A METHOD FOR EARTHQUAKE MOTION-DAMAGE RELATIONSHIPS WITH APPLICATION TO REINFORCED CONCRETE FRAMES

by

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Report No. 119

October 1996
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ABSTRACT

Recent earthquakes have shown their devastating effects on structures. Damage to structures has significant socio-economic consequences. Before the occurrence of an earthquake, planners can use estimates of structural damage to predict the likely extent of building damage, economic loss, and number of casualties. Immediately after an earthquake, damage estimates can be used by emergency response planners to assess the vulnerability of a structure to aftershocks and to decide whether the building is safe to enter or not. Post-earthquake rehabilitation decisions require estimates of structural damage to decide whether to repair or to demolish a damaged structure.

Structural damage to buildings can be estimated by using seismic site hazard along with relationships between earthquake ground motion severity and structural damage. This dissertation deals only with the latter relationships. These relationships are most frequently described in the form of conditional probability distributions of damage at specified ground motion intensities. These motion-damage relationships are usually expressed in terms of fragility curves and damage probability matrices. The development of fragility curves and damage probability matrices requires the characterization of the ground motion and the identification of the different degrees of structural damage.

This study presents a systematic approach for developing motion-damage relationships that does not rely either on heuristics or on empirical data. Instead, the probability of damage is estimated by quantifying the response of a structure subjected to a significant ensemble of ground motions with a wide range of parameter variations. The quantification of the structural response also includes the variability in structural parameters. For this purpose, a Monte Carlo simulation approach is used to determine the probabilities of structural damage, and the ensemble of ground motions is generated using an appropriate model for ground motion simulation. The models for ground motion simulation include the stationary Gaussian model with modulating functions and the autoregressive moving average (ARMA) models. The Latin hypercube technique is used to increase the efficiency of the Monte Carlo simulation.

The approach developed in this study is then applied to obtain fragility curves and damage probability matrices for reinforced concrete moment resisting frames. Reinforced concrete frames are divided into three classes based on the number of stories in the frames.
These include low rise concrete frames that are 1-3 stories tall, mid rise frames that are 4-7 stories tall, and high rise frames that are 8 stories or taller. The ground motion for these three classes of frames is characterized by the average spectral acceleration over period bands corresponding to the three classes of frames. Sample structures for the three classes of frames are used to develop the motion-damage relationships. Parametric studies are performed to assess the effect of geometric variations in the performance of concrete frame structures.

The Bayesian technique is presented that enables the incorporation of observed damage data with the motion-damage relationships. Using damage data from the Northridge earthquake, the fragility curves for low rise frames are updated. It is found that the synthetic fragility curves, obtained by the Monte Carlo simulation, provide the best estimates of the updated probabilities of the different damage states for these frames. The uncertainty associated with the motion-damage relationships is presented in terms of confidence bounds on the fragility curves.
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CHAPTER 1
INTRODUCTION

Earthquakes can have a seriously negative impact on society by causing human suffering and economic losses due to building damage. Earthquakes affect structures in various ways which include damage to structural elements as well as nonstructural components and contents. The main structural components affected are those of the lateral load resisting system. Nonstructural components include exterior curtain walls, interior partition walls, and mechanical and electrical equipment. Nonstructural damage can occur even at low levels of ground shaking when there is little or no structural damage. While nonstructural components are important, this study is confined to evaluating structural damage as a result of earthquakes.

1.1 Background

Disaster planning and post-earthquake rehabilitation decisions require estimation of structural damage. Information on structural damage is of critical importance for reliable economic loss evaluation for a structure or a region that has been or that might be affected by an earthquake. The extent of structural damage is also important in determining expected casualties from collapsed buildings or from falling debris. Relationships between earthquake ground motion severity and structural damage along with seismic site hazard analysis can be used to assess structural damage, casualties, and subsequent long term economic losses due to earthquakes. Planners can use estimates of structural damage before the occurrence of an earthquake to predict the likely extent of building damage, economic loss, and number of casualties. Damage estimates immediately after an earthquake can be used by emergency response planners to assess vulnerability of a structure to aftershocks and to decide whether the building is safe to enter or not. Post-earthquake rehabilitation decisions require estimates of structural damage to decide whether to repair or to demolish a damaged structure. Estimates of structural damage are used by governmental agencies and large corporations in the prioritization process for retrofit of a large stock of structures. This prioritization of structures is needed as in reality there may be a limited stream of funds over a long time period.

Designers can use motion-damage relationships to evaluate the performance levels of structures. The different performance levels of a structure include serviceability,
prevention of casualties, and prevention of collapse of the structure. The serviceability performance level may include immediate occupancy of the building or limited damage. Immediate occupancy of a building is required for emergency response centers and hospitals. Knowing the expected ground motion and the motion-damage relationships for different structural classes, a designer can choose a structural system that fulfills the specified performance requirements.

Relationships between earthquake ground motion severity and structural damage are most frequently used to characterize the damage distribution over a region. These motion-damage relationships are in the form of probability distributions of damage at specified ground motion intensities and are usually expressed in terms of fragility curves or damage probability matrices (DPMs). Currently there are only two studies that provide DPMs (ATC-13, 1985) and fragility curves (NIBS, 1995) for a wide variety of structural classes. The DPMs in ATC-13 are based on expert opinion since actual damage data are very limited. The fragility curves in the standardized earthquake loss estimation methodology (NIBS, 1995) are based on interpretation of test data and engineering judgment.

Fragility curves and DPMs describe the conditional probabilities of sustaining different degrees of damage at given levels of ground motion. Thus, the development of fragility curves and DPMs requires the characterization of the ground motion and the identification of the different degrees of structural damage. Earthquake ground motion amplitude, frequency content, and strong motion duration are some important characteristics that affect structural response and damage. Thus, they need to be taken into consideration in the development of fragility curves and DPMs. Reliable damage estimation requires sufficient information on the degree of structural damage. Structural damage is caused by the maximum inelastic deformation as well as by the cumulative inelastic deformation under repeated stress reversals.

1.2 Objectives

The objective of this dissertation is to develop a methodology for obtaining relationships between ground motion and structural damage for different types of structures. This methodology is then applied to develop motion-damage relationships for
reinforced concrete structures. The formulation of motion-damage relationships is based on analytical models, in contrast to the currently available motion-damage relationships which are subjective in nature. This objective is achieved through the following steps:

- Identification of suitable ground motion parameters,
- Identification of different damage states based on suitable structural response parameters,
- Evaluation of the probability of a concrete structure being in different damage states,
- Parametric study of the motion-damage relationships for different structural attributes, and
- Application of the Bayesian technique to update the analytical motion-damage relationships by incorporating information on buildings damaged during the past earthquakes.

1.3 Scope

This study presents a systematic approach for developing motion-damage relationships that does not rely either on heuristics or on empirical data. Instead, the probability of damage is estimated by quantifying the response of a structure subjected to a significant ensemble of ground motions with a wide range of parameter variations. For this purpose, a Monte Carlo simulation approach is used to determine the probabilities of structural damage, and the ensemble of ground motions is generated using an appropriate model for ground motion simulation. The models for ground motion simulation include the stationary Gaussian model with modulating function and the autoregressive moving average (ARMA) models. This methodology is then applied to obtain fragility curves and DPMs for reinforced concrete moment resisting frames located on firm soils. Damage due to landslides and liquefaction is not considered in the development of the motion-damage relationships for reinforced concrete frames. Only damage due to ground motion is included.

Reinforced concrete frames are divided into three classes based on the number of stories in the frames. These include low rise concrete frames that are 1-3 stories tall, mid rise frames that are 4-7 stories tall, and high rise frames that are 8 stories or taller. This classification is the same as that defined in ATC-13 (1985) and similar to that used in the
standardized earthquake loss estimation methodology (NIBS, 1995). The ground motion for these three classes of frames is characterized by spectral acceleration over period bands corresponding to the three classes of frames. Fragility curves and DPMs are developed for these three classes of structures. Parametric studies are performed to assess the effect of geometric variations in the performance of concrete frame structures.

1.4 Organization of the Dissertation

The first part of Chapter 2 reviews the various parameters used to characterize ground motion levels. In this study, the ground motion is characterized by spectral acceleration and modified Mercalli intensity (MMI). Spectral acceleration is chosen to characterize the ground motion as it is a simple parameter and can easily be used in regional damage evaluation. Furthermore, spectral acceleration provides an approximate estimate of the input seismic energy. MMI is used for developing the DPMs. The second part of this chapter describes the various damage measures. Cumulative damage measures are preferred as structural damage is believed to be caused by high stress excursions as well as repeated stress reversals. Most of the damage indices have been formulated by assuming that the failure in the structural components is governed by flexural behavior. Since it is beyond the scope of this study to develop a new damage index or to modify an existing damage index, shear behavior is assumed not to influence significantly the damage in building structures considered for the development of motion-damage relationships in this study.

Chapter 3 presents a method for the development of fragility curves. In contrast to previous approaches for developing fragility curves and DPMs, the method presented in this chapter does not rely on heuristics or on empirical data. The methodology can be applied to a wide range of structural classes. The methodology is presented for two ground motion parameters: spectral acceleration and root mean square acceleration. However, it is possible to use other ground motion parameters presented in Chapter 2. The Monte Carlo simulation approach is adopted to determine the probabilities of structural damage. The Latin hypercube sampling technique is employed to reduce the number of simulation cycles.
Chapter 3 also presents the estimation of the different damage states for reinforced concrete frames, based on the different damage measures. This study adopts an equivalent form of the Park and Ang damage index to represent structural damage (Bertero and Bertero, 1992). The Park and Ang index is used because it is simple in its computation and because it has been calibrated using experimental data. Estimating structural repair cost due to an earthquake is an important aim of damage evaluation. Such an estimation for reinforced concrete frames can be achieved if information on crack sizes and extent, degree of crushing and spalling, and accumulated strain in reinforcing steel is available. In Chapter 3 a new method is proposed for defining the damage states in terms of crack width.

Furthermore, earthquake ground motion time histories are needed for the analysis. Although there are a large number of recordings obtained from recent earthquakes, a consistent ensemble of time histories that cover all the different parameter ranges that can be discriminated according to distance to the fault, local soil parameters, and spectral characteristics is currently not available. Thus, it is proposed that ensembles of time histories be simulated at each specified ground motion parameter level. A summary of the different simulation techniques is presented in Chapter 3.

Chapter 4 presents the modeling of reinforced concrete frames for the development of motion-damage relationships. The ground motion is characterized by spectral values in the period bands corresponding to the three classes of reinforced concrete frames. The period bands for the three classes of frames are identified, and the relationship between the average spectral acceleration and the average spectral velocity is investigated for the mid rise and the high rise frames. This chapter also describes the modeling of uncertainties in system parameters. The randomness in structural demand and capacities are presented.

Sample structures for the three classes of frames are used to develop the motion-damage relationships. The structural modeling in the computer programs for evaluating the nonlinear response is discussed. This study uses DRAIN-2DX (Prakash and Powell, 1992) for performing the nonlinear dynamic analysis. The results from DRAIN-2DX are compared with those from IDARC2D (Kunnath and Reinhorn, 1994) and CU-DYNAMIX (El-Tawil, 1996).

Chapter 5 presents the motion-damage relationships in terms of fragility curves and DPMs for special moment resisting reinforced concrete frames located on firm sites. The
relationships between spectral acceleration in the three period bands and MMI are developed in this chapter. These relationships are used to obtain the DPMs from the fragility curves.

In addition, Chapter 5 also summarizes the results of the sensitivity analyses carried out to study the influence of the different structural attributes on the nonlinear dynamic behavior of structures. These sensitivity analyses are carried out to study how structural damage is affected by the different structural attributes. The structural attributes included in the sensitivity studies are the number of bays in a structure, the second-order effects, and the site conditions.

Chapter 6 presents the Bayesian technique that enables the incorporation of observed damage data with fragility curves. The uncertainties associated with the motion-damage relationships are also discussed in this chapter. Such uncertainties can be reduced by incorporating observed damage data in the development of fragility curves. Using damage data from the Northridge earthquake, Chapter 6 presents the updated fragility curves for low rise frames presented in Chapter 5. In addition, the uncertainties in the motion-damage relationships are presented as confidence bounds on the fragility curves.

Chapter 7 enumerates the conclusions of this study and the recommendations for future work.
CHAPTER 2
REVIEW OF PARAMETERS FOR CHARACTERIZATION OF GROUND MOTION AND STRUCTURAL DAMAGE

Reliable damage estimation and rehabilitation decisions require information on the degree of structural damage. The motion-damage relationships are usually expressed in terms of fragility curves and damage probability matrices (DPMs). Fragility curves and DPMs describe the conditional probability of a structure reaching a particular damage state at a given level of ground motion. One must characterize ground motion and identify damage states in order to evaluate these conditional probabilities.

This chapter first reviews the various earthquake ground motion parameters that can be correlated to structural damage. Then, the different indices available to describe structural damage are discussed. Structural damage indices are used to identify the different damage states of a structure.

2.1 Ground Motion Characterization

It is difficult to determine a single parameter that best characterizes earthquake ground motion. Recorded time histories, even at the same site, show variations in details. Earthquake ground motion amplitude, frequency content, duration, and the number of peaks in the time history above a certain amplitude are some of the characteristics important for determining structural response and damage. Ground motion amplitude is measured in terms of acceleration, velocity and displacement. The frequency content of an earthquake time history is important in identifying the amount of energy imparted at different frequencies. The strong motion duration of an earthquake time history is the time interval during which most of the energy of that time history is contained. Various measures of strong motion duration are presented in Section 2.1.1.

Numerous parameters have been used to relate ground motion to the degree of damage sustained by a structure. Peak ground acceleration (PGA) has frequently been used as a parameter to characterize ground motion. Other parameters include Housner’s spectral intensity, Arias intensity, root mean square (RMS) acceleration, response spectrum, and modified Mercalli intensity (MMI).
This study uses response spectrum and MMI as the parameters to characterize earthquake ground motion. Although only these two parameters are used in this study, the methodology for developing motion-damage relationships, which is presented in Chapter 3, can be generalized to any ground motion parameter discussed in this chapter.

2.1.1 Strong Motion Duration

Several measures of strong motion duration have been discussed in the literature. The different definitions of strong motion duration include those by Bolt (1973), Trifunac and Brady (1975), McCann and Shah (1979), and Vanmarcke and Lai (1980). The Trifunac and Brady (1975) definition of strong motion duration is used in this study as it is based on the concept of cumulative energy. The Trifunac and Brady strong motion duration is the time interval required to accumulate 90 percent of the total energy.

The following two equations represent the time at which the Trifunac and Brady strong motion starts, $T_1$, and the time at which the strong motion ends, $T_2$:

\[
\int_{0}^{T_1} a^2(t) \, dt = 0.05 \int_{0}^{T_d} a^2(t) \, dt \tag{2.1}
\]

and

\[
\int_{0}^{T_2} a^2(t) \, dt = 0.95 \int_{0}^{T_d} a^2(t) \, dt \tag{2.2}
\]

where:

$T_d$ = total duration of the earthquake and

$a(t)$ = ground acceleration at time $t$.

Trifunac and Brady's strong motion duration is thus given as:

\[
T_s = T_2 - T_1 \tag{2.3}
\]

The strong motion duration is needed for evaluating the root mean square acceleration (discussed in Section 2.1.5).

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*CHAPTER 2. Review of Parameters for Characterization of Ground Motion and Structural Damage* 8
2.1.2 Ground Motion Amplitude

The parameters used to describe ground motion amplitude include PGA, peak ground velocity (PGV), and peak ground displacement (PGD). As the inertia forces depend directly on acceleration, peak ground acceleration is one of the parameters widely used to describe the intensity and damage potential of an earthquake at a given site. However, PGA is a poor indicator of damage, as time histories with the same PGA could be very different in frequency content, strong motion duration, and energy level, thus causing varying amounts of damage. A large recorded PGA associated with a short duration impulse will cause less damage than a more moderate PGA associated with a long-duration impulse. Therefore, PGA represents only a single amplitude and does not incorporate any of the other characteristics considered to be important for damage evaluation.

A better representation of the ground motion can be achieved by using the relationships between the peak ground motion parameters. Mohraz (1976) used two ratios: the ratio of PGV to PGA, \( \frac{v}{a} \), and the ratio of the product of PGA and PGD to the square of PGV, \( \frac{ad}{v^2} \). Mohraz concluded that the \( \frac{v}{a} \) ratios for rock are substantially lower than those for alluvium.

Zhu et al. (1988) suggest that the ratio of PGA to PGV (\( a/v \)) provides information on the frequency content and the strong motion duration of the ground motion. They show that ground motions with a high frequency content correspond to high \( a/v \) ratios, whereas those with long-duration acceleration pulses are associated with low \( a/v \) ratios. The \( a/v \) ratios are high for sites close to the earthquake source and low for sites far from the source. Ground motions at moderate distances from the source have intermediate \( a/v \) ratios. The variation of the \( a/v \) ratio with distance is due to the attenuation of the ground motion velocity being slower than the attenuation of ground motion acceleration.

Zhu et al. (1988) further investigated the effect of the \( a/v \) ratio on structural damage. They found that the effect of the \( a/v \) ratio on damage sustained by different systems depends on the period and the yield strength level of the structure. Flexible systems with low yield strength show significantly different behavior for different \( a/v \) ratios. They found that for systems with low yield strength, the decrease in ductility demands, with increase in period, is more significant for records with high \( a/v \) ratio than for records with low \( a/v \) ratio. They also found that since ground motions with low \( a/v \) ratio tend to have
longer strong motion durations, they impose larger hysteretic energy demands on the system compared to ground motions with high $a/v$ ratios.

### 2.1.3 Housner's Spectral Intensity

Housner (1952) defined a measure for expressing the relative severity of earthquakes in terms of the area under the pseudo-velocity spectrum between 0.1 and 2.5 seconds. Housner's spectral intensity can be defined by the following equation:

$$I_H = \int_{0.1}^{2.5} S_v(T, \xi) \, dT = \frac{1}{2\pi} \int_{0.1}^{2.5} S_a(T, \xi) \, T \, dT$$

(2.4)

where:

- $S_v(T, \xi) = \text{pseudo-velocity at undamped natural period } T \text{ and damping ratio } \xi$
- $S_a(T, \xi) = \text{pseudo-acceleration at undamped natural period } T \text{ and damping ratio } \xi$.

Thus, Housner's spectral intensity is the first moment of the area of $S_a$ $(0.1 \leq T \leq 2.5 \text{ sec})$ about the $S_a$ axis, implying that the Housner spectral intensity is larger for ground motions with a significant amount of low frequency content. Therefore, ground motions with larger Housner's spectral intensity could cause more damage to tall structures. Housner's spectral intensity, however, does not provide information on the strong motion duration.

### 2.1.4 Arias Intensity

Arias (1970) defined the intensity, $I$, of an earthquake as the sum of the energy dissipated (per unit weight) by all the structures having different periods. Thus

$$I = \int_{0}^{\infty} E \, d\omega$$

(2.5)

where:
\( E \) = energy dissipated per unit weight of a structure as a consequence of the motion induced on it by an earthquake and
\( \omega \) = frequency of the structure

Using Parsevaal’s theorem, equation 2.5 can be written as:

\[
I = \frac{\pi}{2g} \int_{0}^{T_d} a^2(t) \, dt
\]

(2.6)

where:
\( I \) = intensity at zero damping,
\( a(t) \) = ground acceleration at time \( t \),
\( T_d \) = total duration of earthquake motion, and
\( g \) = acceleration due to gravity.

As can be seen from the definition, Arias intensity provides an estimate of the total energy of an earthquake. However, Arias intensity does not incorporate any information on the frequency content and strong motion duration of the earthquake.

2.1.5 Root Mean Square Acceleration

RMS acceleration is a parameter incorporating the total intensity and the strong motion duration. RMS acceleration is defined as

\[
\sigma_0 = \left[ \frac{1}{T_s} \int_{T_s}^{T_s} a^2(t) \, dt \right]^{1/2}
\]

(2.7)

where:
\( \sigma_0 \) = RMS acceleration of the strong ground motion,
\( T_s \) = strong motion duration, and
\( a(t) \) = ground motion acceleration at time \( t \).
RMS acceleration is a measure of the average rate of energy input to the structure. However, it does not provide any information about the frequency content as it is the sum of input power at all frequencies.

2.1.6 Destructiveness Potential Factor

Araya and Saragoni (1984) proposed the destructiveness potential factor, \( P_D \), that considers both the Arias intensity and the rate of zero crossings, \( v_0 \). The destructiveness potential factor is defined by the following equation:

\[
P_D = \frac{I_A}{v_0^2}
\]

(2.8)

where:
- \( P_D \) = destructiveness potential factor,
- \( I_A \) = Arias intensity, and
- \( v_0 \) = rate of zero crossings.

The destructiveness potential factor simultaneously considers the effect of the ground motion amplitude, strong motion duration, and frequency content on the relative destructiveness of different ground motion records. The amplitude of ground motion acceleration and strong motion duration are incorporated in the Arias intensity. The frequency content is considered in the rate of zero crossings. Araya and Saragoni (1984) demonstrated that the destructiveness potential correlates very well with the MMI values. However, it is possible that two different time histories could have similar destructiveness potential factors but very different values of the zero crossing rate and Arias intensity. For example, a time history with a small zero-crossing rate would cause less damage to short period structures than a time history with a larger zero crossing rate close to the fundamental period of the structures, although both time histories have the same destructiveness potential factor. Structural damage is determined by seismic energy input into the structure. Thus, the second time history with a larger zero-crossing rate causes more damage as it results in more seismic energy being input into the structures. The concept of input seismic energy is presented in Section 2.1.7.
2.1.7 Response Spectrum

The linear elastic response spectrum represents the maximum acceleration, maximum relative velocity, or maximum relative displacement of a single-degree-of-freedom (SDOF) system subjected to a particular ground motion. Seismic ground motions result in energy being transmitted from the ground into the structure. During an earthquake, the maximum energy transmitted from the ground into a linear, elastic SDOF system can be estimated as:

\[ E = \frac{1}{2} k S_d^2(T, \xi) = \frac{1}{2} m S_{p,v}^2(T, \xi) \]  

(2.9)

where:

- \( E \) = maximum input energy,
- \( m \) = mass of the SDOF system,
- \( k \) = stiffness of the SDOF system,
- \( \xi \) = damping ratio of the SDOF system,
- \( S_d \) = spectral displacement ordinate at the natural period of the SDOF system, and
- \( S_{p,v} \) = pseudo-spectral velocity ordinate at the natural period of the SDOF system.

For a nonlinear SDOF system, the seismic energy input into the system by the ground motion can be estimated by integrating the following equation of motion of the system:

\[ m \ddot{x}_t + c \dot{x} + f_s = 0 \]  

(2.10)

where:

- \( m \) = mass of the SDOF system,
- \( c \) = viscous damping coefficient of the SDOF system,
- \( f_s \) = restoring force for the SDOF system,
- \( x_t = x + x_g \) = absolute displacement of the mass,
- \( x \) = relative displacement of the mass with respect to the ground, and
- \( x_g \) = earthquake ground displacement.

The restoring force, \( f_s \), for an elastic system may be expressed as \( kx \), where \( k \) is the stiffness of the SDOF system. By using the relationship \( \ddot{x}_t = \ddot{x} + \ddot{x}_g \), equation 2.10 may be written as:
\[ m \ddot{x} + c \dot{x} + f_s = -m \ddot{x}_g \]  

(2.11)

Uang and Bertero (1990) show that equations 2.10 and 2.11 lead to different values of the input seismic energy. Integrating equation 2.10 with respect to the relative displacement, \( x \), leads to the following equation:

\[ \frac{m \dot{x}_g^2}{2} + \int c \dot{x} \, dx + \int f_s \, dx = \int m \ddot{x}_g \, dx \]  

(2.12)

The first term in equation 2.12 represents the absolute kinetic energy, the second term represents the damping energy, and the third represents the recoverable strain energy and the irrecoverable hysteretic energy. The right-hand-side term of equation 2.12 is defined as the absolute input energy.

Integrating equation 2.11 with respect to the relative displacement, \( x \), the following equation is obtained:

\[ \frac{m \dot{x}_g^2}{2} + \int c \dot{x} \, dx + \int f_s \, dx = -\int m \ddot{x}_g \, dx \]  

(2.13)

The first term in equation 2.13 represents the relative kinetic energy and is different from the first term in equation 2.12 which represents the absolute kinetic energy. The second and third terms of equations 2.12 and 2.13 are the same. The right-hand-side term of equation 2.13 is the relative input energy due to the ground motion. Whereas the absolute input energy represents the physical energy input, the relative input energy ignores the effect of the rigid body translation of the structure. Akiyama (1985) showed that the relative input energy of a SDOF system can provide a good estimate of the input energy for multistory buildings. Uang and Bertero (1990) concluded that the absolute input energy of a SDOF system can be used to estimate the absolute input energy for multistory buildings.

