the density field

$$\tilde{v}_{c,j=1...3}(\mathbf{k}) = \frac{k_j}{k} \tilde{v}_c [G(0, 1) + iG(0, 1)]$$

$$= -i \frac{k_j}{k} k^{n_s/2-1} \theta_c \times \text{random}.$$  \hspace{1cm} (23)

The same random number field is used for all velocities and densities. The fields are all multiplied with the same normalization factor to fulfill the constraint that the root mean square fluctuation at scales of 8 Mpc/h is consistent with the measured value $\sigma_8 = 0.811$. 

8
Box size and resolution are determined by the need to capture a representative sample of the Universe as well as the scales at which the suppression of growth due to streaming velocity occurs. In general, the box used was long along only one axis to make this feasible. For the final run presented below, box dimensions were $5 \times 5 \times 320$ Mpc/h comoving, with a resolution of $\sim 100$ kpc along all axes (a few times the baryonic Jeans scale).

3 Validation

3.1 Linear growth of dark matter and baryons

A simple but important test of the code is whether it computes linear evolution correctly. As for the fully nonlinear simulation, we initialize a three-dimensional realization based on the isotropic solution provided by LINGER++. Now, however, only the linear terms in the fluid equations are used:

$$\partial_t \tilde{\delta}_c = -a^{-1} (i k \cdot \tilde{\mathbf{v}}_c)$$  (24)
$$\partial_t \tilde{\mathbf{v}}_c = -a^{-1} \left( i k \tilde{\phi} \right) - \frac{\dot{a}}{a} \tilde{\mathbf{v}}_c$$  (25)
$$\partial_t \tilde{\delta}_b = -a^{-1} (i k \cdot \tilde{\mathbf{v}}_b)$$  (26)
$$\partial_t \tilde{\mathbf{v}}_b = -a^{-1} \left( i k \tilde{\phi} \right) - \frac{\dot{a}}{a} \tilde{\mathbf{v}}_b - a^{-1} c_s^2 (i k \tilde{\delta}_b)$$  (27)
$$-a^{-2} |k|^2 \tilde{\phi} = \frac{3}{2} H^2 \left( \frac{\Omega_c}{\Omega_m} \tilde{\delta}_c + \frac{\Omega_b}{\Omega_m} \tilde{\delta}_b \right).$$  (28)

If the initialization and evolution are handled correctly, this linear version of the code will exactly reproduce the isotropic growth in LINGER++ as the two evolve in parallel. This is seen in the evolution of the CDM and baryon power spectra, Figure 1. These simulations are initialized shortly after recombination, $z = 1000$. High resolution is not necessary to verify correctness here, since individual modes grow independently. Since the box is not cubical, the large scale modes are only accounted for in one dimension, and there is some variance at low $k$.

The power spectrum of the baryons is smoothed as the baryons distribute themselves more like the much more abundant dark matter. The dark matter power spectrum also acquires an imprint of the peaks and valleys due to BAO in the baryon power spectrum, although the effect is almost too small to see here. The ratio between the two approaches unity as the distributions become more alike (Figure 2). Although the agreement with the isotropic linear code is perhaps not remarkable—all it means is that we are solving the same equations in three dimensions—this is an effect that N-body codes have difficulty reproducing to this accuracy.

3.2 Nonlinear growth of dark matter: Zel’dovich approximation for 1D collapse

To further test our code, we consider the nonlinear growth of dark matter alone. A pressureless fluid collapsing under self-gravity admits an elegant and practical approximation as a
Figure 1: Linear evolution with our code produces power spectra that match the isotropic linear solution (dashed lines) for both CDM (top) and baryons.
Figure 2: Initially, the baryon power spectrum features peaks and valleys from BAO while the CDM power spectrum varies more smoothly with scale. Over time, the distributions of CDM and baryons become more similar, and the ratio of their power spectra approaches unity.
Figure 3: An initial velocity plane wave grows and steepens, accurately tracing the exact analytical solution (dashed line) close to the crossing time.

collection of fluid elements displaced along linear trajectories. This approximation, proposed by Zel’dovich (1970), assumes that the displacement of a fluid element from its initial position scales with the linear growth factor.