Housner (1956) assumed that the input seismic energy given by equation 2.9 can be used as the energy demand for an inelastic system in his proposed limit design method. Uang and Bertero (1990) compared the energy demand estimated from equation 2.9 and that from equation 2.12 for 5% damping and a ductility ratio of 5 when the structure was
subjected to ground motions from the El Centro, Mexico City, and San Salvador earthquakes. Their results indicate that equation 2.9 can significantly underestimate the input seismic energy. Although it would be preferable to use the absolute input seismic energy as the ground motion parameter, the computation of the absolute energy involves the assumption of the ductility capacities of the structures. For regional damage evaluation, the ductility capacities of the structures could vary greatly. Thus, in this study, the linear response spectral values are used to characterize the ground motion. These spectral values provide a lower bound to the energy input due to seismic ground motion.

2.1.8 Modified Mercalli Intensity

The earthquake intensity at a location is a qualitative measure of the size of the earthquake in terms of observed damage and human reactions at that location. The Rossi-Forel scale is one of the earliest measures of intensity (Richter, 1958). The Rossi-Forel scale has mostly been replaced by the MMI scale. The MMI scale was obtained in 1931 by modifications to the original scale proposed by Mercalli (Richter, 1958). MMI is based on the performance of unreinforced masonry buildings, chimneys and some other older construction. The MMI scale is often used to specify the severity of ground shaking in a given geographic region. It is also used to describe the distribution of damage over a region. The main advantage of the MMI scale is that it has been in use for a long time and that some motion-damage relationships exist in terms of the MMI scale (ATC-13, 1985). However, its use is subjective, and differences in interpretation are substantial.

2.2 Measures of Structural Damage

Structural damage occurs when the deformations of structures under environmental loads are large and permanent. The severity and nature of seismic damage depends on the building material, and the structural configuration. Yao et al. (1986) presented a review of structural damage for different types of buildings. They suggest that structural damage may be defined as a ratio of the demand to the ultimate structural capacity. Extensive studies have been carried to determine the imposed demand, but only limited studies have been carried out to estimate the structural capacity. Structural damage in relatively ductile systems, such as steel frames, depends on the cumulative inelastic deformation. For
relatively brittle systems, such as masonry buildings, shear behavior is dominant, and the damage can be expressed in terms of the maximum deformation. The damage in reinforced concrete structures depends on both the maximum inelastic deformation and the cumulative deformation under repeated stress reversals.

Since damage to reinforced concrete structures is caused by stress reversals as well as high stress excursions, more realistic measures of damage include not only the peak inelastic response but also the effect of reversals of inelastic deformations. The earlier ductility-based measures of damage did not account for cumulative damage under repeated cycles of deformation. The more recent measures of damage include the dissipated hysteresis energy to account for cumulative damage.

There are many damage models which characterize the state of structures after earthquakes in terms of a damage index. A damage model is realistic if the numerical value of the damage index shows correlation with observed seismic damage in structures. This correlation can be used to assess damage to structures by first estimating the damage index under a given ground motion. Structural damage is frequently represented by various local and global damage indices which aim to quantify numerically the damage sustained under earthquake loading. The local indices quantify the damage in individual members, and the global indices describe the state of all or a large part of a structure. Most of the damage models first consider the damage to individual structural elements. Global damage is then defined as a combination of the damage to individual elements. Williams and Sexton (1995) provide a comprehensive review of the damage measures for reinforced concrete structures.

### 2.2.1 Local Damage Indices

Local indices are used to express the damage sustained by individual elements, and usually employ the concepts of ductility and dissipated energy. Ductility and interstory drift are the two earliest forms of damage index which are based only on maximum deformation and fail to account for the effects of repeated cycling under seismic loading. However, they are still widely used because of their simplicity and ease of interpretation. The ductility ratio can be expressed in terms of rotation, curvature or displacement. The rotation ductility, \(\mu_\theta\), is the ratio of the maximum rotation, \(\theta_{\text{max}}\), to the rotation at yielding, \(\theta_y\). The rotation ductility is expressed as follows:
\[ \mu_\theta = \frac{\theta_{\text{max}}}{\theta_y} \] (2.14)

Banon et al. (1981) suggested that the member can be assumed to yield in antisymmetric bending in order to compute the yield rotation, \( \theta_y \). The computation of curvature ductility does not need any assumption on the bending mode of the member as it applies only to a section of the member, usually the section with the maximum stresses. The curvature ductility, \( \mu_\phi \), can be expressed as follows:

\[ \mu_\phi = \frac{\phi_{\text{max}}}{\phi_y} \] (2.15)

Banon et al. (1981) proposed the flexural damage ratio in an attempt to overcome the shortcomings of the ductility ratios. The flexural damage ratio is defined as the ratio of the initial tangent stiffness to a reduced secant stiffness at maximum deformation. Banon et al. also defined two more measures of damage in order to incorporate damage accumulation due to cyclic loading. These are the normalized cumulative rotation and the normalized dissipated energy. The normalized cumulative rotation is defined as the ratio of the sum of all plastic rotations to yield rotation. The normalized dissipated energy, \( E_n \), is the ratio of the energy dissipated by inelastic rotation at one end of the member to half of the maximum elastic energy stored in the member in anti-symmetric bending. The normalized dissipated energy is thus given by the following equation:

\[ E_n(t) = \frac{\int_0^t M(\tau)\theta_d(\tau) \, d\tau}{\frac{1}{2} M_y \theta_y} \] (2.16)

where:

- \( t \) = time elapsed since the beginning of loading and
- \( \theta_d(d\tau) \) = the rotation increment of the inelastic spring at one end of the member during the time interval between \( \tau \) and \( \tau + d\tau \).

For steel components, the damage model proposed by Krawinkler and Zohrei (1983) is often used. There are several models for definitions of local damage in reinforced
concrete components. These measures for reinforced concrete include the measures proposed by Park and Ang (1984), by Chung, Meyer and Shinozuka (1987), and by Bracci et al. (1989). These damage indices are discussed in the following paragraphs.

**Krawinkler and Zohrei’s Damage Index**

Krawinkler and Zohrei (1983) proposed a damage model for steel components. Their damage model is given by the following equation:

\[
D = C \sum_{i=1}^{N} (\Delta \delta p_i)^q
\]  

(2.17)

where:
- \(D\) = damage measure,
- \(C, q\) = structural performance parameters,
- \(N\) = number of inelastic excursions caused by the earthquake, and
- \(\Delta \delta p_i\) = plastic deformation range of excursion \(i\).

The plastic deformation ranges, \(\Delta \delta p_i\), in equation 2.17 are estimated by means of the rainflow cycle counting method. In the rainflow cycle counting method, the ranges are usually reordered so that small excursions constitute only interruptions of the larger cycles. Nassar and Krawinkler (1991) show that for bilinear systems, the hysteretic energy, \(HE\), and the sum of the plastic deformation ranges can be related as follows:

\[
HE = F_y \sum_{i=1}^{N} \Delta \delta p_i
\]  

(2.18)

**Park and Ang’s Local Damage Index**

Park et al. (1984) proposed a damage index which is a linear combination of the damage caused by excessive deformation and that contributed by repeated cyclic loading effect. The Park and Ang damage index is expressed as follows:
\[ D = \frac{\delta_m}{\delta_u} + \frac{\beta}{P_y \delta_u} \int dE \]  \hspace{1cm} (2.19)

or

\[ D = \frac{\delta_m}{\delta_u} + \beta \int \left( \frac{\delta}{\delta_u} \right)^{\alpha} \frac{dE}{E_c(\delta)} \]  \hspace{1cm} (2.20)

where:

- \( \delta_m \) = maximum response deformation under an earthquake shown in Figure 2.1,
- \( \delta_u \) = ultimate deformation capacity under monotonic loading,
- \( P_y \) = calculated yield strength,
- \( E_c(\delta) \) = hysteretic energy per cycle at deformation \( \delta \),
- \( \alpha, \beta \) = non-negative parameters,
- \( \delta \) = amplitude of deformation in each cycle of oscillation, and
- \( dE \) = incremental dissipated hysteretic energy.

\[ \delta_m \text{ is the maximum of all } \delta_p1, \delta_p2 ..., \text{ from all cycles.} \]

Figure 2.1: Definition of \( \delta_m \) in the Park and Ang (1984) index.
The first term in the expression for the damage index (equations 2.19 and 2.20) represents the damage due to maximum deformation experienced during seismic loading, and the second term represents the damage due to cumulative hysteretic energy dissipation. The load deformation terms are shown in Figure 2.1, where \( \delta_{p1}, \delta_{p2}, \ldots, \delta_{pn} \) represent the deformation in each cycle. The parameter \( \delta_m \) is the maximum of \( \delta_{p1}, \delta_{p2}, \ldots, \delta_{pn} \) for all the cycles. The deformation at yield under monotonic loading is represented by \( \delta_y \). The damage index, \( D \), is 0 when there is no damage and is 1 for collapse.

The ultimate deformation capacity of a member under monotonic loading, \( \delta_u \), is an indicator of the ductility capacity of a member. Reinforced concrete members with about the same level of yield deformation and about the same axial load can have different ultimate deformation values depending on the confinement ratios. The ultimate deformation appears to be more important than the yield deformation in predicting damage. Cosenza et al. (1990) found that the value of \( \beta = 0.15 \) correlates closely with results based on other damage models. This study uses this value of \( \beta \).

For reinforced concrete structures, the Park and Ang model has been used widely in recent years because it is simple and because it has been calibrated using data from various structures damaged during past earthquakes. An equivalent form of the Park and Ang index is used in this study (Bertero and Bertero, 1992). The damage index for the plastic hinge locations at the ends of a member is defined as follows:

\[
D = \frac{\theta_m}{\theta_u} + \frac{\beta}{M_y \theta_u} \int dE
\]

(2.21)

where:
- \( \theta_m \) = maximum positive or negative plastic hinge rotation,
- \( \theta_u \) = plastic hinge rotation capacity under monotonic loading,
- \( \beta \) = model parameter (0.15 in this study),
- \( M_y \) = calculated yield strength, and
- \( dE \) = incremental dissipated hysteretic energy.

The damage index for the member is computed as the weighted average of the damage indices at the ends. The weighted average is computed using equation 2.24 where the weighting factor \( \lambda_i \) is the ratio of the energy dissipated at end \( i \) to the sum of the energies dissipated at the two ends.
Chung, Meyer and Shinozuka’s Local Damage Index

Chung et al. (1987) proposed a damage index which combines a modified version of Miner’s Hypothesis with damage modifiers that reflect the effect of the loading history. This index considers the difference in response of members to positive and negative moments and is evaluated by the following expression. The terms in this index are shown in Figure 2.2.

\[ D_c = \sum_i \left( \alpha_i^+ \frac{n_i^+}{N_i^+} + \alpha_i^- \frac{n_i^-}{N_i^-} \right) \]  \hspace{1cm} (2.22)

where:
- \( i \) = indicator of displacement or curvature level,
- \( N_i = \frac{M_i - M_{ni}}{\Delta M_i} \) = number of cycles to cause failure at curvature level \( i \),
- \( n_i \) = number of cycles actually applied at curvature level \( i \),
- \( \alpha_i \) = damage modifier,
- \( M_i \) = initial strength at curvature level \( i \),
- \( M_{ni} \) = final strength at curvature level \( i \),
- \( \Delta M_i \) = strength drop at curvature level \( i \), in a single load cycle, and
- \( +, - \) = loading and unloading, respectively.

The effect of loading history is taken into account by damage modifier, \( \alpha_i \), which for positive moment loading is defined as:

\[ \alpha_i^+ = \frac{\sum k_{ij}^+}{n_i^+ k_i^+} \cdot \frac{\phi_i^+ + \phi_{i-1}^+}{2\phi_i^+} \]  \hspace{1cm} (2.23)

where:
- \( k_{ij}^+ = \frac{M_{ij}^+}{\phi_i^+} \) = stiffness during the \( j^{th} \) cycle up to load level \( i \),
- \( k_i^+ = \frac{1}{N_i^+} \sum_{j=1}^{N_i^+} k_{ij}^+ \) = average stiffness during \( N_i^+ \) cycles up to load level \( i \), and
\[ M_{ij}^+ = M_{il}^+ - (j-1)\Delta M_i^+ \] = moment reached after \( j \) cycles up to load level \( i \).

For negative loading, the damage modifier is defined similarly.

The damage index definition by Chung et al. does not explicitly account for the damage caused by the maximum deformation experienced by the element.

![Diagram](image)

**Figure 2.2:** Damage index definitions by Chung et al. (1987).

**Bracci et al.'s Local Damage Index**

Bracci et al. (1989) defined a damage index in terms of the ratio of the damage consumption to the damage potential of a component. The damage potential is defined as the total area between the monotonic load-deformation curve and the fatigue failure envelope. The damage consumption occurs due to strength damage and deformation damage. Strength damage is caused by strength deterioration and dissipated hysteretic
energy. Strength damage results in the lowering of the upper-bound load-deformation curve. Strength damage is determined as the area between the upper-bound curve and the monotonic load-deformation curve. Deformation damage occurs due to irrecoverable permanent deformations.

2.2.2 Global Damage Indices

Global damage indices provide information about the damage to the overall structure. When a structure is statically determinate, local damage at the most damaged member is enough to determine the damage state of the entire structure. However, a global damage index which accounts for the extent and distribution of localized damage is required for redundant structures. A global damage index can be obtained either by computing a weighted average of the local damage indices for all the members of a structure or by considering some overall structural characteristic like the modal periods. The different types of global damage indices are described below.

Park and Ang’s Global Damage Index

Park and Ang’s (1984) global damage index is defined as a weighted average of the local damage indices of each element. The weighting function for each element is proportional to the energy dissipated in the element. The global damage index is thus given by the following equation:

\[ D_T = \sum_{i=1}^{N} \lambda_i D_i \]  (2.24)

where:
\[ \lambda_i = \frac{E_i}{\sum_{i=1}^{N} E_i} \]

\( N \) = number of elements, and
\( E_i \) = energy dissipated in element i.
In addition to the overall damage index, story-level damage indices can also be obtained by using equation 2.24 except that the summation in that equation is carried out over all the members of that story. The story-level damage indices can be used to identify the story with the highest damage.

The global damage index, as defined by equation 2.24, does not properly account for the local concentration of damage. It is possible for a few structural members of the building to have undergone severe damage without the global index reflecting it. However, in general, the locations having high damage indices will also be the ones that dissipate large amounts of hysteretic energy. Therefore, equation 2.24 assigns higher weighting to the more heavily damaged members of the structure. Thus, the structure damage index reflects the state of the most heavily damaged members in the structure.

**Chung, Meyer and Shinozuka’s Global Damage Index**

Chung et al. (1987) used the damage index from each story to define the global damage index. The story damage index is obtained as a weighted average of the local damage indices of all elements in the story, with the energy dissipated in the member as the weighting function. The story damage index is obtained by the following equation:

\[
D_{sk} = \frac{\sum_{i=1}^{n} D_i^k E_i^k}{\sum_{i=1}^{n} E_i^k}
\]  

(2.25)

where:

- \( D_i^k \) = local damage at location \( i \) on story \( k \),
- \( E_i^k \) = energy dissipated at location \( i \) on story \( k \), and
- \( n \) = number of locations at which the local damage is computed for story \( k \).

This definition of the story damage index is similar to the definition of the Park and Ang global damage index provided by equation 2.24. However, the local damage indices \( D_i \) and \( D_i^k \) in equations 2.24 and 2.25 are the respective local damage indices.
The global damage index is obtained as a weighted average of the story damage indices using a triangular weighting function with the maximum at the base. Thus, the global damage index is given by the following equation:

\[ D_g = \sum_{k=1}^{N} D_{sk} I_k \]  

(2.26)

where:
\[ I_k = \frac{N+1-k}{N} \]

= weighting factor for story k and

\[ N \]

= number of stories

**Bracci et al.'s Global Damage Index**

Bracci et al. (1989) presented a global damage index in terms of the local damage index by means of the following equation:

\[ D_g = \frac{\sum w_i D_i^{(m+1)}}{\sum w_i D_i^m} \]  

(2.27)

where:

\[ D_g \] = global damage index,

\[ D_i \] = local damage index,

\[ w_i \] = importance factor for component i, and

\[ m \] = control weighting factor for component i.

A high value of the exponent, \( m \), results in more emphasis on the most severely damaged elements. Bracci et al. (1989) defined the weight \( w_i \) as the gravity load supported by element i divided by the total weight of the structure. Thus, damage to the columns is assigned larger weights than those to the beams, and the damage at the base of the structure is assigned a much larger weighting factor than damage to the upper stories.
Softening Damage Indices

Softening damage indices relate the changes in the first few natural periods of a structure to the level of damage sustained by the structure. Roufaiel and Meyer (1987) proposed a relationship between a global damage parameter, expressed in terms of deflections at the roof level of a structure, and the change in fundamental frequency of the structure given by the following equation:

\[
D = \frac{\delta_r - \delta_y}{\delta_r - \delta_y} = \frac{14.2\delta_y \left( \sqrt{\omega_c / \omega} - 1 \right)}{\delta_r - \delta_y}
\] (2.28)

where:
- \(\delta_r\) = maximum roof deflection under earthquake excitation,
- \(\delta_y\) = roof displacement at which the first member of the structure reaches yield strength, assuming the frame displaces in its first mode only,
- \(\delta_r\) = roof deflection at which the structure is assumed to fail,
- \(\omega_c\) = fundamental frequency of the undamaged or elastic structure, and
- \(\omega\) = fundamental frequency of the structure after being damaged.

DiPasquale and Cakmak (1990) proposed two softening damage indices, the maximum softening index and the final softening index. These two indices are given by the following two equations:

\[
\delta_m = 1 - \frac{T_0}{T_{\text{max}}}
\] (2.29)

and

\[
\delta_f = 1 - \frac{T_0^2}{T_f^2}
\] (2.30)

where:
- \(\delta_m\) = maximum softening index,
- \(\delta_f\) = final softening index,
\( T_0 \) = initial natural period,
\( T_{\text{max}} \) = maximum natural period of an equivalent linear system, and
\( T_f \) = final natural period of an equivalent linear system.

DiPasquale and Calmik (1990) showed that the final softening index, \( \delta_f \), is approximately equal to the average reduction in stiffness across the structure. The maximum softening index, \( \delta_m \), depends on the combined effect of stiffness degradation and plastic deformations. The response of the structure to input ground motion must be known in order to compute the softening indices. Thus, it is necessary to specify the ground acceleration time history and the structural response at various locations of the structure. The final softening can be assessed on the basis of information on the state of the structure before and after the earthquake, with no need for information on the structural response during the earthquake.

The softening indices provide very little information about the distribution of damage sustained by different members within the structure. Mork (1992) tried to improve this aspect by extending the maximum softening index to include the second mode as shown in the following two equations:

\[
\delta_1 = 1 - \sqrt{\frac{k_{1,\text{max}}}{k_{1,0}}} \tag{2.31}
\]

and

\[
\delta_2 = 1 - \sqrt{\frac{k_{2,\text{max}}}{k_{2,0}}} \tag{2.32}
\]

where:
\( \delta_1 \) = maximum softening index corresponding to the first mode,
\( \delta_2 \) = maximum softening index corresponding to the second mode,
\( k_{1,0} \) = initial stiffness of an equivalent linear system for the first mode,
\( k_{2,0} \) = initial stiffness of an equivalent linear system for the second mode,
\( k_{1,\text{max}} \) = maximum stiffness of an equivalent linear system for the first mode, and
\( k_{2,\text{max}} \) = maximum stiffness of an equivalent linear system for the second mode.
The damage measures $\delta_1$ and $\delta_2$ can be taken to represent damage in the lower and the upper parts of the structure, respectively.

Nielsen et al. (1992) showed that contours for the overall softening index, $\delta_m$, can be obtained in the $\delta_1-\delta_2$ plane by using the following relationship:

$$\delta_2 = 1 - \frac{1}{4\lambda} \sqrt{\frac{\delta^2 - 2\delta(1-\delta_1)^2}{\delta - (1-\delta_1)^2}}$$

(2.33)

where:

$$\lambda = \frac{k_{2,0}}{k_{1,0}} \text{ and }$$

$$\delta = (1-\delta_m)^2 \left(1 + 2\lambda - \sqrt{1 + 4\lambda^2}\right).$$

However, the softening indices do not explicitly account for the dissipated hysteretic energy. Therefore, the softening indices are not strict measures of cumulative damage, though they do approximately account for degradation in strength and stiffness as reflected in the first few modal periods.

2.3 Summary

The first part of this chapter reviewed the various parameters used to characterize ground motion levels. In this study, the ground motion is chosen to be characterized by spectral acceleration and MMI. The spectral acceleration in three period bands corresponding to three classes of reinforced concrete frames is used in the development of fragility curves in Chapters 4 and 5. Spectral acceleration is chosen to characterize the ground motion as it is a simple parameter and can be easily used in regional damage evaluation. Furthermore, spectral acceleration provides a lower bound to the input seismic energy. MMI is used for developing the DPMs in Chapter 5.

The second part of this chapter described the various damage measures. Cumulative damage measures are preferred as structural damage is believed to be caused by high
stress excursions as well as repeated stress reversals. Most of the damage indices have been formulated by assuming that the failure in the structural components is governed by flexural behavior. Since it is beyond the scope of this study to develop a new damage index or to modify an existing damage index, shear behavior is assumed not to influence significantly the damage in building structures considered for the development of motion-damage relationships in Chapters 4 and 5.

However, shear and combined shear-flexure behavior may lead to significant number of failures in structures. The significant difference between shear and flexural behaviors is the usual brittle mode of collapse when shear behavior dominates the structural response. Shear behavior is likely to dominate in short, stocky components. In building structures where slender beams and columns are used, shear behavior is not expected to be significant. Still, shear behavior may dominate in some cases where the effective length of the columns may be reduced due to nonstructural walls adjoining the columns.

The estimation of the different damage states for reinforced concrete frames, based on the different damage measures, is presented in Chapter 3. This study adopts an equivalent form of the Park and Ang damage index to represent structural damage. The Park and Ang index is used because it lends itself to numerical computation and because it has been calibrated using experimental data.
CHAPTER 3
METHODOLOGY FOR DEVELOPING
MOTION-DAMAGE RELATIONSHIPS

Ground-motion-versus-damage relationships characterize the level of damage to a particular class of structures as a function of a ground motion parameter. In order to represent the variability in earthquake ground motion and the uncertainties in structural behavior, these relationships are most frequently described in the form of probabilities of damage conditional on the ground motion parameter. The two most widely used forms of motion-damage relationships are fragility curves and damage probability matrices (DPMs).

3.1 General Framework for Motion-Damage Relationships

A fragility curve describes the probability of reaching a damage state at a specified ground motion level. Thus, a fragility curve for a particular damage state is obtained by computing the conditional probabilities of being in that damage state at various levels of ground motion. A plot of the computed conditional probabilities versus the ground motion parameter describes the fragility curve for that damage state. The conditional probabilities are defined as follows:

\[ p_{ik} = P[D = d_i \mid Y = y_k] \]  

(3.1)

where:

\( p_{ik} \) = probability of being in damage state \( d_i \) given the ground motion is \( y_k \),

\( D \) = damage random variable defined on the damage state vector \( D = \{d_0, d_1, ..., d_n\} \),

\( Y \) = ground motion random variable.

An alternate representation of fragilities is given by the probabilities of reaching or exceeding a specified damage state given a ground motion level. This definition is used to obtain the fragility curves for reinforced concrete frames in Chapter 5. The conditional probabilities can be evaluated from equation 3.1 as follows:

\[ p_{ik} = P[D \geq d_i \mid Y = y_k] = \sum_{j=i}^{n} p_{jk} \]  

(3.2)
Damage states can be defined to characterize the physical state of the structure. A numerical damage scale in terms of the ratio of repair cost to replacement value of the structure can also be specified. Chapter 2 discussed the different damage measures. The segregation of some of the damage measures into damage states is discussed later in this chapter. Several different parameters used to describe ground motion have been discussed in Chapter 2. The methodology can be used with any ground motion parameter. However, root mean square (RMS) acceleration and spectral acceleration, $S_a$, for a specified structural period range are used to develop the methodology in this chapter.

Another commonly used representation of structural damage as a function of the earthquake ground motion is the DPM. A DPM specifies the discrete probabilities of reaching a damage state at different ground motion levels. In this study, modified Mercalli intensity (MMI) is used as the ground motion parameter for the DPMs. Relationships between spectral acceleration, in the relevant period band, and MMI are developed and used to obtain DPMs from fragility curves. The formulation for obtaining DPMs is shown as follows:

$$P_{D|MMI}(d|MMI) = \int_{S_a} P_{D|MMI,S_a}(d|MMI,S_a) f_{S_a|MMI}(S_a|MMI) dS_a$$  \hspace{1cm} (3.3)

where:
- $P_{D|MMI}(d|MMI)$ = probability of being in or exceeding a given damage state at a specified MMI,
- $P_{D|MMI,S_a}(d|MMI,S_a)$ = probability of being in or exceeding a given damage state at specified MMI and spectral acceleration, and
- $f_{S_a|MMI}(S_a|MMI)$ = conditional probability density of spectral acceleration at specified MMI, obtained by assuming this density function to be lognormal with parameters determined later in Chapter 5 for concrete frames.

The above formulation can be simplified by assuming that the probability of reaching or exceeding a given damage state at specified MMI and spectral acceleration is the same as the probability of reaching or exceeding a given damage state at specified spectral acceleration. Both spectral acceleration and MMI are used as ground motion parameters.
Thus, representing the probability of damage as a function only of spectral acceleration should not have significant effect on the conditional probabilities for the DPMs. This assumption can be verified as additional data become available. Equation 3.3 can then be simplified as follows:

$$P_{D|M,M}|[d|M,M] = \int P_{D|S_a}[d|S_a] f_{S_a|M,M}[S_a|M,M] ds_a$$  \hspace{1cm} (3.4)

While simple fragility formulations have been developed and used extensively for components and mechanical assemblies in nuclear power plant safety analyses (Kennedy et al., 1980, Kennedy and Ravindra, 1984), no systematic approach for developing such fragility curves has been presented for complex structural systems. This chapter presents such an approach.

The major components of the proposed methodology consist of (a) characterization of the structure when subjected to extreme dynamic loads, (b) characterization of the potential ground motions, and (c) quantification of the structural response that includes the variability in ground motion and the uncertainty in structural parameters. It is difficult to develop analytical closed-form solutions for motion-damage relationships because neither the ground motion nor the nonlinear behavior of the structure can be described in an analytical form. Thus, a Monte Carlo simulation approach is used to estimate the probabilities of damage conditional on different ground motion levels. Figure 3.1 describes the general framework of this methodology.

Damage to structures subjected to severe earthquake ground shaking depends on their dynamic characteristics and their nonlinear behavior. This evaluation of damage requires that nonlinear dynamic analyses be performed for a wide range of earthquake ground motion time histories. For the nonlinear dynamic analysis, the hysteretic behavior of structural components must be specified. When fragility curves are needed for many different classes of structures, the structural properties need to be representative of a wide range of structures that might fall within that specified structural category. For this purpose, a generic structure should be designed for a specified structural system reflecting a particular design code specification. The behavior of individual structures is likely to differ from the behavior of the generic structure used in the development of fragility
CHAPTER 3: Methodology for Developing Motion-Damage Relationships

Methodology for the Development of Fragility Curves & DPMs

- Structural System Modeling
- Ground Motion Modeling
- Monte Carlo Simulation (Latin Hypercube)
- Simulation of System Parameters
- Artificial Ground Motion Simulation
  - Gaussian Models with Modulating Functions
  - ARMA Models
  - Non-Linear Time History Analysis

Synthetic Fragility Curves
- Relationships between Modified Mercalli Intensity and Spectral Acceleration in the Relevant Period Band

Damage Probability Matrices

Figure 3.1: Steps in the development of fragility curves and damage probability matrices.
curves. However, the damage estimated from the generic structure is expected to be representative on the average over the range of different structures within this structural class.

3.2 Fragility and DPM Simulation

The Monte Carlo simulation technique is used for the computation of the fragility curves defined by equations 3.1 and 3.2. DPMs are obtained from the fragility curves by using equation 3.4. Simulation is a numerical technique for conducting experiments on a digital computer. Rubinstein (1981) defines simulation as a technique that performs sampling experiments on the model of the system. Stochastic simulation, also known as Monte Carlo simulation, includes the sampling of variables from probability distributions. Historically, the Monte Carlo technique has been considered a method for the solution of a model using random numbers.

In damage analysis, the uncertainties associated with structural capacities and demands need to be modeled. Structural capacities and demands can be characterized by a number of parameters which have an important effect on the response statistics and overall reliability of the system. Structural capacities are defined in terms of the capacities of members as part of the structure. Much greater uncertainty is associated with seismic demands than with other demands on the structure. Artificial ground motion simulation is carried out to incorporate this uncertainty. Gaussian models with modulating functions and autoregressive moving average (ARMA) models are used for this purpose. This study treats the uncertainties associated with structural capacities and demands independently.

The Monte Carlo technique as applied for the development of fragility curves and DPMs involves the selection of values of the input capacity random variables required for non-linear dynamic analyses, the generation of artificial ground motion, and the simulation of damage to a structure. An overview of the Monte Carlo simulation technique is presented in Figure 3.2. Examples of input random variables to model capacities for reinforced concrete structures include the strengths of steel and concrete. The procedure for the generation of artificial time histories is discussed later in the chapter. The means, variances, and distribution functions of the output random variable, the quantitative measures of damage in this study, are estimated from the simulations for an ensemble of
time histories corresponding to a given level of ground motion. The probabilities of different damage states are evaluated from the probability distributions of the damage measure.

Figure 3.2: Steps in the Monte Carlo simulation technique.

The direct Monte Carlo technique requires a large number of simulation cycles to achieve an acceptable level of confidence in the estimated probabilities. In this study, the Latin hypercube technique is used to reduce the number of simulation cycles. Iman and Conover (1980) provide a good description of the Latin hypercube sampling technique. Using the Latin hypercube technique for selecting values of the input variables, the estimators from the simulation are close to the real values of the quantities being estimated. The Latin hypercube technique uses stratified sampling of the input variables which usually results in a significant decrease in the variance of estimators. This decrease in variance is accomplished because stratified sampling forces the entire range of the input variables to be represented in the set of values for the input variables. Furthermore, through random permutations, the Latin hypercube technique assures that every stratum of one variable has some possibility of being coupled with each stratum of all other variables.
If there are only two input variables, this method of sampling is known as the Latin square technique.

The Latin hypercube sampling scheme involves the partitioning of the range of each variable into $N$ non-overlapping intervals, corresponding to $N$ simulation cycles, such that all intervals have the same probability of occurrence. The intervals for a general probability density function are shown in Figure 3.3. $N$ different values for each random variable in each of the $N$ non-overlapping intervals are then randomly selected. The generation of the values of the random variable is accomplished by generating $N$ uniform random numbers between 0 and 1 which are transformed to the random numbers in the non-overlapping intervals by using the following equation:

$$U_m = \frac{U}{N} + \frac{m-1}{N}$$  \hspace{1cm} (3.5)

where:
- $m$ = interval number,
- $U$ = uniform random number in the range (0,1), and
- $U_m$ = random number in the $m$th interval.

Only one generated value falls within each interval because $\frac{m-1}{N} < U_m < \frac{m}{N}$. The values of the random variables are generated by evaluating the inverse of the cumulative distribution functions at the generated values (given by equation 3.5). This inverse transformation can be expressed as follows:

$$x_m = F_X^{-1}(U_m)$$  \hspace{1cm} (3.6)

where:
- $x_m$ = $m$th generated value for variable $X$ and
- $F_X^{-1}$ = inverse of the cumulative distribution function for variable $X$.

The Latin hypercube samples are obtained by random permutation of the generated values of all the random variables. A procedure for obtaining a random sequence of the generated values of a random variable is illustrated by the following shuffling algorithm.
Let \( \{x_1, x_2, \ldots, x_N\} \) be the initial order of the values generated for random variable \( X \). The random sequence \( r_1, r_2, \ldots, r_N \) is produced by the following steps:

1. Set \( r_1 = x_1, r_2 = x_2, \ldots, r_N = x_N \) and \( m = N \).
2. Generate an integer \( I \) uniformly distributed between 1 and \( m \). Interchange \( r_I \) and \( r_m \).
3. Set \( m = m-1 \). If \( m = 1 \), return \( r_1, r_2, \ldots, r_N \) and exit. Otherwise go to 2.

The generated values of all the variables are placed in a random sequence. The \( n^{th} \) sample is now obtained by selecting the \( n^{th} \) value of all the random variables.

![Figure 3.3: General probability density function with N intervals used in the Latin hypercube sampling scheme.](image)

**3.3 Characterization of Damage**

Various damage measures were discussed in Chapter 2. The Krawinkler index (1987) is a measure frequently used to quantify damage in steel components. For reinforced concrete structures, the Park and Ang model has been used widely in recent years because it lends itself to simple numerical computation and because it has been calibrated using data from various structures damaged during past earthquakes.

In order to estimate economic loss or casualties as a result of structural damage, the structural damage must be expressed in terms of discrete damage states. Discrete damage states allow the damage sustained by a structure to be expressed in terms of the nature and
extent of the damage suffered by its components. Thus, structural damage, which is a continuous function of building response, is quantified by discrete damage states. The five discrete damage states used in this study are: none, minor, moderate, severe, and collapse. To obtain these discrete damage states, ranges for the damage measures discussed earlier need to be specified.

In this study, the different damage states of a concrete building are identified based on the Park-Ang global damage indices of the overall structure. Park et al. (1984) calibrated the damage index with the observed damage to nine reinforced concrete buildings caused by different earthquakes. The report by Park et al. (1987) gave the semantic definitions for the ranges of damage corresponding to different values of the Park and Ang damage index. Gunturi (1992) further investigated these damage states and simplified them according to his findings. Further calibration of the Park-Ang damage index was performed by De Leon and Ang (1993) using data from eight reinforced concrete buildings that sustained different levels of damage during the 1985 Mexico City earthquake. Stone and Taylor (1994) also calibrated the Park-Ang damage index based on an extensive study of reinforced concrete columns. The ranges of the Park and Ang index for different damage states have been established to reflect damage to concrete frames more realistically and are presented in Table 3.1. Table 3.1 also presents the physical description of the different damage states as proposed by Park et al. (1984)

<table>
<thead>
<tr>
<th>Damage State</th>
<th>Range of the Park and Ang index</th>
<th>Physical Description of the Damage State</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minor</td>
<td>0.1 - 0.2</td>
<td>Minor cracks throughout building, partial crushing of concrete in columns</td>
</tr>
<tr>
<td>Moderate</td>
<td>0.2 - 0.5</td>
<td>Extensive large cracks, spalling of concrete in weaker elements</td>
</tr>
<tr>
<td>Severe</td>
<td>0.5 - 1.0</td>
<td>Extensive crushing of concrete, disclosure of buckled reinforcement</td>
</tr>
<tr>
<td>Collapse</td>
<td>&gt; 1.0</td>
<td>Partial or total collapse of building</td>
</tr>
</tbody>
</table>

Hatamoto et al. (1990) defined four damage states based on the Chung, Meyer and Shinozuka (1987) damage index. The four damage states that they considered are: Minor,
Repairable, Irrepairable and Unsafe. The ranges of the damage index for these four damage states are presented in Table 3.2.

Table 3.2: Chung, Meyer and Shinozuka’s (1987) damage index for different damage states as defined by Hatamoto et al. (1990).

<table>
<thead>
<tr>
<th>Damage State</th>
<th>Range of the damage index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minor</td>
<td>0.0 - 0.2</td>
</tr>
<tr>
<td>Repairable</td>
<td>0.2 - 0.5</td>
</tr>
<tr>
<td>Irrepairable</td>
<td>0.5 - 1.0</td>
</tr>
<tr>
<td>Structure Unsafe</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Another method for the identification of the Minor and Moderate damage states investigated in this study is based on crack width in the elements. The advantage of using crack width as a damage measure is that it can be directly related to the dollar-loss ratio. Various techniques for estimating crack width in concrete members are available. Some are based on empirical relationships (e.g., Gergely and Lutz, 1968) while others (e.g., Bazant and Oh, 1983a and 1983b) are based on analytical formulations using the concept of fracture energy. The prediction of crack width based on the formulation presented by Oh and Kang (1987) is presented below.

The formulas for the prediction of crack width proposed by Oh and Kang are based on the cracking theory developed by Bazant and Oh (1983a and 1983b). Tests on reinforced concrete beams were also carried out by Oh and Kang to check the validity of the proposed formulas. Their equation that gives the best prediction of the maximum crack width in a member is given as:

\[
\frac{w_{\text{max}}}{d} = a_0 (\varepsilon_s - 0.0002) R \tag{3.7}
\]

where:
- \(w_{\text{max}}\) = maximum crack width,
- \(d\) = diameter of reinforcing bar,
- \(\varepsilon_s\) = strain in tensile reinforcement,
- \(R\) = \(h_2/h_3\).
\[ a_0 = 159 \left( \frac{t_b}{h_2} \right)^{4.5} + 2.83 \left( \frac{A_1}{A_{s1}} \right)^{1/3} \]

- \(A_1\) = effective area of concrete surrounding one reinforcing bar,
- \(A_{s1}\) = average area of one tensile reinforcing bar,
- \(t_b\) = concrete cover for tensile reinforcement,
- \(h_2\) = distance from extreme tension fiber to the neutral axis, and
- \(h_3\) = distance from the centroid of steel to the neutral axis.

It may be pointed out that the above formulation is applicable for cracking under static loads. The effect of dynamic loading on the crack widths should be investigated. Beshara (1993) provided a relationship between the dynamic cracking strain and the static cracking strain as a function of effective strain rate. In general, the crack widths can be computed based on the residual strain in the member after dynamic analysis.

The *Manual for Repair Methods of Civil Engineering Structures Damaged by Earthquakes* (1987) suggested that cracks with widths in the range of 0.5 mm - 0.8 mm can be repaired with epoxy injection. That manual also suggested that cracks with widths greater than 0.8 mm require V-cut before repair. Thus, if the maximum crack width in an element is in the range 0.5 mm - 0.8 mm, it is in *Minor* damage state. This definition of *Minor* damage implies that the damage can be repaired by epoxy injection. A crack width larger than 0.8 mm can be defined to be the lower bound of the *Moderate* damage state. A more detailed study is needed in order to arrive at crack widths under dynamic loading and to correlate crack widths with *Minor* and *Moderate* damage states. The damage states for the structure can be estimated based on the proportion of the elements in different damage states, and the importance of those elements. Since elements in the lower part of the structure are vital for the functionality of the entire structure, these elements should be assigned a larger importance factor.

### 3.4 Characterization of Ground Motion

In order to characterize earthquake ground motion for the purposes of evaluating structural performance, the amplitude, frequency content, and duration of ground motion must be described. Thus, it is difficult to specify a single parameter that captures the above important characteristics of ground motion. Various parameters used to
characterize ground motion were discussed in Chapter 2. Although the methodology can be used with any ground motion parameter, spectral acceleration and RMS acceleration are used to characterize the ground motion in the development of the methodology. MMI is used to identify the different levels of ground motion for the DPMs. The relationship between MMI and these parameters needs to be investigated in order to obtain DPMs from fragility curves. The relationship between spectral acceleration and MMI is presented in Chapter 5. DPMs for concrete frame structures are then developed using the relationship between spectral acceleration and MMI.

Furthermore, earthquake ground motion time histories are needed for the analysis. If a large sample of earthquake ground motion time histories that cover all the different parameter ranges is available, these time histories can be discriminated according to distance to the fault, local soil parameters and spectral characteristics, and then can be used for the dynamic analysis of the structure and the evaluation of the fragility curves. Such a consistent ensemble of time histories is not currently available, even though there are a large number of recordings obtained from recent earthquakes. Thus, it is proposed that ensembles of time histories be simulated at each specified ground motion parameter level.

3.4.1 Models for Simulation of Ground Motion

Several procedures are available for the generation of artificial time histories. These include the geophysical models, the stationary Gaussian models with modulating functions, and the ARMA models. Deodatis and Shinozuka (1989) developed a stochastic wave model with evolutionary power to simulate ground motion. This model is useful for simulating ground motions for large scale structures where the spatial variation of ground motion is important.

3.4.1.1 Geophysical Models

Several techniques using geophysical models are available for the simulation of earthquake ground motions. These include ray tracing techniques, Green’s function techniques, and the normal mode method (Suzuki and Kiremidjian, 1989). It is difficult to simulate long duration and wide-band frequency waves with ray tracing methods. Green’s function methods become difficult to apply when a multilayered earth structure is
considered. When the normal mode method is used, it is difficult to generate high frequency waves at intermediate and far distances unless a large number of modes are used. The major difficulty with the normal mode method is the enormous computational effort involved in obtaining the normal modes for the earth. The geophysical models are too complex, computationally involved, and regionally dependent, thus making it difficult to implement them in this study.

3.4.1.2 Stationary Gaussian Models with Modulating Functions

Stationary Gaussian models with modulating functions have been proposed by Shinozuka and Sato (1967), Liu and Jhaveri (1969), and Vanmarcke (1976) among many others. In stationary Gaussian models with modulating functions, the ground motion is expressed as follows:

\[ X(t) = I(t) \sum_{n} A_n \sin(\omega_n t + \phi_n) \]  

(3.8)

where:

- \( A_n \) = amplitude of the \( n \)th sinusoid,
- \( \omega_n \) = frequency of the \( n \)th sinusoid,
- \( \phi_n \) = phase angle of the \( n \)th sinusoid, assumed to be uniformly distributed between 0 and \( 2\pi \), and
- \( I(t) \) = envelope function.

The amplitudes are determined from the power spectral density as follows:

\[ \frac{A_n^2}{2} = G(\omega_n)\Delta\omega \]  

(3.9)

where:

- \( G(\omega) \) = one sided power spectral density.

The product, \( G(\omega_n)\Delta\omega \), can be thought of as the contribution of the sinusoid with frequency \( \omega_n \) to the total power. The nonstationarity is introduced by using an envelope function \( I(t) \). SIMQKE (1976) is one of the programs which uses this procedure.
SIMQKE can be used to generate time histories corresponding to a given response spectrum. The probability distributions of the dynamic amplification factors discussed later in this chapter can be used to obtain an ensemble of response spectra corresponding to a given spectral ordinate. The parameters of the envelope function should be chosen to satisfy the strong motion duration. The relationship between spectral acceleration and strong motion duration is discussed in Section 3.4.2.5.

3.4.1.3 ARMA Models

ARMA models are often applied to generate artificial time histories (e.g., Polhemus and Cakmak, 1981 and Conte et al., 1992). ARMA models consist of a discrete stationary linear transfer function applied to a white noise process. A white noise process is a random process in which all frequencies contribute equally to the mean square value of the process. A white noise process has an infinite variance due to the contribution of all frequencies and therefore is not physically realizable. The autocorrelation and power spectral density functions of white noise process $W(t)$ are expressed mathematically by means of the following two equations:

$$ R_{WW}(\tau) = 2\pi \phi_0 \delta(\tau) \quad (3.10) $$

and

$$ \phi_{WW}(\omega) = \phi_0 \quad (3.11) $$

where:

$\delta(t) = $ Dirac delta function and

$\phi_0 = $ constant power spectral density of the white noise process.

A shot noise process with homogeneous Poisson arrival times tends to a Gaussian white noise as the mean occurrence rate $\lambda$ tends to infinity and $\sigma^2$ tends to zero in such a way that $\lambda\sigma^2$ remains a constant (Housner & Jennings, 1964).

The stationary ARMA model of order (p,q) is represented by the following equation:
\[ a_k - \phi_1 a_{k-1} - \ldots - \phi_p a_{k-p} = e_k - \theta_1 e_{k-1} - \ldots - \theta_q e_{k-q} \] (3.12)

where:

\( a_k = a(k\Delta t), \ k = 0,1,2, \ldots = a \) discrete stationary correlated process,

\( e_k = e(k\Delta t) = a \) zero-mean Gaussian white-noise process with variance \( \sigma_e^2 \),

\( \Delta t = \) sampling time interval,

\( \phi_i, \ i = 1, \ldots, p = autoregressive \) parameters, and

\( \theta_i, \ i = 1, \ldots, q = moving \) average parameters.

A special case of ARMA models is the stationary ARMA(2,1) model defined by the following difference equation:

\[ a_k - \phi_1 a_{k-1} - \phi_2 a_{k-2} = e_k - \theta_1 e_{k-1} \] (3.13)

This model is completely defined by the two autoregressive parameters, the moving average parameter, and the variance of the white noise process.

The process \( a \) in equation 3.13 should be stationary and invertible in order to be physically realizable. The stationarity conditions ensure that the process \( a \) has a finite variance. The stationarity is controlled by the autoregressive part only and is achieved when the following conditions are satisfied:

\[ \phi_1 + \phi_2 < 1 \]
\[ \phi_1 - \phi_2 < 1 \]
\[ |\phi_2| < 1 \] (3.14)

The idea of invertibility is illustrated by means of a first order moving average process represented by the following equation.

\[ a_k = e_k - \theta_1 e_{k-1} \] (3.15)

Equation 3.15 can be written in terms of the previous values of \( e_k \) as shown in the following equation:
\[ a_k = e_k - \theta_1 a_{k-1} - \theta_1^2 a_{k-2} - \cdots - \theta_1^n a_{k-n} - \theta_1^{n+1} e_{k-n-1} \]  

(3.16)

If \( a_k \) is not to depend on some point in the remote past, \( \theta_1 \) must be less than one in absolute value. If \( n \) is allowed to go to infinity, the last term in equation 3.16 vanishes and \( a_k \) can be written as an infinite autoregressive process with declining weights as shown in the following equation:

\[ a_k = \sum_{n=1}^{\infty} -\theta_1^n a_{k-n} + e_k \]  

(3.17)

The reason for excluding the non-invertible processes is that they are not physically realizable. In a non-invertible process a small perturbation in the distant past can have a tremendous effect on the present process \( a_k \).

Conte et al. (1992) show that a linear, viscously damped single-degree-of-freedom (SDOF) system is the underlying physical system for the ARMA(2,1) model. For example, the natural frequency, \( \omega_g \), and the damping, \( \xi_g \), of the underdamped SDOF system can be represented by the following equations when \( \phi_1^2 + 4\phi_2 < 0 \), and \( \phi_2 < 0 \):

\[ \omega_g = \frac{1}{2\Delta t} \sqrt{[\ln(-\phi_2)]^2 + 4\lambda_d^2} \]  

(3.18)

and

\[ \xi_g = \frac{-\ln(-\phi_2)}{\sqrt{[\ln(-\phi_2)]^2 + 4\lambda_d^2}} \]  

(3.19)

where:

\[ \lambda_d = \arccos \left( \frac{\phi_1}{2\sqrt{-\phi_2}} \right), \quad 0 \leq \lambda_d \leq \pi \]

Recorded earthquake time histories exhibit nonstationarities in amplitude and frequency content. In order to incorporate these two nonstationarities, a dynamic version of the ARMA model is used. This model is represented by the following equation:
\[ a_k - \phi_{1,k} a_{k-1} - \ldots - \phi_{p,k} a_{k-p} = e_k - \theta_{1,k} (\sigma_{k-1} e_{k-1}) - \ldots - \theta_{q,k} (\sigma_{k-q} e_{k-q}) \]  

(3.20)

The nonstationarity in amplitude is represented by the variance envelope of the underlying white-noise process, \( \sigma_{e,k}^2 \), and the nonstationarity in frequency content is modeled by the time varying ARMA parameters, \( \phi_{i,k} \) and \( \theta_{i,k} \). The uncoupling of these two nonstationarities is possible if the standard deviation envelope, \( \sigma_{e,k} \), is slowly varying in time compared to the periods of the earthquake motion.

### 3.4.2 Modeling of Uncertainty in Ground Motion

The uncertainty associated with seismic demands on a structure is much larger than the uncertainties associated with demands imposed by dead and live loads. Ground motion can be simulated from power spectral density functions, or from normalized spectral shapes, or from recorded time histories. The procedure for incorporating the uncertainties when simulating earthquake ground motion is discussed in the following sections. In addition, the relationships between RMS acceleration and strong motion duration and those between spectral acceleration and strong motion duration are also examined. These relationships are necessary when simulating ground motions corresponding to a specified level of RMS acceleration or spectral acceleration.

#### 3.4.2.1 Uncertainties in Kanai-Tajimi Parameters

The Kanai-Tajimi (Tajimi, 1960) power spectral density is one of the most commonly used functions to characterize the power of the ground motion at different frequencies. This function is defined by the following equation:

\[
S(\omega) = \frac{1 + 4\xi_g^2 \left( \frac{\omega}{\omega_g} \right)^2}{\left[ 1 - \left( \frac{\omega}{\omega_g} \right)^2 \right]^2 + 4\xi_g^2 \left( \frac{\omega}{\omega_g} \right)^2} \cdot S_0 \tag{3.21}
\]

where:

- \( S_0 \) = intensity of the ideal white noise excitation at the bedrock level,
\( \omega_g \) = predominant ground natural frequency, and
\( \xi_g \) = effective damping coefficient of the ground.