The Zel’dovich approximation is exact in one dimension, and accordingly makes a good test of the accuracy of our code. The simulation is initialized with a flat distribution of dark matter with an initial plane wave in velocity sending parallel sheets of matter toward the center (along the long axis of the box). In the linear model, the velocity plane wave simply grow in amplitude, as one that Fourier mode does not couple to any others. In the nonlinear simulation, the velocity plane wave grows and steepens (Figure 3). The density at the center grows until the peaks of the wave cross at the center. The pseudospectral code produces the correct evolution until very near the crossing time, at which point a nonphysical high-frequency component is introduced. This is also illustrated by the increase in the matter power spectrum at high $k$ (Figure 4).

This is not a problematic failure mode for our purposes. The onset time of the spurious high-frequency component is quite sensitive to resolution. Even with the low resolution ($256 \times 64 \times 64$ grid points) used for the run in the figure, our code is accurate in the fairly
Figure 4: Approaching the crossing time, a recognizably nonphysical high-frequency component appears in the matter power spectrum.
nonlinear regime, with the central density peak tens of times the mean density. This lasts us long enough to investigate the effect of interest. Additionally, this nonphysical behavior is easy to identify and always propagates from small scales to large scales; thus in general we can trust our results to be robust to this error.

We can expect that collapse in a three-dimensional realization of the cosmological density fields will not generally look too different from this one-dimensional case. Even a nearly-spherical overdensity will collapse faster along some direction, effectively decreasing its dimensionality as time goes on. Thus although the Zel’dovich approximation is no longer exact in three dimensions, it has nonetheless been found practical (Yoshisato et al. 2005).

4 Nonlinear growth of dark matter and baryons

Preliminary results concerning the fully nonlinear evolution of coupled dark matter and baryons already demonstrate the expected streaming velocity effect. Figure 5 illustrates a significant reduction in the power of baryonic fluctuations at $\sim 100$ kpc scales as compared with linear theory. The discrepancy is noticeable by $z \sim 400$, and we are in agreement with O’Leary & McQuinn (2012) that previous numerical studies starting at later times missed much of the effect. In fact, the effect we find a larger effect at larger scales than originally suggested by Tseliakhovich & Hirata, with a 40% reduction in power at $k = 80$ h/Mpc and $z \sim 100$.

The effect on the ratio of the power spectra is clearer (Figure 6). The baryons no longer follow the dark matter at small scales, but are instead comparatively smoothed out.

The CDM power spectrum also features a small deviation from linear theory at small scales, but in the opposite direction. It is likely that this is a numerical artifact of the kind discussed in the previous section. Ideally, we could compare it with an N-body simulation in the same regime, but those are unreliable in their own ways. In any case, this is as late as we had hoped for our code to remain reliable, as we are approaching the highly nonlinear regime where N-body codes are most useful.

The velocity power spectra for CDM and baryons, Figures 7 and 8, behave similarly to the matter power spectra. The baryon velocity field, like the density, is smoothed out over larger scales than pressure alone would accomplish, due to the supersonic relative velocities between baryons and the CDM.

A number of steps remain before we can make general quantitative claims about the effect. Since large-scale modes are only included in one dimension, there are fewer of them and they will show more variance between simulations. Large-scale modes in the velocity should be the main determinant of the magnitude of the effect, so a suite of simulations will be necessary to quantify typical power suppression and its variance. Additionally, some validation of numerical results remains to be performed, in particular a study of how results are affected by resolution and inclusion of large-scale modes.