The Kanai-Tajimi power spectral density is defined by two random variables \( \omega_g \) and \( \xi_g \). Lai (1982) proposed a gamma probability density function for \( \omega_g \) and a lognormal probability density function for \( \xi_g \). He used 22 rock site records to arrive at the means and standard deviations of \( \omega_g \) and \( \xi_g \). The mean and standard deviation of \( \omega_g \) are estimated as 26.7 rad/sec and 10.6 rad/sec, respectively. The mean and standard deviation of \( \xi_g \) are found to be 0.35 and 0.14, respectively. These distributions can be used to arrive at different power spectral density functions for each simulation. A time history can then be simulated for each power spectral density function.

3.4.2.2 Uncertainties in Dynamic Amplification Factors

The spectral values at different periods are frequently represented by means of dynamic amplification factors. The dynamic amplification factors represent the normalized spectral values at specified damping, obtained as a ratio of the spectral acceleration to the peak ground acceleration. Figure 3.4 shows the dynamic amplification factors obtained at a damping ratio of 5% of the critical damping from the firm site records of the Loma Prieta, Whittier Narrows, and Morgan Hill earthquakes. Although the average spectral shapes shown in Figure 3.4 appear to be smooth in each period band, the individual time histories may have sharp peaks in their spectra. Our interest, however, is in the average response of the structures over all the time histories. Since a structure is likely to be subjected to many different ground motions during its economic life, it is important to consider an ensemble of ground motions that have a wide range of characteristics.

Kiremidjian and Shah (1980) demonstrated the applicability of lognormal distribution to dynamic amplification factors. The firm site records from the Loma Prieta, Whittier Narrows, and Morgan Hill earthquakes are used for obtaining the parameters of the lognormal distributions of the dynamic amplification factors at different periods. The computed mean and standard deviations of the dynamic amplification factors are shown in Figure 3.4. As a part of this study, Kolmogorov-Smirnov analysis was performed on the dynamic amplification data at four periods: 0.5, 1.0, 1.5 and 2.0 seconds. The sample cumulative frequencies and the theoretical distribution functions are shown in Figures 3.5 through 3.8. The lognormal model was verified at the 5% significance level.

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Figure 3.4: Parameters of the dynamic amplification factors for firm sites.

Figure 3.5: Comparison of the empirical and fitted probability distributions of DAF at $T = 0.5$ seconds.
Figure 3.6: Comparison of the empirical and fitted probability distributions of DAF at $T = 1.0$ seconds.

Figure 3.7: Comparison of the empirical and fitted probability distributions of DAF at $T = 1.5$ seconds.
3.4.2.3 Moving Window Technique for Estimating ARMA Parameters

Another approach for incorporating the uncertainties in earthquake ground motion is to estimate ARMA parameters from an ensemble of recorded time histories. The estimated ARMA parameters then can be used to simulate ground motion. The moving-window technique is frequently used to estimate ARMA parameters. This technique assumes that the time history is stationary within a time window. In this study, the moving-window technique is used to estimate the ARMA parameters of recorded ground motion from the Loma Prieta, Whittier Narrows, and Morgan Hill earthquakes. Details on ground motion simulation using the moving-window technique are presented later in this chapter.

3.4.2.4 Relationship Between RMS Acceleration and Strong Motion Duration

Trifunac and Brady's (1975) definition of strong motion duration, given by equations 2.1 through 2.3, is used in this study. The probability distributions of the strong motion duration \( T_s \) for firm sites given the RMS acceleration, \( f_{T_s|RMS}(t_s|RMS) \), are derived.
using the firm site data from the Loma Prieta, Whittier Narrows, and Morgan Hill earthquakes. Only the free-field records are considered in this study to avoid possible soil-structure interaction effects. The conditional probability distributions of strong motion duration at a given RMS acceleration are obtained by assuming the RMS acceleration and strong motion duration to be jointly lognormally distributed. The distributions of the strong motion duration given the RMS acceleration are therefore lognormal with parameters $\lambda_{Ts}'$ and $\xi_{Ts}'$ defined by the following two expressions:

$$\lambda_{Ts}' = \mathbb{E}[\ln(T_s) | \text{RMS} = r] = \lambda_{Ts} + \rho \frac{\xi_{Ts}}{\xi_{\text{RMS}}} (\ln(r) - \lambda_{\text{RMS}})$$  \hspace{1cm} (3.22)

$$\xi_{Ts}'^2 = (1 - \rho^2) \xi_{Ts}^2$$  \hspace{1cm} (3.23)

where:

- $T_s$ = strong motion duration,
- $\lambda_{\text{RMS}}$ = expected value of $\ln$ RMS acceleration,
- $\lambda_{Ts}$ = strong motion duration,
- $\lambda_{Ts}$ = expected value of $\ln$ strong motion duration,
- $\xi_{\text{RMS}}^2$ = variance of $\ln$ RMS acceleration,
- $\xi_{Ts}^2$ = variance of $\ln$ strong motion duration, and
- $\rho$ = correlation coefficient of $\ln$ RMS acceleration and $\ln$ strong motion duration.

The expected value of the strong motion duration given the RMS acceleration can thus be written as:

$$\mathbb{E}[T_s | \text{RMS} = r] = \exp\left(\lambda_{Ts}' + \frac{1}{2} \xi_{Ts}'^2\right)$$

$$= r^{\frac{\xi_{Ts}^2}{\xi_{\text{RMS}}^2}} \exp\left[\frac{1}{2} (1 - \rho^2) \xi_{Ts}^2 + \lambda_{Ts} - \rho \lambda_{\text{RMS}} \frac{\xi_{Ts}}{\xi_{\text{RMS}}}\right]$$  \hspace{1cm} (3.24)

and the variance of $T_s$ given the RMS acceleration, can be written as:

$$\text{var}(T_s | \text{RMS} = r) = \omega(\omega - 1)r^{2\frac{\xi_{Ts}^2}{\xi_{\text{RMS}}^2}} \exp\left[2\left(\lambda_{\text{RMS}} - \rho \lambda_{\text{RMS}} \frac{\xi_{Ts}}{\xi_{\text{RMS}}}\right)\right]$$  \hspace{1cm} (3.25)
where:
\[ \omega = \exp\left(1 - \rho^2 \right) \xi_T^2 \]  
(3.26)

The dependence of the parameters of the distributions on the distance from the rupture zone is taken into account by dividing the recording stations into two groups: one with distance to the rupture zone less than 50 km and the other with distance greater than 50 km. Due to the limited amount of data available for each group, further subdivision into more groups based on the distance from the rupture zone will result in a very small data set and consequently the parameters determined from each group will be quite unreliable. Moreover, further subdivision would require more simulations which would be economically prohibitive. The data considered for computing the parameters for the distributions are presented in Tables 3.3 and 3.4. These tables show the different recording stations located on firm sites for the Loma Prieta, Whittier Narrows and Morgan Hill earthquakes. These tables also show the Trifunac and Brady strong motion duration and the corresponding RMS acceleration values for the two directions of recorded ground motion at each recording station.

The parameters of the distributions are determined using the method of maximum likelihood. For RMS acceleration, the parameters are determined using the following two equations:

\[ \lambda_{RMS} = \frac{\sum_{i=1}^{n} \ln(RMS_i)}{n} \]  
(3.27)

and

\[ \xi_{RMS}^2 = \frac{1}{n} \sum_{i=1}^{n} (\ln(RMS_i) - \lambda_{RMS})^2 \]  
(3.28)

where RMS\(_i\) is the RMS acceleration value of the \(i^{th}\) ground motion and \(n\) is the number of samples in the data.

The parameters \(\lambda_{T_s}\) and \(\xi_{T_s}\) are computed in a similar manner. Figures 3.9 and 3.10 show the plot of the data. As can be seen from these figures, there is a strong negative correlation between natural log of RMS acceleration and natural log of strong motion duration, \(T_s\). Table 3.5 presents the values of the parameters of the distributions.
Table 3.3: Trifunac and Brady's (1975) strong motion duration and the RMS acceleration values for the Loma Prieta, Whittier Narrows, and Morgan Hill earthquakes for sites with distances to rupture zones less than 50 km.

<table>
<thead>
<tr>
<th>SITE NAME AND NUMBER</th>
<th>EARTHQUAKE</th>
<th>AZIMUTH</th>
<th>DURATION (sec)</th>
<th>RMS (cm/sec²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corralitos - Eureka Canyon 57007</td>
<td>Loma Prieta</td>
<td>360</td>
<td>6.86</td>
<td>163.240</td>
</tr>
<tr>
<td>Crystal Springs - Pulgas 58378</td>
<td>Loma Prieta</td>
<td>0</td>
<td>15.12</td>
<td>32.871</td>
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<tr>
<td>Crystal Springs - Skyline 58373</td>
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<td>90</td>
<td>16.10</td>
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<td>Gilroy #1 - G.C. Water Tank 47379</td>
<td>Loma Prieta</td>
<td>0</td>
<td>16.76</td>
<td>26.600</td>
</tr>
<tr>
<td>Gilroy #6 - San Ysidro 57383</td>
<td>Loma Prieta</td>
<td>360</td>
<td>3.70</td>
<td>159.600</td>
</tr>
<tr>
<td>Monterey City Hall 47377</td>
<td>Loma Prieta</td>
<td>360</td>
<td>6.64</td>
<td>94.680</td>
</tr>
<tr>
<td>SAGO South 47189</td>
<td>Loma Prieta</td>
<td>0</td>
<td>12.66</td>
<td>44.473</td>
</tr>
<tr>
<td>Saratoga - Aloha Ave. 58065</td>
<td>Loma Prieta</td>
<td>0</td>
<td>13.28</td>
<td>18.542</td>
</tr>
<tr>
<td>Stanford Linear Accelerator 1601</td>
<td>Loma Prieta</td>
<td>351</td>
<td>14.80</td>
<td>18.065</td>
</tr>
<tr>
<td>Santa Cruz - UCSC 58135</td>
<td>Loma Prieta</td>
<td>0</td>
<td>16.84</td>
<td>16.979</td>
</tr>
<tr>
<td>Woodside - Fire Station 58127</td>
<td>Loma Prieta</td>
<td>360</td>
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<tr>
<td>Mt. Wilson 24399</td>
<td>Whittier Narrows</td>
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<td>9.70</td>
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<td>Whittier Narrows</td>
<td>360</td>
<td>8.86</td>
<td>86.349</td>
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<tr>
<td>LA-116th St. School 14403</td>
<td>Whittier Narrows</td>
<td>90</td>
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<tr>
<td>LA - Baldwin Hills 24157</td>
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<td>50.524</td>
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<tr>
<td>Long Beach Park (14241)</td>
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<td>270</td>
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<td>52.197</td>
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Table 3.4: Trifunac and Brady’s (1975) strong motion duration and the RMS acceleration values for the Loma Prieta and Whittier Narrows earthquakes for sites with distances to rupture zones greater than 50 km.

<table>
<thead>
<tr>
<th>SITE NAME AND NUMBER</th>
<th>EARTHQUAKE</th>
<th>AZIMUTH</th>
<th>DURATION (sec)</th>
<th>RMS (cm/sec²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Berkeley - LB Lab</td>
<td>Loma Prieta</td>
<td>90</td>
<td>8.16</td>
<td>32.149</td>
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<tr>
<td>58471</td>
<td></td>
<td>0</td>
<td>18.22</td>
<td>14.115</td>
</tr>
<tr>
<td>Hayward - CSUH Stadium</td>
<td>Loma Prieta</td>
<td>90</td>
<td>19.30</td>
<td>17.421</td>
</tr>
<tr>
<td>58219</td>
<td></td>
<td>0</td>
<td>19.06</td>
<td>15.371</td>
</tr>
<tr>
<td>Piedmont Jr. High School</td>
<td>Loma Prieta</td>
<td>45</td>
<td>11.98</td>
<td>15.043</td>
</tr>
<tr>
<td>58338</td>
<td></td>
<td>315</td>
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<td>15.667</td>
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<td>Point Bonita</td>
<td>Loma Prieta</td>
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<td>58043</td>
<td></td>
<td>207</td>
<td>10.08</td>
<td>18.947</td>
</tr>
<tr>
<td>S.F. - Cliff House</td>
<td>Loma Prieta</td>
<td>90</td>
<td>7.26</td>
<td>32.441</td>
</tr>
<tr>
<td>58132</td>
<td></td>
<td>360</td>
<td>10.28</td>
<td>22.420</td>
</tr>
<tr>
<td>S.F. - Diamond Heights</td>
<td>Loma Prieta</td>
<td>90</td>
<td>9.42</td>
<td>25.080</td>
</tr>
<tr>
<td>58130</td>
<td></td>
<td>360</td>
<td>8.78</td>
<td>29.915</td>
</tr>
<tr>
<td>S.F. - Pacific Heights</td>
<td>Loma Prieta</td>
<td>270</td>
<td>11.10</td>
<td>16.557</td>
</tr>
<tr>
<td>58131</td>
<td></td>
<td>360</td>
<td>12.40</td>
<td>13.467</td>
</tr>
<tr>
<td>S.F. - Presidio</td>
<td>Loma Prieta</td>
<td>90</td>
<td>8.56</td>
<td>41.682</td>
</tr>
<tr>
<td>58222</td>
<td></td>
<td>360</td>
<td>10.54</td>
<td>28.719</td>
</tr>
<tr>
<td>S.F. - Rincon Hill</td>
<td>Loma Prieta</td>
<td>90</td>
<td>11.52</td>
<td>18.173</td>
</tr>
<tr>
<td>58181</td>
<td></td>
<td>360</td>
<td>13.88</td>
<td>14.943</td>
</tr>
<tr>
<td>S.F. - Telegraph Hill</td>
<td>Loma Prieta</td>
<td>90</td>
<td>9.48</td>
<td>16.586</td>
</tr>
<tr>
<td>58133</td>
<td></td>
<td>360</td>
<td>11.46</td>
<td>11.312</td>
</tr>
<tr>
<td>S.S.F. Sierra Point</td>
<td>Loma Prieta</td>
<td>205</td>
<td>9.48</td>
<td>21.470</td>
</tr>
<tr>
<td>58539</td>
<td></td>
<td>115</td>
<td>11.68</td>
<td>15.039</td>
</tr>
<tr>
<td>Yerba Buena Island</td>
<td>Loma Prieta</td>
<td>90</td>
<td>8.32</td>
<td>16.986</td>
</tr>
<tr>
<td>58163</td>
<td></td>
<td>360</td>
<td>21.66</td>
<td>6.509</td>
</tr>
<tr>
<td>Vagupark (24047)</td>
<td>Whittier Narrows</td>
<td>0</td>
<td>9.22</td>
<td>12.877</td>
</tr>
</tbody>
</table>

Table 3.5: Parameters for the estimation of the mean and the variance of the conditional strong motion duration given the RMS acceleration.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Distance less than 50 km.</th>
<th>Distance greater than 50 km.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_{\text{RMS}}$</td>
<td>3.633</td>
<td>2.916</td>
</tr>
<tr>
<td>$\xi_{\text{RMS}}$</td>
<td>0.751</td>
<td>0.398</td>
</tr>
<tr>
<td>$\lambda_{T_e}$</td>
<td>2.378</td>
<td>2.413</td>
</tr>
<tr>
<td>$\xi_{T_e}$</td>
<td>0.361</td>
<td>0.293</td>
</tr>
<tr>
<td>Correl. coeff. $\rho_{\text{RMS}, \text{in RMS}}$</td>
<td>-0.705</td>
<td>-0.686</td>
</tr>
</tbody>
</table>

CHAPTER 3. Methodology for Developing Motion-Damage Relationships 54
**Figure 3.9:** Correlation between Trifunac and Brady's (1975) strong motion duration and the RMS acceleration for firm sites with distances to rupture zones less than 50 km.

**Figure 3.10:** Correlation between Trifunac and Brady's (1975) strong motion duration and the RMS acceleration for firm sites with distances to rupture zones greater than 50 km.
3.4.2.5 Relationship Between Spectral Acceleration and Strong Motion Duration

The spectral acceleration in three period bands is used in order to investigate its relationship with strong motion duration. The three period ranges used are representative of three classes of reinforced concrete frames and are based on the study reported in FEMA 223 (1992). Details on arriving at the three ranges are provided in Chapter 4. The spectral acceleration appears to be poorly related to strong motion duration. Table 3.6 shows the average spectral acceleration in the three ranges along with the MMI values at the recording stations. The strong motion duration for the horizontal components of ground motion listed in Table 3.6 can be obtained from Tables 3.3 and 3.4. At a recording station, the two components of ground motion in Table 3.6 are listed in the same order as in Tables 3.3 and 3.4. Figures 3.11 through 3.13 show the plot of the data and the correlation between the strong motion duration and the spectral acceleration in the three period bands. A lognormal probability density independent of the spectral acceleration is assumed for the strong motion duration when generating time histories for a given spectral acceleration. The parameters of the distribution are $\lambda_{T_s} = 2.391$ and $\xi_{T_s} = 0.331$. Figure 3.14 presents the comparison of the lognormal distribution with these parameters and the observed distribution.

**Figure 3.11:** Correlation between Trifunac and Brady’s (1975) strong motion duration and the average $S_a$ in the period range 0.1-0.5 sec for firm sites.
**Figure 3.12:** Correlation between Trifunac and Brady's (1975) strong motion duration and the average $S_a$ in the period range 0.5-0.9 sec for firm sites.

**Figure 3.13:** Correlation between Trifunac and Brady's (1975) strong motion duration and the average $S_a$ in the period range 0.9-2.5 sec for firm sites.
Table 3.6: Average spectral acceleration and MMI values for the Loma Prieta, Whittier Narrows, and Morgan Hill earthquakes.

<table>
<thead>
<tr>
<th>SITE NAME AND NUMBER</th>
<th>EARTHQUAKE</th>
<th>MMI</th>
<th>Average $S_a$ (0.1≤T≤0.5) (g)</th>
<th>Average $S_a$ (0.5&lt;T≤0.9) (g)</th>
<th>Average $S_a$ (0.9&lt;T≤2.5) (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Berkeley - LB Lab 58471</td>
<td>Loma Prieta</td>
<td>7</td>
<td>0.228</td>
<td>0.306</td>
<td>0.129</td>
</tr>
<tr>
<td>Corralitos - Eureka Canyon 57007</td>
<td>Loma Prieta</td>
<td>8</td>
<td>1.506</td>
<td>0.934</td>
<td>0.228</td>
</tr>
<tr>
<td>Crystal Springs - Pulgas 58378</td>
<td>Loma Prieta</td>
<td>7</td>
<td>0.348</td>
<td>0.302</td>
<td>0.094</td>
</tr>
<tr>
<td>Crystal Springs - Skyline 58373</td>
<td>Loma Prieta</td>
<td>7</td>
<td>0.225</td>
<td>0.216</td>
<td>0.121</td>
</tr>
<tr>
<td>Gilroy #1 - G.C. Water Tank 47379</td>
<td>Loma Prieta</td>
<td>7</td>
<td>1.363</td>
<td>0.454</td>
<td>0.184</td>
</tr>
<tr>
<td>Gilroy #6 - San Ysidro 57383</td>
<td>Loma Prieta</td>
<td>6</td>
<td>0.457</td>
<td>0.222</td>
<td>0.113</td>
</tr>
<tr>
<td>Hayward - CSUH Stadium 58219</td>
<td>Loma Prieta</td>
<td>6</td>
<td>0.199</td>
<td>0.124</td>
<td>0.056</td>
</tr>
<tr>
<td>Monterey City Hall 47377</td>
<td>Loma Prieta</td>
<td>6</td>
<td>0.155</td>
<td>0.075</td>
<td>0.028</td>
</tr>
<tr>
<td>Piedmont Jr. High School 58338</td>
<td>Loma Prieta</td>
<td>7</td>
<td>0.150</td>
<td>0.098</td>
<td>0.054</td>
</tr>
<tr>
<td>Point Bonita 58043</td>
<td>Loma Prieta</td>
<td>6</td>
<td>0.151</td>
<td>0.165</td>
<td>0.136</td>
</tr>
<tr>
<td>SAGO South 47189</td>
<td>Loma Prieta</td>
<td>7</td>
<td>0.139</td>
<td>0.199</td>
<td>0.076</td>
</tr>
<tr>
<td>Santa Cruz - UCSC 58135</td>
<td>Loma Prieta</td>
<td>8</td>
<td>1.080</td>
<td>0.271</td>
<td>0.119</td>
</tr>
<tr>
<td>Saratoga - Aloha Ave. 58065</td>
<td>Loma Prieta</td>
<td>8</td>
<td>0.731</td>
<td>0.567</td>
<td>0.356</td>
</tr>
<tr>
<td>S.F. - Cliff House 58132</td>
<td>Loma Prieta</td>
<td>7</td>
<td>0.169</td>
<td>0.203</td>
<td>0.163</td>
</tr>
<tr>
<td>S.F. - Diamond Heights 58130</td>
<td>Loma Prieta</td>
<td>6</td>
<td>0.238</td>
<td>0.199</td>
<td>0.081</td>
</tr>
<tr>
<td>S.F. - Pacific Heights 58131</td>
<td>Loma Prieta</td>
<td>7</td>
<td>0.096</td>
<td>0.146</td>
<td>0.099</td>
</tr>
<tr>
<td>S.F. - Presidio 58222</td>
<td>Loma Prieta</td>
<td>7</td>
<td>0.403</td>
<td>0.380</td>
<td>0.178</td>
</tr>
</tbody>
</table>
Table 3.6: (cont'd) Average spectral acceleration and MMI values for the Loma Prieta, Whittier Narrows, and Morgan Hill earthquakes.

<table>
<thead>
<tr>
<th>SITE NAME AND NUMBER</th>
<th>EARTHQUAKE</th>
<th>MMI</th>
<th>Average $S_a$ (0.1≤T≤0.5) (g)</th>
<th>Average $S_a$ (0.5&lt;T≤0.9) (g)</th>
<th>Average $S_a$ (0.9&lt;T≤2.5) (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S.F. - Rincon Hill 58181</td>
<td>Loma Prieta 7</td>
<td>0.131</td>
<td>0.156</td>
<td>0.090</td>
<td></td>
</tr>
<tr>
<td>S.F. - Telegraph Hill 58133</td>
<td>Loma Prieta 7</td>
<td>0.145</td>
<td>0.120</td>
<td>0.073</td>
<td></td>
</tr>
<tr>
<td>S.S.F. Siera Point 58539</td>
<td>Loma Prieta 7</td>
<td>0.219</td>
<td>0.154</td>
<td>0.049</td>
<td></td>
</tr>
<tr>
<td>Stanford Linear Accelerator 1601</td>
<td>Loma Prieta 7</td>
<td>0.666</td>
<td>0.512</td>
<td>0.196</td>
<td></td>
</tr>
<tr>
<td>Woodside - Fire Station 58127</td>
<td>Loma Prieta 7</td>
<td>0.184</td>
<td>0.175</td>
<td>0.099</td>
<td></td>
</tr>
<tr>
<td>Yerba Buena Island 58163</td>
<td>Loma Prieta 7</td>
<td>0.134</td>
<td>0.145</td>
<td>0.069</td>
<td></td>
</tr>
<tr>
<td>Mt. Wilson 24399</td>
<td>Whittier Narrows 6</td>
<td>0.251</td>
<td>0.034</td>
<td>0.012</td>
<td></td>
</tr>
<tr>
<td>Inglewood 14196</td>
<td>Whittier Narrows 6</td>
<td>0.412</td>
<td>0.418</td>
<td>0.072</td>
<td></td>
</tr>
<tr>
<td>LA-116th St. School</td>
<td>Whittier Narrows 6</td>
<td>0.786</td>
<td>0.343</td>
<td>0.072</td>
<td></td>
</tr>
<tr>
<td>LA - Baldwin Hills 24157</td>
<td>Whittier Narrows 6</td>
<td>0.358</td>
<td>0.163</td>
<td>0.035</td>
<td></td>
</tr>
<tr>
<td>Long Beach Park (14241)</td>
<td>Whittier Narrows 6</td>
<td>0.099</td>
<td>0.087</td>
<td>0.021</td>
<td></td>
</tr>
<tr>
<td>Pacoima (24088)</td>
<td>Whittier Narrows 5</td>
<td>0.283</td>
<td>0.175</td>
<td>0.039</td>
<td></td>
</tr>
<tr>
<td>Ranchoff (23497)</td>
<td>Whittier Narrows 5</td>
<td>0.069</td>
<td>0.029</td>
<td>0.008</td>
<td></td>
</tr>
<tr>
<td>Vasqpark (24047)</td>
<td>Whittier Narrows 5</td>
<td>0.105</td>
<td>0.031</td>
<td>0.005</td>
<td></td>
</tr>
<tr>
<td>Corralitos - Eureka Canyon 57007</td>
<td>Morgan Hill 6</td>
<td>0.254</td>
<td>0.195</td>
<td>0.062</td>
<td></td>
</tr>
<tr>
<td>Gilroy #1 - G.C. Water Tank 47379</td>
<td>Morgan Hill 6</td>
<td>0.147</td>
<td>0.036</td>
<td>0.017</td>
<td></td>
</tr>
<tr>
<td>Gilroy #6 - San Ysidro 57383</td>
<td>Morgan Hill 6</td>
<td>0.710</td>
<td>0.605</td>
<td>0.261</td>
<td></td>
</tr>
<tr>
<td>Gilroy Gavilan College 47006</td>
<td>Morgan Hill 6</td>
<td>0.116</td>
<td>0.035</td>
<td>0.018</td>
<td></td>
</tr>
</tbody>
</table>

CHAPTER 3. Methodology for Developing Motion-Damage Relationships
Figure 3.14: Comparison of the observed and the estimated log normal distribution ($\lambda_{T_s} = 2.391$ and $\xi_{T_s} = 0.331$) of strong motion duration for firm sites.

3.4.3 Simulation of Ground Motion

Artificial time histories are required for the Monte Carlo simulation technique of obtaining motion-damage relationships. This section describes the simulation of ground motion time histories for two ground motion parameters: RMS acceleration and spectral acceleration. Figure 3.15 provides the overview of the procedure for ground motion simulation. For simulating time histories for a specified level of RMS acceleration, the ARMA parameters are computed for a Kanai-Tajimi power spectrum. There are two ways of simulating ground motion when spectral acceleration is used to characterize the ground motion. These include the stationary Gaussian models with modulating functions and the ARMA models.

3.4.3.1 Simulation of Time Histories with Specified RMS Acceleration and Duration

Artificial time histories corresponding to a specified RMS acceleration are generated using the ARMA model. The ARMA parameters are computed for the Kanai-Tajimi
For given RMS

Generation of values of Kanai-Tajimi parameters: $\omega_g$ and $\xi_g$, and simulation of strong motion variables

Stationary ARMA models with modulating functions

Simulated Ensemble of Earthquake Time Histories for Specified Ground Motion Parameter

For given $S_a$

Generation of ensemble of dynamic amplification factors, and simulation of strong motion variables

Gaussian stationary models with modulating functions

Nonstationary ARMA model

Estimation of ARMA parameters using the moving-window technique

Figure 3.15: Steps in the simulation of time histories.
power spectral density function. The parameters of the ARMA(2,1) model corresponding to the Kanai-Tajimi stochastic earthquake model (equation 3.21) can be computed from the following equations when the system is assumed to be underdamped (Conte et al., 1992):

\[
\phi_1 = 2 e^{-\xi_g \omega_g \Delta t} \cos \left( \omega_g \sqrt{1 - \xi_g^2} \Delta t \right) \tag{3.29}
\]

\[
\phi_2 = -e^{-2\xi_g \omega_g \Delta t} \tag{3.30}
\]

\( \theta_1 \) is the solution of:

\[
\theta_1^2 + \frac{2\rho_1 \phi_1 - \phi_1^2 + \phi_2^2 - 1}{\phi_1 - \rho_1 (1 - \phi_2)} \theta_1 + 1 = 0, \quad |\theta_1| < 1 \tag{3.31}
\]

where:

\[
\rho_1 = \rho(\Delta t) = \frac{1}{2} \frac{1 - 4\xi_g^2}{1 + 4\xi_g^2} \frac{\xi_g}{\sqrt{1 - \xi_g^2}} \sqrt{(\phi_1^2 + 4\phi_2)} + \frac{1}{2} \phi_1 \tag{3.32}
\]

The ARMA(2,1) spectrum is defined for all frequencies smaller or equal to the Nyquist frequency. Therefore the following condition should be satisfied:

\[
0 < \omega_g \sqrt{1 - \xi_g^2} \leq \frac{\pi}{\Delta t} \tag{3.33}
\]

The parameters \( \omega_g \) and \( \xi_g \) are the parameters used to define the Kanai-Tajimi power spectral density.

Thus, the following steps are involved in the generation of artificial time histories using ARMA models:

1. Generation of a stationary discrete white-noise \( \{e_k, k = 1, \ldots, N\} \) where \( e_k \) is the shot noise impulse at time \( t_k \).

2. Time modulation of the stationary white-noise by means of the following equation:
\[ w_k = \psi(t_k) e_k \quad k = 1, \ldots, N \quad (3.34) \]

where \( \psi(t_k) \) is the envelope function.

3. ARMA filtering of the non-stationary white-noise. The ARMA (2,1) model given by equation 3.14 is used in the simulation process.

The time enveloping function suggested by Shinozuka and Sato (1967) may be used in step 2. This enveloping function is given by:

\[ \psi(t) = e^{-\alpha t} - e^{-\beta t} \quad \beta > \alpha > 0 \quad (3.35) \]

The parameters \( \alpha \) and \( \beta \) in the above function need to be determined to correspond to the known strong motion duration.

**Duration Calibration**

The parameters \( \alpha \) and \( \beta \) of the envelope function can be determined by means of the following two equations based on the Trifunac and Brady definition of strong motion duration:

\[
0.05 \int_0^{T_d} |\psi(t)|^2 \, dt = \int_0^{T_1} |\psi(t)|^2 \, dt \quad (3.36)
\]

and

\[
0.95 \int_0^{T_d} |\psi(t)|^2 \, dt = \int_0^{T_2} |\psi(t)|^2 \, dt \quad (3.37)
\]

where:

- \( T_1 \) = start of the strong motion duration,
- \( T_2 \) = end of the strong motion duration,
- \( T_s \) = strong motion duration, and
- \( T_d \) = total duration of the motion.

Using equation 3.35, equations 3.36 and 3.37 can be expressed as follows:
\[
\frac{\alpha^2 - 2\alpha\beta + \beta^2 + 4\alpha\beta e^{-(\alpha+\beta)T_d} - \alpha\beta e^{-2\alpha T_d} - \alpha\beta e^{-2\beta T_d} - \alpha^2 e^{-2\alpha T_d} - \beta^2 e^{-2\alpha T_d}}{20 \alpha\beta(\alpha + \beta)} = 0
\]

and

\[
\frac{\alpha^2 - 2\alpha\beta + \beta^2 + 4\alpha\beta e^{-(\alpha+\beta)T_1} - \alpha\beta e^{-2\alpha T_1} - \alpha\beta e^{-2\beta T_1} - \alpha^2 e^{-2\beta T_1} - \beta^2 e^{-2\alpha T_1}}{19\alpha\beta(\alpha + \beta)} = 0
\]  

(3.38)  

(3.39)

Applying the conditions that \(T_2 - T_1\) is a known strong motion duration and \(\alpha \succ \beta \succ 0\), equations 3.38 and 3.39 are solved by a predictor-corrector method for the parameters \(\alpha\) and \(\beta\).

The values of \(\alpha\) and \(\beta\) obtained by solving equations 3.38 and 3.39 would satisfy the given strong motion duration if the time history is ergodic. However, in reality, the time history is not ergodic. Therefore, iterations need to be performed in the neighborhood of the values of \(\alpha\) and \(\beta\) until the duration of the generated ground motion is close to the desired duration.

**RMS Acceleration Calibration**

The ARMA parameters are computed for the Kanai-Tajimi power spectral density function using equations 3.29 through 3.31. The procedure to determine the variance of the shot noise process, \(e_k\) in equation 3.13, is presented in this section.

Let \(X(t)\) represent the stationary filtered process obtained after ARMA filtering and \(Y(t)\) be the process obtained after \(X(t)\) is modulated by the enveloping function. Thus

\[
Y(t) = \psi(t) X(t)
\]  

(3.40)

The Arias intensity, \(I\), of the final process \(Y(t)\) is given by:
\[ I = \int_0^{T_d} Y^2(t) dt \]  
(3.41)

where:

\[ T_d = \text{total duration of the simulated process.} \]

The expected value of the Arias intensity can be written as follows:

\[ E[I] = \int_0^{T_d} E[Y^2(t)] dt = \int_0^{T_d} \sigma_Y^2 dt \]  
(3.42)

Since \( Y(t) \) is a process with zero mean, \( E[Y^2] = \sigma_Y^2 \).

Nigam (1983) shows that

\[ \sigma_Y^2 = \sigma_X^2 |\psi(t)|^2 \]  
(3.43)

where:

\[ \sigma_X^2 = E[X^2(t)] \]

Equations 3.42 and 3.43 can be combined to yield:

\[ E[I] = \int_0^{T_d} E[\psi(t)X(t)]^2 dt = \sigma_X^2 \int_0^{T_d} |\psi(t)|^2 dt \]  
(3.44)

The variance \( \sigma_X \) can be related to the expected value of RMS acceleration based on equations 2.7 and 3.44. To a first order approximation for the expected values, the relationship between the two parameters is expressed as follows:

\[ E[\text{RMS}] = \sqrt{\frac{0.9}{T_s} E[I]} = \frac{\sigma_X}{\sqrt{T_s}} \sqrt{0.9 \int_0^{T_d} |\psi(t)|^2 dt} \]  
(3.45)
or

\[
\sigma_X = \text{E}[\text{RMS}] \sqrt{\frac{T_s}{t_d}} \int_0^{t_d} |\psi(t)|^2 \, dt
\]  

(3.46)

where:

- \( T_d \) = total duration of the simulated process and
- \( T_s \) = Trifunac-Brady strong motion duration.

A factor of 0.9 is used in equation 3.46 as the Trifunac-Brady definition of strong motion is the time interval to accumulate 90% of the total energy.

The variance of the stationary, filtered process \( X(t) \) can be related to the variance of the input shot noise process by means of the following equation (Conte et al., 1992):

\[
\sigma_X^2 = \frac{(1-\phi_2)(1+\phi_1^2)-2\phi_1\theta_1}{(1+\phi_2)((1-\phi_2)^2-\phi_1^2)} \sigma_c^2
\]  

(3.47)

where:

- \( \sigma_c^2 \) = variance of the shot noise process,
- \( \phi_1, \phi_2, \theta_1 \) = ARMA parameters, and
- \( \sigma_X^2 \) = variance of the stationary filtered process \( X(t) \).

The value of the variance of the shot noise process, \( \sigma_c^2 \), has to match the expected value of RMS acceleration. This value is determined by solving equations 3.46 and 3.47 simultaneously. It is obvious that using this value of the variance of the shot noise process will lead to the RMS acceleration being satisfied only in the ensemble mean.

In order to match the desired RMS acceleration, the process generated by using the calculated value of \( \sigma_c^2 \) needs to be modified. Let the generated process be denoted by \( Y_1(t) \) and its RMS acceleration by \( \text{RMS}_1 \). The ratio of the desired RMS acceleration to \( \text{RMS}_1 \) is then used to scale the amplitudes of the time history \( Y_1(t) \) to yield \( Y(t) \). The resulting time history \( Y(t) \) has the desired strong motion duration and RMS acceleration.
3.4.3.2 Simulation of Time Histories with Specified Spectral Acceleration

There are different ways of simulating the ground motion for a specified level of spectral acceleration. These include the stationary Gaussian models with modulating functions and the ARMA models. The input required to obtain time histories using the stationary Gaussian models with modulating functions is relatively simple, the target response spectrum and the envelope function are required. However, these models produce time histories which are nonstationary in amplitude but stationary in frequency content. Yeh and Wen (1990) show that the nonstationarity in frequency content has significant effect on the response of non-linear systems. To capture the nonstationarity in the frequency content of the ground motion, nonstationary ARMA models may be used to simulate time histories. However, in comparison to the Gaussian models with modulating functions, the nonstationary ARMA models require more input in terms of the time-varying ARMA parameters at different instants of time.

The response spectra for simulating time histories using the stationary Gaussian models with modulating functions are obtained from the dynamic amplification factors. The parameters of the lognormal distributions of the dynamic amplification factors at different periods are obtained from the mean and standard deviations shown in Figure 3.4. These lognormal distribution functions are used to generate values of the dynamic amplification factors at different periods. These dynamic amplifications factors are then scaled to obtain an ensemble of response spectra corresponding to a given average ordinate of spectral acceleration in the period range corresponding to each structure class (for example, \(0.1 \leq T \leq 0.5\) sec for low rise reinforced concrete frames). Earthquake time histories are then generated corresponding to these response spectra.

The moving-window technique or the Kalman filtering technique may be used to estimate the parameters of the nonstationary ARMA model. In this study, the moving time-window technique is used to estimate the parameters of the nonstationary ARMA model from ground motions recorded during the Loma Prieta, Whittier Narrows, and Morgan Hill earthquakes. The program MATLAB (1994) is used to estimate the ARMA parameters within each window. The ARMA parameters estimated for each window are assumed to be representative of the center point of the window. The parameter estimation is repeated for successive equidistant window positions. Based on a parametric study conducted to estimate the size of the ARMA model and the window size, it was found that
the ARMA(2,1) model with a window size of 3 seconds gives reasonable results in terms of the spectral acceleration of the simulated time histories. The window is moved by 0.2 seconds between successive window positions. The moving-window technique for an acceleration time history is shown in Figure 3.16. When generating time histories for a particular level of spectral acceleration, each simulated time history is scaled to match the desired spectral acceleration.

In addition, baseline correction is also performed using a high-pass, bi-lateral Butterworth filter to remove the low frequency components. The bi-lateral Butterworth filter is a pure amplitude filter and does not cause any phase shift in the resulting time history. Hamming (1987) provides a good description of different filters. The velocity and displacement time histories obtained by integrating the acceleration time history are also corrected. Using the least squares method, a straight line is fit to the displacement time history to obtain the corrected displacement. The slope of this straight line is added to the velocity, obtained by integration of the acceleration, to get the corrected velocity time history.

![Diagram showing the moving-window technique with time history and window positions](image)

- ARMA parameters estimated from each window are representative of these points

**Figure 3.16:** Representation of the moving-window technique.
RMS acceleration is used to normalize the spectral shapes because it is insensitive to isolated peaks in the ground motion and is an average statistic for the entire time history. A comparison of the mean normalized spectral shapes computed from recorded time histories, obtained from the Loma Prieta, Whittier Narrows, and Morgan Hill earthquakes, and an ensemble of simulated time histories is presented in Figure 3.17.

![Figure 3.17: Comparison of the recorded and simulated spectral shapes.](image)

3.5 Bayesian Technique for Updating Fragility Curves

Fragility curves and DPMs, ideally, should be developed from observed data on seismic damage. Since such data are very limited at the present time, they can at best be used to revise or update the analytical fragility curves developed using the methodology presented earlier in this chapter. Bayesian analysis is used to combine the analytical estimates with observed data on damage. The remaining portion of this chapter provides a general description of the Bayesian analysis method. The application of the Bayesian
analysis to update fragility curves is presented in Chapter 6. Chapter 6 also presents the probability distributions relevant to the fragility curves for reinforced concrete frames.

Bayes’ theorem provides an approach for updating subjective knowledge with experimental results. If the experimental outcome is a set of observed values \( x_1, x_2, \ldots, x_n \), from a population \( X \) with underlying probability density function \( f_{X|\Theta}(x|\Theta) \), the parameters of the distribution, represented by the vector \( \Theta \), are revised in light of the experimental results by the following expression:

\[
f_{\tilde{\Theta}}(\Theta) = \frac{\prod_{i=1}^{n} f_{X}(x_i \mid \Theta)}{\int_{\Theta} \prod_{i=1}^{n} f_{X}(x_i \mid \Theta) f_{\tilde{\Theta}}(\Theta) \, d\Theta}
\]

(3.48)

where

- \( f_{\tilde{\Theta}}(\Theta) \) = prior density function of the parameters \( \Theta \),
- \( f_{\tilde{\Theta}}(\Theta) \) = posterior density function of \( \Theta \), and
- \( f_{X|\Theta}(x|\Theta) \) = probability distribution function of the basic random variable \( X \).

The density \( f_{\tilde{\Theta}}(\Theta) \) incorporates all prior knowledge about the unknown parameters. The prior knowledge can be in the form of subjective information. Equation 3.48 can be written as:

\[
f_{\tilde{\Theta}}(\Theta) = kL(X|\Theta)f_{\tilde{\Theta}}(\Theta)
\]

(3.49)

where:

- \( k = \) normalizing constant = \( \left[ \int_{\Theta} \left( \prod_{i=1}^{n} f_{X|\Theta}(x_i \mid \Theta) \right) f_{\tilde{\Theta}}(\Theta) \, d\Theta \right]^{-1} \)

and

- \( L(X \mid \Theta) = \) likelihood function = \( \prod_{i=1}^{n} f_{X|\Theta}(x_i \mid \Theta) \)

CHAPTER 3. Methodology for Developing Motion-Damage Relationships 70
The likelihood function is proportional to the probability of making specific observations, \( X = x \), given the values \( \theta \) of the parameters. The initial belief about the stochastic behavior of the parameters of the distributions is thus updated using the observations.

Considerable mathematical simplification can be achieved if the distributions of the parameters are appropriately chosen with respect to the underlying random variable \( X \). Such pairs of distributions are known as conjugate distributions. By choosing prior distributions that are conjugate of the distribution of the underlying random variable, one thereby obtains convenient posterior distributions, which are usually of the same mathematical form as the prior.

The uncertainty associated with parameters \( \Theta \) is combined with the inherent variability of the underlying random variable, \( X \), to obtain the total uncertainty associated with \( X \). Using the total probability theorem, the posterior probability density function of \( X \) is expressed as follows:

\[
f''_X(x) = \int \limits_\Theta k \, L(X|\Theta) f'_X(x | \Theta) \, f(\Theta) \, d\Theta \tag{3.50}
\]

### 3.6 Summary

This chapter presented a method for the development of fragility curves. In contrast to previous approaches for developing fragility curves and DPMs, the method presented in this chapter does not rely on heuristics or on empirical data. The methodology can be applied to a wide range of structural classes. The methodology is presented for two ground motion parameters: spectral acceleration and RMS acceleration. However, it is possible to use one of the other ground motion parameters presented in Chapter 2. The methodology is used to obtain fragility curves and DPMs for reinforced concrete frames in Chapters 4 and 5. Chapter 6 presents the application of Bayesian analysis to update the fragility curves using actual data on building damage.

This chapter also reviewed the various techniques for simulating ground motion. Although there are a large number of recordings obtained from recent earthquakes, a consistent ensemble of time histories that cover all the different parameter ranges that can
be discriminated according to distance to the fault, local soil parameters and spectral characteristics is currently not available. Thus, it is proposed that ensembles of time histories be simulated at each specified ground motion parameter level. The different models for ground motion simulation include the geophysical model, the stationary Gaussian models with modulating functions, and the ARMA models. Whereas the stationary Gaussian models with modulating functions incorporate nonstationarity of the amplitudes of the time histories, the nonstationary ARMA models are capable of accounting for nonstationarities in both the amplitude and the frequency content of the time histories. As these two nonstationarities significantly influence the response of nonlinear systems, the non-stationary ARMA(2,1) model is used to simulate time histories in this study.
CHAPTER 4
MODELING OF REINFORCED CONCRETE FRAMES FOR FRAGILITY ANALYSIS

This chapter presents the application of the methodology presented in Chapter 3 to develop motion-damage relationships for reinforced concrete (RC) frames. Three classes of RC frames are considered. These include low rise frames that are 1-3 stories tall, mid rise frames that are 4-7 stories tall, and high rise frames that are 8 stories or taller. This classification is consistent with that defined in ATC-13 (1985) and is similar to that used in the standardized earthquake loss estimation methodology (NIBS, 1995). The motion-damage relationships for RC frames are obtained by using a representative building in each building class. The fragility curves are developed by using the Monte Carlo simulation technique. The nonstationary ARMA(2,1) model is used to generate artificial time histories corresponding to a specified value of average spectral acceleration.

4.1 Ground Motion Characterization for RC Frames

The average spectral acceleration ordinate in the period range corresponding to the three classes of reinforced concrete frames is used to characterize the ground motion for fragility curves. The three period bands used in this study are 0.1-0.5 seconds, 0.5-0.9 seconds, and 0.9-2.5 seconds. These period bands are based on the study reported in FEMA 223 (1992) and are estimated to reflect the natural periods of buildings belonging to the three classes of reinforced concrete frames. FEMA 223 suggests the following equation for estimating the mean value of the periods of RC frames:

\[ T = 0.035 h_n^{0.75} \]  \hspace{1cm} (4.1)

where \( h_n \) is the height of the building in feet. Equation 4.1 is based on periods computed from accelerograph records obtained from RC frames during the 1971 San Fernando earthquake. The design codes (e.g., SEAOC, 1990) estimate the periods of reinforced concrete buildings conservatively using a coefficient of 0.03 in contrast to the value of 0.035 used in this study. FEMA 223 also estimates the average story height in reinforced concrete frames to be 9.65 feet. However, an integer value of 10 feet is used for the story height in this study.
Although the acceleration spectrum is likely to vary considerably over the long-period bands, the pseudo-velocity spectrum is expected to show much less variation as shown in Figures 4.1 and 4.2. The normalized spectral curves are computed from the firm site records of six earthquakes. In comparison to Chapter 3 where ground motions recorded during the Loma Prieta, Whittier Narrows, and Morgan Hill earthquakes were used, recorded motions from three more earthquakes, Landers, Petrolia, and Northridge, are used here to arrive at the normalized spectral shapes. The average spectral acceleration in the three period bands for these ground motions are shown in Table 4.1. The additional ground motions are considered here so as to have a larger number of ground motion records which are used in the sensitivity analysis later in this chapter. However, the number of recorded ground motions is still not large enough to be used in the development of fragility curves. Therefore, ground motion simulation is carried out to develop the fragility curves.

### Table 4.1: Average spectral acceleration values for the Landers, Petrolia, and Northridge earthquakes.

<table>
<thead>
<tr>
<th>SITE NAME AND NUMBER</th>
<th>EARTHQUAKE</th>
<th>( \text{Average } S_a ) (0.1( \leq T \leq 0.5 )) (g)</th>
<th>( \text{Average } S_a ) (0.5( &lt; T \leq 0.9 )) (g)</th>
<th>( \text{Average } S_a ) (0.9( &lt; T \leq 2.5 )) (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amboy 21081</td>
<td>Landers</td>
<td>0.356</td>
<td>0.236</td>
<td>0.163</td>
</tr>
<tr>
<td>Joshua Tree 22170</td>
<td>Landers</td>
<td>0.533</td>
<td>0.700</td>
<td>0.355</td>
</tr>
<tr>
<td>Puerta La Cruz 12168</td>
<td>Landers</td>
<td>0.084</td>
<td>0.023</td>
<td>0.014</td>
</tr>
<tr>
<td>Silent Valley - Poppet Fl. 12206</td>
<td>Landers</td>
<td>0.086</td>
<td>0.033</td>
<td>0.020</td>
</tr>
<tr>
<td>Twenty Nine Palms 22161</td>
<td>Landers</td>
<td>0.128</td>
<td>0.043</td>
<td>0.020</td>
</tr>
<tr>
<td>Cape Mendocino 89005</td>
<td>Petrolia</td>
<td>2.320</td>
<td>1.111</td>
<td>0.395</td>
</tr>
<tr>
<td>Shelter Cove Airport 89530</td>
<td>Petrolia</td>
<td>1.124</td>
<td>0.366</td>
<td>0.257</td>
</tr>
<tr>
<td>LA - 116th St. School 14403</td>
<td>Northridge</td>
<td>0.390</td>
<td>0.042</td>
<td>0.013</td>
</tr>
<tr>
<td>LA - Baldwin Hills 24157</td>
<td>Northridge</td>
<td>0.396</td>
<td>0.215</td>
<td>0.083</td>
</tr>
<tr>
<td>LA - Hollywood Storage 24303</td>
<td>Northridge</td>
<td>0.373</td>
<td>0.250</td>
<td>0.060</td>
</tr>
</tbody>
</table>

CHAPTER 4. Modeling of Reinforced Concrete Frames for Fragility Analysis
The dynamic amplification factors shown in Figure 4.1 show very minor variations from those in Figure 3.4. The ratio of the largest to the smallest mean dynamic amplification factors in the period band corresponding to high rise frames is about 4. The same ratio for normalized spectral velocity is only about 1.25. For mid rise frames, the differences between the normalized spectral acceleration and spectral velocity are less significant. The largest to the smallest dynamic amplification factors have a ratio of almost 1.35 compared to a ratio of 1.25 for the normalized spectral velocity in the mid rise period band.

The spectral acceleration used for the fragility curves can easily be converted into equivalent pseudo-spectral velocity at the centroidal periods in each period band. The relationship between the average spectral acceleration and the average spectral velocity for mid rise and high rise frames is shown in Figures 4.3 and 4.4. As expected, the correlation coefficient between the spectral acceleration and velocity is more than 99%. The slope of the regression line is equal to $\overline{T}/2\pi$, where $\overline{T}$ is the centroidal period used to convert spectral velocity into spectral acceleration in each period band. From the slopes of the regression lines shown in Figures 4.3 and 4.4, the centroidal periods for the mid rise and high rise frames are estimated as 0.70 and 1.7 seconds, respectively. These centroidal periods correspond to the midpoint in the respective period bands.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure41.png}
\caption{Dynamic amplification factors for firm sites.}
\end{figure}
Figure 4.2: Normalized spectral velocity curves for firm sites.