5 Future applications

The code and results presented here should be useful in studying other question in cosmology. It can potentially resolve the difficulties in initializing N-body codes at late times in a realistic
Figure 5: Dashed lines: isotropic linear solution. Solid lines: simulated nonlinear evolution. The dark matter (top) evolves like the linear solution, while the baryonic power spectrum is suppressed at high $k$ (small scales) due to supersonic streaming velocity.
Figure 6: Dashed lines: isotropic linear solution. Solid lines: simulated nonlinear evolution. The baryons clearly no longer follow the dark matter at small scales, but are instead comparatively smoothed out.
The dark matter evolves like the linear solution, while the baryonic velocity power spectrum is suppressed at high $k$ (small scales) due to supersonic streaming velocity.

Figure 7: Dashed lines: isotropic linear solution. Solid lines: simulated nonlinear evolution.
Figure 8: Baryon velocities are smoothed over larger scales than pressure alone would accomplish.
way, which will be important in numerical studies of highly nonlinear processes like structure formation. Another potential application is in studies of inflation, the rapid initial expansion of the Universe discussed briefly in Section 1.1.

Various models of inflation have distinct signatures in the primordial density perturbations they create. One relevant parameter is the spectral index (Equation 3), which has been fairly well constrained by observation and is not exactly 1, as the simplest model predicts [5].

The different models of inflation can be more strongly characterized by the Gaussianity of the primordial density perturbation. The simplest models produce a perturbation which is a Gaussian random field; that is, its Fourier components are uncorrelated and have random phases. In this case, the power spectrum (or equivalently the two-point correlation function) fully describes the perturbation. Our analysis, like most so far, focuses on such a scenario. Indeed, our initialization procedure (22) explicitly makes this assumption. Other models produce perturbations with non-Gaussian properties, described by nonzero three-point and higher-order correlations. Experiments which detect or constrain primordial non-Gaussianity will allow us to rule out large classes of models.

Primordial non-Gaussianity will be an especially important question for the next decade, as the coming generation of observations will be sensitive to its effects. The Cosmic Microwave Background (CMB) is one useful observational probe, since it provides a map of matter density at an early enough time that we can treat it as having evolved linearly from the primordial density. The best data so far from the Wilkinson Microwave Anisotropy Probe (WMAP) has provided weak constraints, and the Planck mission will improve on these [5].

A second observational probe of primordial non-Gaussianity is the large-scale structure we observe today. Current observational uncertainties are too large to use large-scale structure for this purpose, but near-future missions including the Dark Energy Survey and the Large Synoptic Survey Telescope will provide galaxy cluster counts sufficient for sensitivity to non-Gaussianity [2]. This makes it more important that we understand the precise consequences of non-Gaussianity for large-scale structure. Since the formation of structure involves matter overdensities large enough that linear approximation fails, the “inverse problem” of inferring the primordial matter distribution (and hence the physics of inflation) is much more difficult than the same inference from CMB data. Large-scale, accurate simulations will be instrumental in relating observation to theory.

It is especially important for these simulations to accurately model the non-linear coupled dynamics of baryons and dark matter up to the onset of structure formation. For example, these dynamics introduce a scale-dependent bias in the large-scale clustering of early galaxies [10], but so does primordial non-Gaussianity [2]; we must model both effects to relate the observed bias to non-Gaussianity. We have demonstrated that our code successfully models the coupled dynamics, and in fact generating non-Gaussian initial conditions requires only a small modification in which our initial random Gaussian field is convolved with a function characteristic of a given model of inflation.
References


[5] E. Komatsu et al. Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Interpretation. Astrophys.J.Suppl., 192:18, 2011. 57 pages, 20 figures. Accepted for publication in ApJS. (v2) References added. The SZ section expanded with more analysis. The discrepancy between the KS and X-ray derived profiles has been resolved. (v3) New analysis of the SZ effect on individual clusters added (Section 7.3). The LCDM parameters have been updated using the latest recombination history code (RECFAST version 1.5).