Figure 4.3: Relationship between spectral acceleration and velocity for $0.5 \leq T \leq 0.9$ seconds.
Figure 4.4: Relationship between spectral acceleration and velocity for $0.9 \leq T \leq 2.5$ seconds.

Another way of determining the relationship between the average spectral acceleration and average spectral velocity is by assuming that spectral acceleration is equal to the pseudo-spectral acceleration and that the spectral velocity is constant in the respective period band. Thus, the average spectral acceleration can be expressed by means of the following two equations for mid rise and high rise frames, respectively.

$$S_a = \frac{1}{0.4} \int_{0.5}^{0.9} \frac{2\pi}{T} S_v \,dT = 9.233 \, S_v, \quad 0.5 \leq T \leq 0.9 \quad (4.2)$$

$$S_a = \frac{1}{1.6} \int_{0.9}^{2.5} \frac{2\pi}{T} S_v \,dT = 4.012 \, S_v, \quad 0.9 \leq T \leq 2.5 \quad (4.3)$$

The average values of spectral acceleration as functions of spectral velocity computed by using the regression analysis, presented above, are about 97% and 90% of the values given in equations 4.2 and 4.3 for the mid rise and the high rise frames, respectively.
4.2 Computation of Section Properties Needed for Damage Evaluation

The two section properties needed for evaluating the Park-Ang member damage are the ultimate rotation capacity, \( \theta_u \), and the yield moment, \( M_y \). In this study, these properties are computed in the program IDARC2D (Kunnath et al., 1994). A brief description of the procedure adopted to compute these properties is presented in the following sub-sections. The computation of the properties for an example section is presented in Appendix A.

4.2.1 Computation of Ultimate Rotation Capacity of a Section

The following assumptions are made in IDARC2D (Kunnath and Reinhorn, 1994) when arriving at the ultimate rotation capacity. The ultimate curvature capacity of a member is assumed to be reached when a specified ultimate compressive strain in the extreme concrete fiber is reached or when the specified ultimate strength of one of the reinforcement bars is reached. The ultimate compressive strain in concrete is specified as the level of strain when the stress has dropped to 20% of the compressive strength of concrete. The details on arriving at this level of strain are presented below. The ultimate curvature is converted into ultimate rotation by assuming the plastic hinge length at each member end to be equal to 9% of the member length.

The behavior of the reinforcing steel is specified in terms of a trilinear stress-strain relationship with an initial elastic portion, a horizontal yield plateau, and a linear strain hardening portion. The stress-strain relationship for concrete as defined by the Kent and Park (1971) relationship and summarized by Park and Paulay (1975) is used in IDARC2D. This relationship is presented in Figure 4.5. The expressions for stress at different levels of strain are given below.

For concrete strain, \( \varepsilon_c \leq 0.002 \), the stress, \( f_c \), is given by:

\[
f_c = f'_c \left[ \frac{2 \varepsilon_c}{0.002} - \left( \frac{\varepsilon_c}{0.002} \right)^2 \right]
\]

(4.4)

where:
\[ f'_c = \text{concrete cylinder strength.} \]

Equation 4.4 is the expression for the ascending portion of the stress-strain curve in Figure 4.5. The Kent and Park model assumes that the confining steel has no effect on the shape of this portion of the curve.

When the concrete strain exceeds 0.002, the stress, \( f_c \), is given by:

\[ f_c = f'_c \left[ 1 - Z(\varepsilon_c - 0.002) \right] \tag{4.5} \]

where:
\[ Z = \frac{0.5}{\varepsilon_{50c} - 0.002}. \]

Equation 4.5 describes the descending portion of the stress-strain curve when the concrete strain is greater than 0.002 and the concrete stress has not dropped to 20\% of the concrete compressive strength. The parameter \( Z \) specifies the slope of this linear portion. The parameter \( \varepsilon_{50c} \) in the expression for \( Z \) specifies the strain in confined concrete when the stress has dropped to 50\% of the compressive strength of concrete. \( \varepsilon_{50c} \) is defined by the following expression:

\[ \varepsilon_{50c} = \frac{3 + 0.002f'_c}{f'_c - 1000} + \frac{3}{4} \rho_s \sqrt{\frac{b''}{s_h}} \tag{4.6} \]

where:
\[ \rho_s = \text{ratio of volume of transverse reinforcement to volume of concrete core measured to outside of hoops,} \]
\[ b'' = \text{width of confined core measured to outside of hoops, and} \]
\[ s_h = \text{spacing of hoops.} \]

The first term in equation 4.4 is the strain in unconfined concrete when the stress has dropped to 50\% of the concrete strength. This term is represented as \( \varepsilon_{50u} \) in Figure 4.5. The second term in equation 4.6 provides the additional ductility capacity due to confinement provided by the hoop reinforcement. However, the maximum stress reached by both the confined and the unconfined concrete is the cylinder strength \( f'_c \).
Figure 4.5: Kent and Park’s stress-strain relationship for concrete (from Park and Paulay, 1975).

Paulay and Priestley (1992) suggest an increase in the compressive strength of concrete due to confinement. They also suggest that the ultimate compressive strain in confined concrete may be obtained by equating the strain energy capacity of the transverse steel at fracture to the increase in energy absorbed by the concrete. According to them, the ultimate compressive strain is given by the following expression:

\[ \varepsilon_{cu} = 0.004 + 1.4\rho_s f_{yh} \varepsilon_{sm} / f'_{cc} \]  \hspace{1cm} (4.7)

where:

- \( \rho_s \) = volumetric ratio of confining steel,
- \( f_{yh} \) = yield strength of confining steel,
- \( \varepsilon_{sm} \) = steel strain at maximum tensile stress, and
- \( f'_{cc} \) = compressive strength of confined concrete.
According to Paulay and Priestley, typical values of the ultimate compression strain in confined concrete range from 0.012 to 0.05, an increase of 4 to 16 times the values assumed for unconfined concrete.

When computing the ultimate curvature for a section, Bertero and Bertero (1992) also check that the buckling of compression steel has not occurred when the ultimate strain is reached in concrete or the ultimate strength of tensile steel is reached. They use the following expression to compute the critical buckling stress:

$$f_{cr} = 2E_t(e_s)\pi^2 \frac{(0.25\phi_L)^2}{s^2}$$

(4.8)

where:

$E_t(e_s)$ = tangent modulus of steel stress-strain relation,
$\phi_L$ = diameter of longitudinal bar, and
$s$ = spacing of stirrups.

### 4.2.2 Computation of Yield Moment for a Section

In this study, the yield moment capacity of a section is defined as the moment required to produce yielding of the tensile steel. If a section has a very high reinforcement ratio, or is subjected to a high axial load, yielding of the tensile steel may not occur till a high compressive strain has been developed in concrete. For these cases, Paulay and Priestley (1992) suggest that that the yield moment may be defined as the applied moment required to produce a compressive strain in concrete equal to $0.0015$.

### 4.3 Evaluation of Structural Damage for Buildings on Firm Sites

The fragility curves and damage probability matrices (DPMs) are first developed for buildings located on firm sites. The dynamic amplification factors for these sites were
presented in Chapter 3. However, the influence of soil sites is presented in the later part of this chapter.

4.3.1 Description of the Sample Structures

For the purposes of this study, a typical structure was considered to have five bays in the longitudinal direction and one bay in the transverse direction. The sample building for each class of concrete frames was designed according to the 1990 SEAOC Recommendations for special moment resisting frames. The thickness of the floor slab in these buildings was assumed to be 7 in. A uniformly distributed dead load of 30 psf was superimposed on the self weight of the structure and used in the design of the members. In addition, reduced live loads for member design were represented by a uniformly distributed load of 25 psf. The plans of the three structures are the same, and the plan of a typical structure is shown in Figure 4.6. A typical interior frame for each of the three structures was used in the nonlinear time history analysis to estimate damage at different levels of ground shaking. As explained in Section 4.1, a story height of 10 feet is used for these buildings. Figure 4.7 shows the elevations for the three frames used in the analysis.

![Figure 4.6: Plan of the three frames.](image-url)
Figure 4.7: Elevation of the frames used in the analyses.
4.3.2 Modeling of Uncertainties in Structural Capacities and Demands

The structural capacities and demands need to be specified stochastically before the Monte Carlo simulation can be performed. The following two paragraphs discuss the uncertainties associated with the capacity and demand parameters for reinforced concrete frames.

**Capacity parameters:** The different parameters which affect the resistance of the structure include the compressive strength of concrete, the yield strength of reinforcing steel, hysteretic behavior, damping ratio, physical dimensions of the different components, and the amount of reinforcing steel. The compressive strength of concrete and the yield strength of steel are the only parameters treated as the strength random variables in this study. Following Galambos et al. (1982), a normal probability distribution for concrete strength and a lognormal probability distribution for steel strength are used in this study. Concrete strength has a mean of 1.14 times the nominal concrete strength and a coefficient of variation of 0.14. Steel strength has a mean of 1.05 times the nominal strength and a coefficient of variation of 0.11.

**Demand parameters:** The uncertainty associated with dead and live loads is considerably smaller compared to the uncertainty in seismic load. In this study, only the earthquake load is modeled as a nonstationary stochastic process. The different models for simulating ground motion were discussed in Chapter 3.

4.3.3 Structural Modeling in DRAIN-2DX

The beam-column element (Type 02) available in DRAIN-2DX is used to model the beams and columns in a reinforced concrete frame. This element as modeled in DRAIN-2DX consists of two fibers. One of the fibers has an elastoplastic moment-curvature relationship, and the other always remains elastic to model the strain hardening in the moment-curvature relationship of the member. Thus, this element is able to capture “lumped” plasticity at the ends of the member. In reality, the member has distributed plasticity, i.e., the member can have nonlinearities along the length as well as across the depth of the member. The effect of distributed plasticity on nonlinear dynamic response is evaluated by considering the sample five story frame discussed earlier. Throughout
this research, a bilinear hysteretic model is used for the nonlinear dynamic analysis performed by DRAIN-2DX.

Distributed plasticity is considered by subdividing each member into three elements before performing nonlinear analysis using DRAIN-2DX. Each member consists of two short elements at the ends and one long element in the middle. The length of the short elements is 15% of the member length. This approach captures the spread of plasticity along the length of a member. The spread of plasticity across the depth of a member can be considered by using more fibers to represent the cross section of the member. The present approach uses two fibers, one of which remains elastic and the other is elastoplastic.

The effect of subdividing the member into shorter elements on the nonlinear dynamic response of the structure was studied. The two cases used in the study are shown graphically in Figure 4.8. Nonlinear time history analyses were carried out for each of the two cases. Ensembles of ground motion corresponding to different values of spectral acceleration in the period range 0.5-0.9 seconds were used in the analyses.

![Diagram of two cases](image)

*Figure 4.8: Two cases used to study the effect of distributed plasticity.*

The Monte Carlo method, with random sampling, would make independent runs at different levels of discretization and compare the results obtained. The aim is to determine the differences among the different cases. Therefore, correlated sampling, one of the most powerful variance reduction techniques, was used in the Monte Carlo technique. If the simulations use the same random numbers, their results can have a high positive correlation, and a reduction in variance of the differences between two simulation results can be achieved. The aim of correlated sampling is to produce a high positive correlation between two similar processes so that the variance of the difference
is much smaller than the case where the processes are statistically independent. Therefore, the recorded ground motions from the six earthquakes, discussed earlier in Section 4.1, are scaled to different levels of spectral acceleration and used for the purpose of the comparison between the two cases. Using three elements to model each member, DRAIN-2DX produced results similar to those from IDARC2D (Kunnath and Reinhorn, 1994) and CU-DYNAMIX (EL-Tawil, 1996). The details on the comparison of the results are presented in Section 4.3.4.

Drift Ratios

The interstory drift ratios for the two cases are shown in Figures 4.9 for the first story where the largest difference was observed. The overall drift ratios, obtained as the ratio of the roof displacement to the structure height, for the two cases are shown in Figure 4.10. The interstory drift ratios for all stories decrease from Case 1, where only one element is used, to Case 2 where 3 elements are used. The effect of accounting for the spread of plasticity along the length of members decreases from bottom to top of the structure. Thus, for the top story there are no significant differences between the two cases because of the decrease in seismic demand from bottom to top of the structure.

![Drift Ratios Graph](figure.png)

**Figure 4.9:** Comparison of the interstory drift ratios for the first story of the five story frame.

*CHAPTER 4. Modeling of Reinforced Concrete Frames for Fragility Analysis*
Figure 4.10: Comparison of the overall drift ratios for the five story frame.

Park-Ang Damage Index

This section examines how damage, in terms of the Park-Ang index, is affected by subdividing the member into three elements. The Park-Ang damage index is computed at the member end irrespective of the number of elements. The hysteretic energies dissipated and the plastic rotations at the element ends are accumulated at the member end closest to the element end for the computation of the damage index. Figure 4.11 shows the hysteretic energies dissipated by the frame elements of the first story where the maximum dissipation of hysteretic energy occurs. The total dissipated energies are similar for all cases and thus independent of the level of discretization.

Figures 4.12 shows the overall Park-Ang story damage indices for the two cases. The overall Park-Ang damage index for Case 1 is significantly larger than that Case 2. When the member is discretized, plastic deformations can occur inside the member. The damage indices computed using three elements are found to be consistent with those from IDARC2D. The comparison between DRAIN-2DX and IDARC2D is presented in Section 4.3.4.1.
Figure 4.11: Comparison of the hysteretic energy dissipation in the first story of the five story frame.

Figure 4.12: Comparison of the overall Park-Ang damage index for the five story frame.
4.3.4 Comparison of DRAIN-2DX with Other Computer Programs

Several computer programs are available for evaluating the nonlinear dynamic response of a structure, including DRAIN-2DX (Prakash and Powell, 1993), IDARC2D (Kunnath and Reinhorn, 1994), and CU-DYNAMIX (El-Tawil, 1996). In this study, the program DRAIN-2DX is used for performing the nonlinear dynamic analysis, the results of which are then used to evaluate the Park-Ang damage index. However, it is useful to compare the results obtained from DRAIN-2DX to the those obtained from the other two programs in order to examine the bounds within which the response of the structures may lie. The comparisons of the results are presented in the next two sections.

4.3.4.1 Comparison Between DRAIN-2DX and IDARC

The program IDARC2D uses a trilinear moment-curvature relationship in contrast to the bilinear relationship used in DRAIN-2DX. The trilinear relationships in IDARC2D are shown in Figure 4.13. Furthermore, IDARC2D uses a general distributed flexibility model to model distributed plasticity, whereas DRAIN-2DX uses a lumped plasticity model. However, the effect of the spread of plasticity along the length of a member is captured in DRAIN-2DX by discretizing a member into smaller elements as discussed in the previous section. Also, the hysteretic model in IDARC2D is able to incorporate stiffness degradation, strength deterioration, and pinching. The parameters used to describe these characteristics of the hysteretic model are shown in Figure 4.13.

The results of the nonlinear dynamic analyses performed using IDARC2D and DRAIN-2DX are compared in terms of the Park-Ang damage index for the twelve story, one bay frame shown in Figures 4.6 and 4.7. Correlated sampling is again used to compare the results from these two programs. Whereas IDARC2D computes the Park-Ang damage index, the member outputs from DRAIN-2DX are used to compute the damage index. Thus, the plastic rotations and the dissipated hysteretic energies of each member obtained from DRAIN-2DX are used to compute the damage index. As only mass-proportional damping is implemented in IDARC2D, the mass-proportional damping coefficient is computed so as to provide 5% damping in the first vibrational mode. Furthermore, IDARC2D does not use an event-to-event strategy used in DRAIN-2DX. Therefore, an integration step of 0.0001 seconds is used when performing the nonlinear dynamic analysis in IDARC2D.
Figure 4.13: Hysteretic parameters in IDARC2D.

Figure 4.14 shows the comparison of the damage indices computed from the two programs when no deterioration in the hysteretic behavior is considered. Figure 4.15 shows the comparison when nominal degradation is considered in IDARC2D. Kundnath et al. (1992) suggest that for nominal degradation in stiffness, a value of 2 be used for the stiffness degradation parameter, $\alpha$. They also suggest that a value of 0.1 be used for the strength degradation parameter, $\beta$, to obtain nominal degradation in strength. In the present study, the parameter $\beta$ used for strength degradation is different from the parameter $\beta$ in the expression for the Park-Ang damage index. As mentioned in Chapter 2, a value of 0.15 is used for the parameter $\beta$ to compute the Park-Ang damage index.

From Figures 4.14 and 4.15, it is observed that the damage index computed using the analysis results from DRAIN-2DX are very similar to those obtained from IDARC2D. At present there is some numerical instability in IDARC2D when performing nonlinear dynamic analysis in certain cases. These instabilities are expected to be due to errors in modeling of some segments of the hysteretic behavior. During the simulations, it was observed that if one of these segments was traversed, the response of the structure would "blow up". Therefore, the program is not used to generate fragility curves in this study.
Figure 4.14: Comparison of the Park-Ang damage index for the twelve story frame with no deterioration in the hysteretic behavior.

Figure 4.15: Comparison of the Park-Ang damage index for the twelve story frame with nominal deterioration in the hysteretic behavior.
4.3.4.2 Comparison Between DRAIN-2DX and CU-DYNAMIX

The computer program CU-DYNAMIX (El-Tawil, 1996) is capable of performing two- and three-dimensional analyses on building systems. The stiffness of the elements are based on a flexibility approach, i.e., the flexibility matrix is used to obtain the stiffness matrix of each member. The element flexibility matrix is calculated from the sectional flexibility matrix which relates sectional forces and strains. A bounding strength surface defined for bisymmetric sections is used to determine the stiffnesses in the principal bending and axial directions of a section. The bounding surface is estimated based on the interaction among the axial force and the bending moments in the two principal directions. The stiffness in a principal direction is defined as a function of the distance between the current location inside the bounding surface and the bounding surface in that direction. Stiffness degradation is defined as a function of the accumulated plastic strain energy per unit length of the member.

Currently, the program CU-DYNAMIX can only be used in an interactive mode, which makes the implementation of the program for the development of fragility curves difficult. Also, the program requires that the sections be bisymmetric. In reinforced concrete frames, the beams are almost never bisymmetric. However, it is still worth comparing the responses from CU-DYNAMIX with DRAIN-2DX. For this purpose, a five story frame with bisymmetric beams is subjected to two ground motions recorded during the Loma Prieta earthquake. The ground motion time histories used were recorded at Gavilan College in Gilroy and the UCSC Lab in Santa Cruz. The time histories along with the comparisons of the displacement time histories at the roof and the third floor are presented in Figures 4.16 through 4.21. In the nonlinear dynamic analyses in both the programs, the interaction between the moment and the axial force in the members was not considered. Furthermore, three elements were used to model each member in CU-DYNAMIX.

Figures 4.17 and 4.18 show that DRAIN-2DX predicts a permanent deformation, whereas CU-DYNAMIX does not. Otherwise, the peak displacements and the number of cycles that the structure goes through are similar for the two programs. The permanent deformation shown in Figure 4.17 is due to cumulative permanent deformations experienced at the lower stories as the permanent deformation shown in Figure 4.18 is smaller than in Figure 4.17. The permanent deformation for the first story is much smaller than in Figures 4.17 and 4.18.
Figure 4.16: Ground motion time history recorded at Gavilan College in Gilroy during the Loma Prieta earthquake.

Figure 4.17: Lateral displacement at the roof of the five story frame subjected to the ground motion time history recorded at Gavilan College in Gilroy.
Figure 4.18: Lateral displacement at the third floor of the five story frame subjected to the ground motion time history recorded at Gavilan College in Gilroy.

Figure 4.19: Ground motion time history recorded at the UCSC Lab in Santa Cruz during the Loma Prieta earthquake.
Figure 4.20: Lateral displacement at the roof of the five story frame subjected to the ground motion time history recorded at the UCSC Lab in Santa Cruz.

Figure 4.21: Lateral displacement at the third floor of the five story frame subjected to the ground motion time history recorded at the UCSC Lab in Santa Cruz.
Figures 4.20 and 4.21 show that the responses from DRAIN-2DX and CU-DYNAMIX are very similar in terms of the peak displacements and number of cycles. However, the responses from the two programs are slightly out of phase. The differences in the response predicted by the two programs are likely due to the modeling of strain-hardening and the formulation of the basic inelastic model in the two programs. Whereas CU-DYNAMIX uses a flexibility approach to model the stiffness of a member by monitoring the flexibility of the member at the Gauss-points, DRAIN-2DX uses a direct stiffness approach to arrive at the stiffness of an element. The Park-Ang damage index computed from the results of the two programs should be similar as the damage index is based on the maximum deformation and dissipated hysteretic energy which are similar for the two programs.

4.4 Summary

This chapter presented application of the methodology for the development of motion-damage relationships for RC frames. The general methodology for the development of these relationships was presented in Chapter 3. In contrast to previous approaches for developing fragility curves and DPMs, the method used in this study does not rely on heuristics or on empirical data. The ground motion is characterized by spectral values in the period bands corresponding to the three classes of RC frames. The period bands for the three classes of frames are identified, and the relationship between average spectral acceleration and the average spectral velocity is investigated for the mid rise and the high rise frames.

Sample structures for the three classes of frames were described. The structural modeling in the computer programs for evaluating the nonlinear response was discussed. In this study, DRAIN-2DX was used for performing the nonlinear dynamic analysis. The results from DRAIN-2DX were compared with those from IDARC and CU-DYNAMIX. The motion-damage relationships for RC frames are presented as fragility curves and DPMs in Chapter 5.
CHAPTER 5
FRAGILITY CURVES AND DPMs FOR RC FRAMES

This chapter presents the fragility curves and damage probability matrices (DPMs) for special moment resisting frames located on firm soils. The general methodology for developing the motion-damage relationships was presented in Chapter 3, and the application of that methodology for concrete frames was presented in Chapter 4. In addition, sensitivity analyses are carried out to study the influence of different structural attributes which include the plan layout of the buildings, the second-order effects, and the effect of site conditions on the response of buildings.

5.1 Sample Fragility Curves

The computer programs IDARC2D (Kunnath and Reinhorn, 1994) and DRAIN-2DX (Prakash and Powell, 1992) are used for damage analysis. The member properties in terms of moment-rotation relationships are evaluated in IDARC2D. These properties are then used for the nonlinear dynamic analyses performed in DRAIN-2DX. The spread of plasticity along the length of each member is captured by a discretization of the members into smaller elements. Each member is divided into three elements with one small element of length equal to 15% of the member length at each end along with a larger middle element.

Nonlinear dynamic analysis is performed for 100 simulated ground motions generated at each value of spectral acceleration. An integration time step of 0.002 seconds is used in the analysis. The damping matrix was obtained as a linear combination of the mass and stiffness matrices. The coefficients for the mass and stiffness matrices were selected to give 5% of critical damping in the first two vibrational modes. The Park-Ang damage index given by equations 2.21 and 2.24 is computed from the results of the time history analysis performed in DRAIN-2DX. Only the length of a time history corresponding to its strong motion portion was used in the dynamic analysis.

The statistics of the Park and Ang damage index, obtained at each spectral acceleration value, are used to obtain the parameters of a lognormal probability distribution function at that ground motion level. Figures 5.1 through 5.3 show the
comparison between the empirical probability distributions obtained from simulation results and the fitted lognormal distributions at spectral acceleration value of 2g. These lognormal distributions are verified at a 5% significance level based on the Kolmogorov-Smirnov significance test.

The lognormal probability functions at each level of ground motion are then used to obtain the probabilities of the different damage states by computing the probabilities of the damage index being in the ranges given in Table 3.1. Smooth fragility curves are obtained by arbitrarily fitting lognormal distribution functions to the simulation results. Smooth fragility curves facilitate their use in regional loss-estimation and vulnerability assessment studies. The simulation results and the fitted curves are shown as discrete points and smooth curves respectively, in Figures 5.4 through 5.6.

The confidence bounds on the fragility curves shown in Figures 5.4 through 5.6 are established in Chapter 6. The fragility curves of Figures 5.4 through 5.6 represent the maximum likelihood curves. These curves are referred to as the median fragility curves in Chapter 6. The parameters of these fragility curves are given in Tables 6.9 through 6.11.

![Figure 5.1: Comparison between the empirical and fitted probability distribution functions for the low rise frame at $S_a = 2g$.](image)

*CHAPTER 5. Fragility Curves and DPMs for RC Frames*
Figure 5.2: Comparison between the empirical and fitted probability distribution functions for the mid rise frame at $S_a = 2g$.

Figure 5.3: Comparison between the empirical and fitted probability distribution functions for the high rise frame at $S_a = 2g$. 

CHAPTER 5. Fragility Curves and DPMs for RC Frames
Figure 5.4: Fragility curves for low rise frames.

Figure 5.5: Fragility curves for mid rise frames.

CHAPTER 5. Fragility Curves and DPMs for RC Frames
5.2 Sample Damage Probability Matrices

The DPMs for the sample buildings are developed from the fragility curves presented in the previous section along with the relationship between modified Mercalli intensity (MMI) and spectral acceleration. The relationship between MMI and spectral acceleration is presented in the next section. The integral in equation 3.4 is evaluated numerically by using the subroutine QDAGI in IMSL (1991). This subroutine uses a 21-point Gauss-Kronrod rule to estimate the integral. The probability of the structure being in a particular damage state at a specified MMI is estimated by taking the difference in probabilities of two adjacent damage states evaluated by means of equation 3.4. For example, the probability of the structure being in the moderate damage state is given by the probability of the structure reaching or exceeding the severe damage state minus the probability of the structure reaching or exceeding the moderate damage state. These probabilities are computed at the ground motion level of interest.

Tables 5.1 through 5.3 show the DPMs evaluated for the representative buildings. Even though not appreciably different, these matrices show an increase in the probability
of the collapse damage state at higher MMI values as the building height is increased. The collapse damage state as defined by Park et al. (1987) includes total or partial collapse of the building. Thus, as the building height is increased, it is reasonable to expect the probability of partial collapse anywhere in the building to increase.

Table 5.1: Damage probability matrix for the sample low rise building.

<table>
<thead>
<tr>
<th>Damage state</th>
<th>VI</th>
<th>VII</th>
<th>VIII</th>
<th>IX</th>
<th>X</th>
<th>XI</th>
<th>XII</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>99.5</td>
<td>97.0</td>
<td>85.4</td>
<td>52.9</td>
<td>14.1</td>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>Minor</td>
<td>0.3</td>
<td>1.6</td>
<td>6.9</td>
<td>16.9</td>
<td>15.5</td>
<td>3.4</td>
<td>0.1</td>
</tr>
<tr>
<td>Moderate</td>
<td>0.2</td>
<td>1.1</td>
<td>5.4</td>
<td>18.5</td>
<td>30.5</td>
<td>17.6</td>
<td>2.8</td>
</tr>
<tr>
<td>Severe</td>
<td>0.2</td>
<td>1.4</td>
<td>7.0</td>
<td>20.7</td>
<td>28.0</td>
<td>14.6</td>
<td></td>
</tr>
<tr>
<td>Collapse</td>
<td>0.1</td>
<td>0.9</td>
<td>4.7</td>
<td>19.2</td>
<td>50.1</td>
<td>82.5</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.2: Damage probability matrix for the sample mid rise building.

<table>
<thead>
<tr>
<th>Damage state</th>
<th>VI</th>
<th>VII</th>
<th>VIII</th>
<th>IX</th>
<th>X</th>
<th>XI</th>
<th>XII</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>99.0</td>
<td>93.5</td>
<td>70.0</td>
<td>24.5</td>
<td>1.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minor</td>
<td>0.7</td>
<td>4.2</td>
<td>15.8</td>
<td>23.0</td>
<td>6.1</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>Moderate</td>
<td>0.3</td>
<td>2.1</td>
<td>12.0</td>
<td>36.6</td>
<td>33.7</td>
<td>4.9</td>
<td>0.2</td>
</tr>
<tr>
<td>Severe</td>
<td>0.2</td>
<td>1.9</td>
<td>12.5</td>
<td>34.6</td>
<td>22.3</td>
<td>1.8</td>
<td></td>
</tr>
<tr>
<td>Collapse</td>
<td>0.3</td>
<td>3.4</td>
<td>24.0</td>
<td>72.7</td>
<td>98.0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.3: Damage probability matrix for the sample high rise building.

<table>
<thead>
<tr>
<th>Damage state</th>
<th>VI</th>
<th>VII</th>
<th>VIII</th>
<th>IX</th>
<th>X</th>
<th>XI</th>
<th>XII</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>100.0</td>
<td>99.7</td>
<td>93.0</td>
<td>35.3</td>
<td>0.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minor</td>
<td></td>
<td>0.3</td>
<td>5.7</td>
<td>35.3</td>
<td>3.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Moderate</td>
<td></td>
<td>1.3</td>
<td>26.9</td>
<td>45.9</td>
<td>0.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Severe</td>
<td></td>
<td>2.4</td>
<td>38.9</td>
<td>11.8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Collapse</td>
<td></td>
<td>0.1</td>
<td>11.7</td>
<td>87.5</td>
<td>100.0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

CHAPTER 5. Fragility Curves and DPMs for RC Frames
5.2.1 Relationship Between MMI and Spectral Acceleration

The firm site records of the Loma Prieta, Whittier Narrows, and Morgan Hill earthquakes are used for estimating the relationship between the average spectral acceleration and MMI. The average spectral acceleration of the larger of the two horizontal components of the ground motions recorded on firm sites and the MMI values from these earthquakes at the respective recording stations are used to develop these relationships. Figures 5.7 through 5.10 show the MMI contours along with the station numbers of the sites where ground motion was recorded. The data used to develop these relationships were also presented in Table 3.6.

The average spectral acceleration in each period range is assumed to have a conditional lognormal probability density function at given values of MMI. Regression analysis is performed between the natural logarithm of the mean of the average spectral acceleration and MMI. Similar regression analysis is performed between the standard deviation of the average spectral acceleration and MMI. The resulting regression curves are used to estimate the means and standard deviations of the average spectral acceleration at higher MMI values for which observed data are not available. The regression equations for the mean and standard deviation of the average spectral acceleration, expressed in cm/sec\(^2\) in each period band, are shown as follows:

\[
\begin{align*}
\mu_{S_a|MMI} &= 7.49 \times 10^{0.59 \text{ MMI}} \quad \text{for } 0.1 \leq T \leq 0.5 \quad (5.1) \\
\sigma_{S_a|MMI} &= 16.35 \times 10^{0.41 \text{ MMI}} \quad \text{for } 0.1 \leq T \leq 0.5 \quad (5.2) \\
\mu_{S_a|MMI} &= 2.78 \times 10^{0.68 \text{ MMI}} \quad \text{for } 0.5 \leq T \leq 0.9 \quad (5.3) \\
\sigma_{S_a|MMI} &= 5.95 \times 10^{0.52 \text{ MMI}} \quad \text{for } 0.5 \leq T \leq 0.9 \quad (5.4) \\
\mu_{S_a|MMI} &= 0.31 \times 10^{0.85 \text{ MMI}} \quad \text{for } 0.9 \leq T \leq 2.5 \quad (5.5) \\
\sigma_{S_a|MMI} &= 1.54 \times 10^{0.55 \text{ MMI}} \quad \text{for } 0.9 \leq T \leq 2.5 \quad (5.6)
\end{align*}
\]

The curves representing the mean values, mean plus and minus one standard deviation values, and the median values of the average spectral acceleration are shown in Figures 5.11 through 5.13. These figures also show the conditional distributions for the average spectral acceleration, with MMI in the V to VIII range.
Figure 5.7: Map showing the MMI contours and the California Strong Motion Instrumentation Program (CSMIP) station numbers (Shakal et al. 1985) for the Morgan Hill earthquake of April 24, 1984 (M_L of 6.2).
Figure 5.8: Map showing the MMI contours and the CSMIP station numbers (Shakal et al., 1987) for the Whittier Narrows earthquake of October 1, 1987 ($M_L$ of 6.1).
Figure 5.9: Map showing the MMI contours and the CSMIP station numbers (Shakal et al., 1989) for the Loma Prieta earthquake of October 18, 1989 (M_L of 7.0 and M_S of 7.1).
Figure 5.10: Map showing the MMI contours and the United States Geological Survey (USGS) station numbers (Maley et al., 1989) for the Loma Prieta earthquake of October 18, 1989, (ML of 7.0 and MS of 7.1).
Figure 5.11: Relationships for the mean, median, standard deviation, and conditional probability distribution functions of $S_a$ at given MMI, for $0.1 \leq T \leq 0.5$ seconds.

Figure 5.12: Relationships for the mean, median, standard deviation, and conditional probability distribution functions of $S_a$ at given MMI, for $0.5 \leq T \leq 0.9$ seconds.
Figure 5.13: Relationships for the mean, median, standard deviation, and conditional probability distribution functions of $S_a$ at given MMI, for $0.9 \leq T \leq 2.5$ seconds.

5.2.2 Comparison of DPMs with those in ATC-13

The DPMs of ATC-13 (1985) were transformed to correspond to the damage states used in this study. The mapping of the ATC-13 damage states to the damage states used in this study is presented in Table 5.4. Tables 5.5 through 5.7 present the transformed DPMs for ductile reinforced concrete frames. The DPMs of ATC-13 show significant probabilities only for a few damage states, whereas those from this study show significant non-zero probabilities for more damage states at a given level of MMI. Moreover, the DPMs of ATC-13 show much less damage at higher levels of MMI. This result may be due to some differences in the definitions of damage states used in ATC-13 and in the current study. The negligible probability of collapse of the frames at MMI values of XI and XII given in ATC-13 appear rather unrealistic, particularly in view of the performance of concrete frame structures in recent large earthquakes. It is also possible that the spectral accelerations predicted at higher MMI using the MMI-spectral acceleration relationships developed above may be exaggerated due to the fact that the relationships are derived using observations at lower MMI values. Ground motion recordings at larger MMI values are needed in order to accurately predict the spectral acceleration values at higher MMI levels.
**Table 5.4:** Mapping of ATC-13 damage states to those used in this study.

<table>
<thead>
<tr>
<th>ATC-13 Damage State</th>
<th>Damage Factor Range (%)</th>
<th>Present Study Damage State</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>0</td>
<td>None</td>
</tr>
<tr>
<td>Slight</td>
<td>0 - 1</td>
<td>Minor</td>
</tr>
<tr>
<td>Light</td>
<td>1 - 10</td>
<td>Moderate</td>
</tr>
<tr>
<td>Moderate</td>
<td>10 - 30</td>
<td>Moderate</td>
</tr>
<tr>
<td>Heavy</td>
<td>30 - 60</td>
<td>Severe</td>
</tr>
<tr>
<td>Major</td>
<td>60-100</td>
<td>Complete</td>
</tr>
<tr>
<td>Destroyed</td>
<td>100</td>
<td>Complete</td>
</tr>
</tbody>
</table>

**Table 5.5:** ATC-13 DPM for low rise, ductile moment resisting frames.

<table>
<thead>
<tr>
<th>Damage state</th>
<th>Modified Mercalli Intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>VI</td>
</tr>
<tr>
<td>None</td>
<td></td>
</tr>
<tr>
<td>Minor</td>
<td>97.5</td>
</tr>
<tr>
<td>Moderate</td>
<td></td>
</tr>
<tr>
<td>Severe</td>
<td></td>
</tr>
<tr>
<td>Collapse</td>
<td></td>
</tr>
</tbody>
</table>

**Table 5.6:** ATC-13 DPM for mid rise, ductile moment resisting frames.

<table>
<thead>
<tr>
<th>Damage state</th>
<th>Modified Mercalli Intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>VI</td>
</tr>
<tr>
<td>None</td>
<td></td>
</tr>
<tr>
<td>Minor</td>
<td>99.7</td>
</tr>
<tr>
<td>Moderate</td>
<td></td>
</tr>
<tr>
<td>Severe</td>
<td></td>
</tr>
<tr>
<td>Collapse</td>
<td></td>
</tr>
</tbody>
</table>

*CHAPTER 5. Fragility Curves and DPMs for RC Frames*
Table 5.7: ATC-13 DPM for high rise, ductile moment resisting frames.

<table>
<thead>
<tr>
<th>Damage state</th>
<th>VI</th>
<th>VII</th>
<th>VIII</th>
<th>IX</th>
<th>X</th>
<th>XI</th>
<th>XII</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minor</td>
<td>100.0</td>
<td>100.0</td>
<td>83.6</td>
<td>27.6</td>
<td>3.1</td>
<td>0.4</td>
<td>0.1</td>
</tr>
<tr>
<td>Moderate</td>
<td></td>
<td></td>
<td>16.4</td>
<td>72.4</td>
<td>96.9</td>
<td>99.2</td>
<td>96.4</td>
</tr>
<tr>
<td>Severe</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.4</td>
<td>3.5</td>
</tr>
<tr>
<td>Collapse</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5.3 Sensitivity Analysis

The fragility curves and DPMs for reinforced concrete frames developed earlier in the chapter were obtained by designing a representative building in each of the three classes. However, a class of frames is likely to consist of many frames having different characteristics. This section attempts to address some of the issues associated with the different structural characteristics of frames composing a particular class. Sensitivity studies are conducted for the following properties:

- plan layout of the buildings,
- second-order effects, and
- site condition.

These properties are studied with respect to the buildings used earlier in this chapter for the development of motion-damage relationships. Structural damage is primarily caused by high stress excursion as well as repeated stress reversals. Most damage measures, including those discussed in Chapter 2, depend on one or both of these response variables. Therefore in this sensitivity study, the interstory drift ratio and the dissipated hysteretic energies are used to characterize the damage variables: stress excursions and repeated stress reversals. The comparisons presented in the following sub-sections are in terms of the mean values of the drift ratios, the dissipated hysteretic energy, and the Park-Ang damage index obtained for ensembles of time histories.
The aim in this sensitivity study is to determine the influence of the different parameters. Correlated sampling is again used for these sensitivity analyses. As mentioned in Section 4.3.3, the aim of correlated sampling is to produce a high positive correlation between two similar processes so that the variance of the difference is much smaller than the case where the processes are statistically independent. Therefore, the recorded ground motions from the six earthquakes, discussed earlier in Section 4.1, are scaled to different levels of spectral acceleration and used for the purpose of these sensitivity studies. The time histories recorded on firm sites are used to study the sensitivity to number of bays and to second-order effects. To study the sensitivity to site conditions, a subset is selected from these time histories to correspond to rock sites. A different set of time histories is used for soil sites. Only the lengths of the time histories corresponding to their strong motion portions were used for the nonlinear dynamic analyses.

5.3.1 Sensitivity to Number of Bays

The sample buildings selected in each of the three classes of concrete frames are one bay frames. In reality, the buildings in each class could have a different number of bays. Therefore, it is worthwhile to examine the effect of the number of bays on structural damage. To study this effect, a three bay, five story building is designed using the recommendations of SEAOC (1990). The plan of this building is shown in Figure 5.14. The story height in this building is also 10 feet.

The responses of the three bay and the one bay frames are compared in terms of the overall drift ratio defined as the ratio of the maximum drift at the roof level to the height of the building, the total dissipated hysteretic energy, and the Park-Ang structure damage index. The comparisons of the responses of the two frames presented in Figures 5.15 through 5.17 show that the responses of the two frames are very similar. In this study, we assumed that the different members in the structure have the same strengths of steel and concrete. In reality, the strengths of steel and concrete in the different elements may not be the same but a high correlation coefficient is expected between the strengths of steel and concrete for members of the same structure. If the strengths of steel and concrete in the members are not the same, some redistribution of the internal forces is expected as members progressively yield.
Figure 5.14: Plan of the three bay, five story frames.

Figure 5.15: Comparison of the overall drift ratios for the two five story frames.
Figure 5.16: Comparison of the total dissipated hysteretic energies for the two five story frames.

Figure 5.17: Comparison of the Park and Ang damage indices for the two five story frames.
5.3.2 Sensitivity to Second-Order Effects

The dynamic analyses considered in developing the fragility curves, shown in Figures 5.4 through 5.6, are first-order dynamic analyses as the distribution of internal forces in the structures ignored the effect of sway deformation on the equilibrium equations and the influence of axial forces on the stiffnesses of members. Second-order effects in building systems are associated with the movement of the structural mass to a deformed position which give rise to second-order overturning moments due to the lateral displacement. Thus, second-order effects refer to the effect of geometric nonlinearity on the behavior of a structure. In geometric nonlinear analysis of a structure, the equilibrium equations are formulated in the deformed configuration of the structure. Geometric nonlinear analyses may be carried out at the structure level only or may also include member curvature effects. When carried out at the structure level only, the geometric nonlinear analyses, also referred to as P-Δ analyses, only include the effect of member chord rotation. P-Δ analyses on the other hand include nonlinearity arising out of member curvature. While the P-Δ effect reduces the element flexural stiffness against sidesway, the P-δ effect reduces the member stiffness in both sidesway and non-sidesway modes of deformation.

Strictly speaking, the solution of second-order effects is iterative, as the second-order moments depend on the lateral displacements which in turn depend on the total applied moments. For building structures, Wilson and Habibullah (1987) suggest that the P-Δ problem can be linearized and the solution to the problem obtained directly and exactly without iteration as the weight of the structure is constant during lateral motions and the structural displacements can be assumed to be small compared to the building dimensions. They assume that the total axial force at a story level is equal to the weight of the building above that level, a weight which does not change during the application of the lateral loads. Therefore, the sum of the column geometric stiffness terms associated with the lateral loads cancel, and only the axial forces due to the weight of the structure need to be included in the evaluation of the negative stiffness terms for the entire building.

In this study, the P-Δ effects are investigated for the five story and the twelve story frames using the program DRAIN-2DX. The geometric stiffnesses used for dynamic analyses are based on the axial forces resulting from gravity loads. The program's manual suggests that considering only the gravity axial loads will often be accurate for building frames as the translational geometric stiffness for a story depends only on the sum of the
axial forces in all columns of a story. The sum of the axial forces in all columns of a story is a constant equal to the gravity load if there are no vertical inertial forces.

The responses of the one bay, five story frame are compared in terms of the overall drift ratio, total dissipated hysteretic energy, and the Park-Ang structure damage index. The comparisons of the responses with and without P-Δ effect are presented in Figures 5.18 through 5.20. Furthermore, the P-Δ effect was also investigated for the sample high rise building. The results for the twelve story frame are presented in Figures 5.21 through 5.23. Figures 5.18 through 5.23 suggest that the P-Δ effect is negligible for lower levels of ground motion. Furthermore, the P-Δ effect is more significant for the high rise frame compared to the mid rise frame. Wilson and Habibullah (1987) suggest that if the lateral displacements obtained from analyses with and without P-Δ effects at the design level differ by more than 10 or 15%, the basic design is too flexible. As the ground motion level is increased, the P-Δ effect becomes significant. The structures become unstable at still larger levels of ground motion as the geometric stiffness begins to dominate and reduces the effective stiffness of the system. The P-Δ effect for the low rise frame was not expected to be significant and therefore was not investigated.

![Graph showing Drift Ratio vs Spectral Acceleration](image)

**Figure 5.18:** Comparison of the overall drift ratio for the one bay, five story frame.
Figure 5.19: Comparison of the total dissipated hysteretic energies for the one bay, five story frame.

Figure 5.20: Comparison of the Park-Ang damage index for the one bay, five story frame.
Figure 5.21: Comparison of the overall drift ratio for the one bay, twelve story frame.

Figure 5.22: Comparison of the total dissipated hysteretic energies for the one bay, twelve story frame.
Figure 5.23: Comparison of the Park-Ang damage index for the one bay, twelve story frame.

5.3.3 Sensitivity to Site Conditions

The response of buildings can be influenced by the soil on which the building is founded. The response spectra of ground motion recorded on firm sites is different from those recorded on softer sites. Furthermore, the ground motion recorded close to a building would be different from that which would have been recorded if the building had not been present. This modification of the ground motion is referred to as soil-structure interaction. Jennings and Bielak (1973) suggest that the soil-structure interaction effects depend on the relative stiffness of the building and its foundation. They also conclude that in most instances, the soil-structure interaction will reduce the fundamental frequency of the structure from the case where the structure is assumed to be located on a firm base. Soil-structure interaction has been considered by various researchers, including Veletsos and Prasad (1989), Luco and Mita (1987), and Veletsos and Meek (1974).

Although the consideration of the soil-structure interaction effect is beyond the scope of the current study, it is still useful to consider the effect of different mean spectral shapes on the dynamic response of structures. Thus, ground motions recorded on soil sites during the Landers, Loma Prieta, Morgan Hill, Northridge, and Whittier Narrows
earthquakes are used. These ground motions are selected on the basis of the average shear wave velocities at the recording stations. These ground motions correspond to site class SC-III in the classification proposed by Borchertd (1994). The time histories used, and the average spectral acceleration in the three period bands are shown in Table 5.8. The statistics on the dynamic amplification factors are presented in Figure 5.24. Figure 5.25 shows the comparison of the mean dynamic amplification factors corresponding to firm and soil sites. As expected, the dynamic amplification for soil sites has a large plateau in the short-period range. However, in the long-period range, the dynamic amplification factors are similar for firm and soil sites. In this study, the two story frame described in Chapter 4 is used. The two story frame is selected as it is likely to be effected most by the change in the spectral shapes. The mid rise and the high rise frames lie in the descending portions of the dynamic amplification factors, where the dynamic amplification factors for firm and soil sites are similar. The comparison of the results in terms of the overall drift ratios, dissipated hysteretic energies, and the Park-Ang damage index are presented in Figures 5.26 through 5.28, respectively.

Figure 5.24: Dynamic amplification factors for soil sites.
Table 5.8: Average spectral acceleration values for ground motions recorded on soil sites.

<table>
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<tr>
<th>SITE NAME AND NUMBER</th>
<th>EARTHQUAKE</th>
<th>Average $S_a$ (0.1≤T≤0.5) (g)</th>
<th>Average $S_a$ (0.5&lt;T≤0.9) (g)</th>
<th>Average $S_a$ (0.9&lt;T≤2.5) (g)</th>
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Table 5.8: (cont’d) Average spectral acceleration values for ground motions recorded on soil sites.

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<th>SITE NAME AND NUMBER</th>
<th>EARTHQUAKE</th>
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Table 5.8: (cont’d) Average spectral acceleration values for ground motions recorded on soil sites.

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<th>SITE NAME AND NUMBER</th>
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<th>Average $S_a$ (0.5&lt;T≤0.9) (g)</th>
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</table>

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Figure 5.25: Comparison of the mean dynamic amplification factors for firm and soil sites.

Figure 5.26: Comparison of the overall drift ratio for the one bay, two story frame subjected to ground motions recorded on firm and soil sites.
Figure 5.27: Comparison of the total dissipated hysteretic energies for the one bay, two story frame subjected to ground motions recorded on firm and soil sites.

Figure 5.28: Comparison of the Park-Ang damage index for the one bay, two story frame subjected to ground motions recorded on firm and soil sites.
The site conditions in this study so far have been determined based on the shear wave velocity at that site. However, this relationship results in very similar dynamic amplification factors for firm and soil sites as shown in Figure 5.25. Using ground motions recorded on rock and alluvium sites during the Loma Prieta earthquake, Mohraz and Tiv (1991) arrive at the conclusion that the amplification factors for these two sites are very similar. Seed et al. (1976) and Mohraz (1976) arrive at very different amplification factors for rock and alluvial sites, those for alluvial sites being quite similar to those shown in Figure 5.25 and to those obtained by Mohraz (1991). It is worthwhile to investigate the effect of the spectral shape on nonlinear dynamic response. To obtain spectral shapes similar to those obtained by Seed et al. (1976) and Mohraz (1976), about forty time histories were selected from the six earthquakes mentioned before. The sites on which these forty time histories were recorded are termed as rock sites for the purpose of this study. The statistics on the dynamic amplification factors for rock sites are presented in Figure 5.29, and the comparison of these factors with those for firm sites in Figure 5.30. The mean dynamic amplification factors for rock sites shown in Figure 5.30 are very similar to the amplification factors for rock sites provided by Seed et al. (1976) and Mohraz (1976).

Figure 5.29: Dynamic amplification factors for rock sites.
Figure 5.30: Comparison of the mean dynamic amplification factors for firm, rock, and soil sites.

Figures 5.31 through 5.33 respectively present the comparisons of the overall drift ratios, total dissipated energy, and the Park-Ang damage index for the low rise frame when subjected to ground motions recorded on rock and firm sites. These figures show that there are considerable differences in the responses when the frame is subjected to motions recorded on firm and rock sites. These differences are due to the reduction in seismic demand for the frame subjected to ground motions recorded on rock as the members of the frame progressively yield resulting in the lengthening of the period of the structure. Figure 5.30 depicts the reduction of seismic demand at larger periods. Figures 5.34 through 5.36 respectively present the comparisons of the overall drift ratios, total dissipated energy, and the Park-Ang damage index for the mid rise frame when subjected to ground motions recorded on rock and firm sites. Figures 5.37 through 5.39 respectively present the comparisons of the overall drift ratios, total dissipated energy, and the Park-Ang damage index for the high rise frame when subjected to ground motions recorded on rock and firm sites. Figures 5.34 through 5.39 show that the responses are very similar at lower levels of ground motion, some differences being observed at larger levels of ground motion. The differences in the responses of the frames when subjected to ground motions recorded on firm and rock sites are due to higher mode effects. Compared to ground motions recorded on firm sites, those recorded on rock sites have more demands at smaller periods for the same level of demands in the period ranges for the mid rise and the high rise frames.
Figure 5.31: Comparison of the overall drift ratio for the one bay, two story frame subjected to ground motions recorded on firm and rock sites.

Figure 5.32: Comparison of the total dissipated energy for the one bay, two story frame subjected to ground motions recorded on firm and rock sites.
Figure 5.33: Comparison of the overall Park-Ang damage index for the one bay, two story frame subjected to ground motions recorded on firm and rock sites.

Figure 5.34: Comparison of the overall drift ratio for the one bay, five story frame subjected to ground motions recorded on firm and rock sites.
Figure 5.35: Comparison of the total dissipated energy for the one bay, five story frame subjected to ground motions recorded on firm and rock sites.

Figure 5.36: Comparison of the overall Park-Ang damage index for the one bay, five story frame subjected to ground motions recorded on firm and rock sites.
Figure 5.37: Comparison of the overall drift ratio for the one bay, twelve story frame subjected to ground motions recorded on firm and rock sites.

Figure 5.38: Comparison of the total dissipated energy for the one bay, twelve story frame subjected to ground motions recorded on firm and rock sites.
Figure 5.39: Comparison of the overall Park-Ang damage index for the one bay, twelve story frame subjected to ground motions recorded on firm and rock sites.

5.4 Relationship Between Damage Caused by Deformation and Dissipated Hysteretic Energy

Structural damage is primarily caused by excessive deformation and by the dissipation of hysteretic energy. The Park-Ang damage index is estimated as a linear combination of damage due to deformation and dissipated energy. This section examines the relative values of the two terms of the Park-Ang damage index for the three sample frames when subjected to different levels of ground motion. The comparison is carried out for the typical frames in each structural class: low rise, mid rise, and high rise. Figures 5.40 through 5.42 show the relative contributions of the deformation and the dissipated energy terms to the Park-Ang damage index for the structure. The results presented in these figures are obtained by using the ground motions recorded on firm sites during the six earthquakes discussed earlier. The results from simulated time histories show the same trends except that they have more fluctuations about the values plotted in Figures 5.40 through 5.42. Thus, these figures are the mean contributions of the two terms of the damage index. These figures show that the two terms of the damage index contribute almost equally to the damage index.
Figure 5.40: Relative contributions of the two terms in the Park-Ang damage index for the low rise frame.

Figure 5.41: Relative contributions of the two terms in the Park-Ang damage index for the mid rise frame.
5.5 Relationship Between Drift Ratios and Park-Ang Damage Index

The drift ratios are often utilized as measures of damage, (e.g., NIBS 1995). Thus, it is worthwhile to examine the correlation between story-drift and the damage index. For the purpose of estimating this correlation, the simulated results are used for the low rise, the mid rise, and the high rise frames. The results for the overall drift ratios and the overall Park-Ang damage index are presented in Figures 5.43 through 5.45. A correlation coefficient greater than 99% is observed in all cases.

Figures 5.43 through 5.45 show that the Park-Ang damage index is zero up to a certain level of drift. This observation can be explained by the fact that for very small values of the drift ratio, there is no yielding in the members and so no plastic deformation or hysteretic energy dissipation in the members. Another interesting observation from these figures is that the drift ratios corresponding to the Park-Ang damage index of unity are 0.054, 0.050, and about 0.042 for the low rise, the mid rise, and the high rise frames, respectively. This decrease in the drift ratios is partly explained by the fact that higher modes become significant in the response of the structure as the structure height is increased. Due to the participation of the higher modes, the overall drift ratio decreases.
Figure 5.43: Relationship between the overall drift ratio and the overall Park-Ang damage index for the low rise frame.

Figure 5.44: Relationship between the overall drift ratio and the overall Park-Ang damage index for the mid rise frame.
Figure 5.45: Relationship between the overall drift ratio and the overall Park-Ang damage index for the high rise frame.

5.6 Summary

This chapter presented fragility curves and DPMs for special moment resisting reinforced concrete frames. In contrast to previous approaches for developing fragility curves and DPMs, the method used does not rely on heuristics or on empirical data. Details on the application of the methodology to reinforced concrete frames were presented in Chapter 4.

Sensitivity analyses were carried out to study the influence of the different structural attributes on the nonlinear dynamic analysis of structures. These sensitivity analyses were carried out to study how structural damage is affected by the different structural attributes because a sample building is taken to represent a building class in the development of fragility curves. The structural attributes included in the sensitivity studies were the number of bays in a structure, the second-order effects, and the site conditions. The influence of the number of bays is not found to be very significant on structural response. However, the second-order effects has a significant influence on structural response at larger ground motion levels. The second-order effects are found to lead to instability of
the structure at higher ground motion levels. Rock ground motions were also found to influence the response of the structures compared to the firm site ground motions, the most significant influence being observed for the low rise frame.

This chapter developed sample fragility curves for the three classes of reinforced concrete frames using a sample frame in each class. The uncertainty due to representation of a structural class by a sample frame is discussed in Chapter 6. The uncertainty associated with representation of a structural class by a single frame can be reduced by taking more sample buildings in each structural class. A better way to reduce this uncertainty is to incorporate observed damage data in the development of fragility curves. The procedure to update the fragility curves on the basis of observed damage data is demonstrated in Chapter 6 by using damage data from the Northridge earthquake.
CHAPTER 6
BAYESIAN UPDATING OF MOTION-DAMAGE RELATIONSHIPS

This chapter presents the concepts of randomness and uncertainty associated with the motion-damage relationships. Bayesian analysis is used to update the fragility curves which were presented in Chapter 5 based on the data about buildings damaged during the Northridge earthquake. The uncertainty in these relationships is incorporated in terms of the confidence bounds on the fragility curves for each damage state.

6.1 Randomness and Uncertainty

Randomness is associated with the inherent variability in the characteristics of a structure and the environmental demands that are imposed on that structure. The randomness in these structural characteristics includes the variability associated with the material properties of the structure. The randomness in demands includes the variability in the loads to which a structure is subjected. Whereas randomness is intrinsic in nature and beyond our control, uncertainty is extrinsic and to some extent reducible. Geyskens et al. (1993) suggest the following three sources of uncertainty:

- errors of ignorance and simplifications,
- measurement errors, and
- statistical errors.

Errors of ignorance and simplifications arise due to incomplete knowledge and understanding of the physical phenomena governing the behavior of the system. Often, the behavior of the system is idealized and represented by means of simplified functional forms. Measurement errors arise due to the use of uncertain measured values to determine the model parameters. Statistical errors are introduced in the estimation of the model parameters due to the use of a finite data-set to determine the model parameters. Since the model is uncertain, an infinite data-set is required to determine the model parameters exactly. Further information on randomness and uncertainty is provided by Der Kiureghian (1989).
6.2 Randomness and Uncertainty in the Response of RC Frames

The randomness and uncertainty in the response of reinforced concrete (RC) structures has been treated by Park et al. (1984) in terms of their damage index. They examined the variability in the response of RC structures as a result of the randomness and uncertainty associated with the structural parameters and modeling errors. The structural parameters they considered which are relevant to this study include the stiffness and strength, damping, and mass. The variabilities associated with these parameters are summarized in the following sub-sections.

6.2.1 Stiffness and Strength

The variability associated with the stiffness and strength of the different members of a structure consists of the inherent material randomness and the modeling error. The randomness associated with the material properties has already been considered in Section 4.3.2. However, modeling error needs to be incorporated here. Based on test data from 260 RC components, Park et al. (1984) suggest the modeling error, expressed as a coefficient of variation, is equal to 0.29 for stiffness and 0.12 for the yield strength.

The variability in the stiffness and strength of RC members depends on the uncertainty in the member dimensions and the placement of reinforcing steel. Mirza and MacGregor (1979) recommend normal distributions for modeling the geometric properties of in-situ RC beams. For concrete cover to main reinforcement in beams, they recommend a normal distribution with the lower tail truncated at one stirrup or tie diameter. The overall depth of members is important in determining the stiffnesses of the members. The effective depth of the main reinforcement is very important from the point of view of strength. The position of the bottom bars in beams is affected by the height of chairs used to support the bars, and the overall depth of the beams. The factors that lead to the variability in the position of the top bars include the variations in the height of the bottom bars, uncertainty in the vertical dimensions of the stirrups, and conflict with other top bars at beam intersections. The location of vertical bars in columns is influenced by the variabilities in the dimensions of ties and forms, alignment of the columns from floor to floor, and accuracy in the alignment of the reinforcement cage within the forms. The parameters for
the normal distributions for the geometrical properties of beams and columns are shown in Tables 6.1 and 6.2, respectively.

**Table 6.1:** Estimated distributions for geometrical properties of beams (from Mirza and Macgregor, 1979).

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Nominal range (in.)</th>
<th>Mean deviation from nominal (in.)</th>
<th>Standard deviation (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width</td>
<td>11-12</td>
<td>+3/32</td>
<td>3/16</td>
</tr>
<tr>
<td>Overall depth</td>
<td>18-27</td>
<td>-1/8</td>
<td>1/4</td>
</tr>
<tr>
<td>Cover to top reinforcement</td>
<td>1-1/2</td>
<td>+1/8</td>
<td>5/8</td>
</tr>
<tr>
<td>Cover to bottom reinforcement</td>
<td>3/4-1</td>
<td>+1/16</td>
<td>7/16</td>
</tr>
</tbody>
</table>

**Table 6.2:** Estimated distributions for column dimensions (from Mirza and Macgregor, 1979).

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Nominal range (in.)</th>
<th>Mean deviation from nominal (in.)</th>
<th>Standard deviation (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lateral dimension of rectangular columns</td>
<td>11-30</td>
<td>+1/16</td>
<td>1/4</td>
</tr>
<tr>
<td>Diameter of circular columns</td>
<td>11-13</td>
<td>0</td>
<td>3/16</td>
</tr>
</tbody>
</table>

6.2.2 Damping

Park et al. (1984) suggest that the viscous damping in structures arises due to the energy dissipation of the soil and the friction between the foundation and the soil. They suggest a coefficient of variation of 0.52 in the damping coefficient of the fundamental mode of RC structures. They also suggest that the large scatter in the damping coefficient is due to the variability of the soil properties.

6.2.3 Mass

Park et al. (1984) suggest a coefficient of variation of 0.12 in the mass of buildings.
6.2.4 Uncertainty in the Park-Ang Damage Index

Park et al. (1984) conclude that the total variability associated with the mean of their damage index has a coefficient of variation equal to 0.6, whereas the inherent randomness in their damage index has a coefficient of variation of 0.5. Thus for this study, the coefficient of variation due to uncertainty in structural response is equal to $\sqrt{0.6^2 - 0.5^2} = 0.33$. Kennedy et al. (1980) provide the uncertainties associated with the capacity of and the demand on reactor buildings. They suggest that the logarithmic standard deviation due to uncertainty associated with the overall response of a reactor building may be taken as 0.38. This value of uncertainty is close to the value adopted in this study. Furthermore, the uncertainty due to representation of a structural class by a sample frame is assumed to be included in the value of uncertainty used in this study. Such uncertainty can be reduced by taking more sample buildings in each structural class. In addition, this uncertainty can be reduced by incorporating observed damage data in the development of the fragility curves.

6.3 Bayesian Updating

The overview of Bayesian analysis was presented in Section 3.5. The Bayesian updating of the Park-Ang damage index, at specified levels of ground motion in terms of spectral acceleration, is presented in this section. The simulations performed in Chapter 5 provide the prior distributions of the Park-Ang damage index. The observed damage data from the Northridge earthquake provide the likelihood functions required in equation 3.49.

It was shown in Chapter 5 that a lognormal distribution can be used to represent the probability distribution of the Park-Ang damage index at a specified level of ground motion. The variation of the coefficient of variation of the Park-Ang damage index are plotted in Figures 6.1 through 6.3 for the three sample frames of Chapter 5 subjected to simulated ground motion. The coefficient of variation is larger for the low rise frame in Figure 6.1 compared to the case when the frame is subjected to recorded ground motions scaled to different levels of spectral accelerations. These figures show that the coefficient of variation for the three frames converges to constant values at larger levels of ground motion. Therefore, in this study, only the median of the underlying lognormal process is treated as a random variable.
Figure 6.1: Variability of the coefficient of variation of the Park-Ang damage index for the low rise frame at different levels of ground motion.

Figure 6.2: Variability of the coefficient of variation of the Park-Ang damage index for the mid rise frame at different levels of ground motion.
6.3.1 Estimation of the Likelihood Functions

A lognormal random variable can be treated as a normal variable by using the natural logarithm of the lognormal variable. Therefore, the estimation of the likelihood functions is presented for the case where the sampling is from a normal process. The likelihood of obtaining observations, $x_1, \ldots, x_i, \ldots, x_n$, is given as:

$$L(X|\mu, \sigma) = \frac{1}{n} e^{-\frac{1}{2\sigma^2} \sum (x_i - \mu)^2}$$

(6.1)

where:

$X = \text{vector of observations, } x_1, \ldots, x_i, \ldots, x_n$,
$\mu = \text{mean of the underlying normal process, and}$
$\sigma = \text{standard deviation of the underlying normal process.}$
As mentioned earlier, the variance of the underlying process is assumed to be known. Then, the likelihood function is proportional to:

\[ L(X|\mu, \sigma) \propto e^{\frac{1}{2\sigma^2} n(m-\mu)^2} \]  

(6.2)

where:

\[ m = \frac{1}{n} \sum x_i = \text{mean of the observations.} \]

### 6.3.2 Posterior Distribution of the Mean

As mentioned in Section 3.5, considerable mathematical simplification can be achieved if conjugate distributions are used. When the likelihood function is given by equation 6.2, a normal distribution is the conjugate distribution for the underlying random variable, the mean, \( \mu \), in this study (Raiffa and Schlaifer, 1964). Therefore, if the prior distribution of \( \mu \) is a normal distribution, the posterior distribution will also be a normal distribution. The prior distribution of \( \mu \) is given by the following equation:

\[ f_M^{\prime}(\mu) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{(\mu-m')^2}{2\sigma^2}} \]  

(6.3)

where:

- \( f_M^{\prime}(\mu) = \) prior distribution of \( \mu \),
- \( m' = \) prior mean of \( \mu \), and
- \( \sigma^2 = \) prior variance of \( \mu \).

Multiplying the prior distribution by the likelihood function of equation 6.2 and normalizing the product, the posterior distribution of \( \mu \) is a normal density function given by:
\[ f_{\mu|X}(\mu|x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(\mu-m')^2}{2\sigma^2}} \]  \hspace{1cm} (6.4)

where the posterior mean and variance are expressed by the following two equations, respectively,

\[ m'' = \frac{(1/\sigma'^2)m' + (n/\sigma^2)m}{(1/\sigma'^2) + (n/\sigma^2)} \]  \hspace{1cm} (6.5)

and

\[ \frac{1}{\sigma''^2} = \frac{1}{\sigma'^2} + \frac{n}{\sigma^2} \]  \hspace{1cm} (6.6)

where:

\[ \frac{\sigma^2}{n} = \text{variance of the sample mean.} \]

Equation 6.6 shows that as the variance of the prior distribution decreases, the prior information is given more weight in the determination of the posterior distribution.

### 6.4 Bayesian Analysis With Damage Data from Northridge

The Northridge earthquake is currently the best documented disaster in the history of the United States (OES, 1995). This earthquake caused about 20 billion dollars of direct losses in terms of damage to buildings, utilities, and lifelines. The focus of this study is the structural damage to buildings, specifically to RC frame buildings.

#### 6.4.1 Building Inventory

In order to estimate the likelihood functions in the Bayesian analysis, a description of the inventory of RC frames is required. The Governor's Office of Emergency Services for
the State of California (OES, 1995) provided information about the inventory for RC frame buildings in the city of Los Angeles in an electronic format. From the tax assessor database for the city of Los Angeles, OES inferred that there are a total of about 1240 RC frame buildings, including both residential and non-residential buildings. Figure 6.4 shows the geographical locations of these RC frame buildings in the city of Los Angeles. In addition, the Governor’s Office of Emergency Services for the State of California also provided data on damage in terms of “tagging” and estimated dollar loss data. Immediately after the earthquake, building inspections were carried out. The building inspectors affixed a color tag on an inspected structure according to the following guidelines (OES, 1995):

Green Tag: No apparent hazard found, although repairs may be required. Original lateral load capacity not significantly decreased. No restriction on use or occupancy.

Yellow Tag: Dangerous condition believed to be present. Entry by owner permitted only for emergency purposes and only at own risk. No usage on continuous basis. Entry by public not permitted. Possible major aftershock hazard.

Red Tag: Extreme hazard, may collapse. Imminent danger of collapse from an aftershock. Unsafe for occupancy or entry, except by authorities.

The distribution of the red- and yellow-tagged RC frame buildings within the city of Los Angeles is shown in Figures 6.5 and 6.6, respectively. These figures show 22 red-tagged and 35 yellow-tagged buildings. Although these figures give an idea of the location and extent of damage to RC frame buildings, the extent of damage is only very approximate as the process of judging the safety of a building is subjective. In this study, the extent of damage is inferred from the dollar loss data for the buildings.

The data provided by the Governor’s Office of Emergency Services for the State of California (OES, 1995) also contained information on the age of construction, and the number of stories in the buildings. As the number of mid rise and high rise RC frame buildings is very small, Bayesian analysis is only presented for low rise frames. Furthermore, as defined by Anagnos et al. (1995), low rise frames constructed after 1976
Figure 6.4: Geographical location of all the RC frame buildings in the city of Los Angeles. There are about 1240 buildings.

Figure 6.5: Geographical location of all the yellow-tagged RC frame buildings in the city of Los Angeles. There are 35 yellow-tagged buildings.
Figure 6.6: Geographical location of all the red-tagged RC frame buildings in the city of Los Angeles. There are 22 red-tagged buildings.

may be treated as ductile moment resisting frames. Thus, low rise RC frames built after 1976 are used to update the fragility curves presented in Chapter 5.

6.4.2 Posterior Distribution of the Park-Ang Damage Index

In order to evaluate the posterior distribution of the Park-Ang damage index at a specified level of ground motion, one must estimate the level of seismic excitation as well as the corresponding Park-Ang damage index of all the RC frame buildings. The level of seismic excitation is measured in terms of spectral acceleration in the relevant period band. In this study, the site conditions at the building locations is not taken into account in the Bayesian analysis. For the Northridge earthquake, Somerville et al. (1995) developed spectral acceleration contours at five different periods: 0.3, 0.5, 1.0, 2.0 and 3.0 seconds. The spectral acceleration contours for the east-west and the north-south components of ground motion at a period of 0.3 seconds are shown in Figures 6.7 and 6.8, respectively. The ground motion levels at the building sites are obtained by overlaying the buildings on the spectral acceleration contours. Figures 3.4 and 4.1 show that for low rise frames, the spectral acceleration at 0.3 seconds may be taken to represent the average value. In fact,
the ratio of the dynamic amplification factor at 0.3 seconds to the average value of the
dynamic amplification factor in the range of 0.1 - 0.5 seconds is about 0.97. Thus, it is
reasonable to assume the average value to be equal to the value of the dynamic
amplification factor at 0.3 seconds. For the mid rise frames, the ground motion level may
be obtained from the values of spectral acceleration at periods of 0.5 and 1.0 seconds, by
assuming a suitable variation of spectral acceleration between these two periods.
Similarly, the ground motion level for the high rise frames may be obtained by using the
spectral acceleration values at periods of 1.0, 2.0, and 3.0 seconds. Alternatively, as
suggested in Section 4.1, spectral velocity contours may be used to arrive at the ground
motion level in terms of spectral acceleration for these two classes of frames. However,
the ground motion levels for the mid rise and the high rise frames is not explored as the
number of buildings belonging to the mid rise and high rise frames is very limited in order
to perform the Bayesian analysis. Figures 6.9 and 6.10 respectively present all the low
rise, ductile moment resisting RC frames, and the low rise ductile RC frames for which the
damage was observed. These figures also present the average spectral acceleration
contours used to determine the ground motion level for the frames.

Figure 6.7: Spectral acceleration contours from the Northridge earthquake for the east-
west component at T = 0.3 seconds. The contours are spaced at an interval
of 0.05g, and the value of the innermost contour is 1.4g.
Figure 6.8: Spectral acceleration contours from the Northridge earthquake for the north-south component at $T = 0.3$ seconds. The contours are spaced at an interval of 0.05g, and the value of the innermost contour is 1.4g.

Figure 6.9: Low rise, ductile RC frame buildings and the average spectral acceleration contours from the Northridge earthquake at $T = 0.3$ seconds. There are 184 buildings. The contours are spaced at an interval of 0.05g, and the value of the innermost contour is 1.4g.
Figure 6.10: Seventeen low rise, ductile RC frame buildings for which damage was observed and the average spectral acceleration contours from the Northridge earthquake at $T = 0.3$ seconds. The contours are spaced at an interval of 0.05g, and the value of the innermost contour is 1.4g.

The extent of damage sustained by the buildings, in terms of the Park-Ang damage index, is determined from the damage factors for these buildings. There are two studies which relate damage factors to damage states. The relationship provided in ATC-13 (1985) is given in Table 5.4. Whitman et al. (1974) proposed another relationship which provides a finer discretization at the lower damage levels when compared to the ATC-13 relationship. Table 6.3 provides Whitman et al.'s relationship for the structural damage states.

Table 6.3: Whitman et al.'s relationship between damage factors and damage states.

<table>
<thead>
<tr>
<th>Damage State</th>
<th>Damage Range (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minor</td>
<td>3.5 - 7.5</td>
</tr>
<tr>
<td>Substantial</td>
<td>7.5 - 20</td>
</tr>
<tr>
<td>Major</td>
<td>20 - 65</td>
</tr>
<tr>
<td>Building Condemned</td>
<td>65 - 100</td>
</tr>
</tbody>
</table>

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In this study, the lower bound of the Minor damage state is assumed to have a damage factor of 3.5% as shown in Table 6.3. The other damage states are assumed to have the damage factors given in Table 5.4. De Leon and Ang (1993) arrived at a linear relationship between the damage factor and the Park-Ang global damage index for buildings damaged during the Mexico City earthquake. Therefore, in this study, a piecewise linear relationship is assumed between the damage factor and the Park-Ang damage, the relationship being linear for each damage state. Thus, Tables 3.1, 5.4, and 6.3 are used to estimate the damage index for each building. The Park-Ang damage index can be expressed by means of the following expression:

$$\text{EDI}_i = \text{LDI}_i + \left( \frac{\text{DF} - \text{LDF}_i}{\text{UDF}_i - \text{LDF}_i} \right) \cdot (\text{UDI}_i - \text{LDI}_i)$$

(6.7)

where:

- **i** = damage state from the vector\{none, minor, moderate, severe, and collapse\},
- EDI = estimate of the Park-Ang damage index,
- LDI = minimum value of the Park-Ang damage index for damage state i as given in Table 3.1,
- UDI = maximum value of the Park-Ang damage index for damage state i as given in Table 3.1,
- DF = damage factor, defined as the ratio of repair to replacement cost, obtained from the OES database,
- LDF = minimum value of the damage factor for damage state i, obtained from Tables 5.4 or 6.3, and
- UDF = maximum value of the damage factor for damage state i, obtained from Table 5.4.

The relationships between the damage factor and the Park-Ang damage index inferred from the relationships between the damage states and the damage factor, proposed by Whitman et al. and ATC-13 are shown in Figure 6.11. Figure 6.11 also shows the relationship between the damage factor and the damage index used in this study.

Table 6.4 shows the total number of ductile, low rise RC frame buildings subjected to different levels of ground motion. Although the contours shown in Figures 6.7 through
6.10 are at an interval of 0.05g, the ground motion levels for the buildings have been rounded to the nearest 0.1g level in order to account for some uncertainties associated with the geographical location of the buildings as well as the uncertainty in the contours themselves. Furthermore, the fragility curves in Chapter 5 are obtained from simulations performed for spectral acceleration values at intervals of 0.1g. Thus, the results from Chapter 5 can be directly used in the Bayesian analysis. Table 6.5 lists the buildings for which damage was observed. The spectral acceleration values and the estimated Park-Ang damage indices for these buildings are also listed in Table 6.5.

![Figure 6.11: Mapping of damage factor to the Park-Ang damage index.](image)

Table 6.6 provides the logarithmic standard deviation of the Park-Ang damage index at different levels of ground motion. In this study, this logarithmic standard deviation is considered to be a constant. Table 6.7 provides the parameters of the conjugate prior distribution on the logarithmic mean of the Park-Ang damage index. Table 6.8 lists the sample logarithmic mean of the Park-Ang damage index at different levels of ground motion. This mean was calculated based on the information provided in Tables 6.4 and 6.5. The buildings listed in Table 6.4 for which damage was not observed were assumed to have a Park-Ang damage index of zero. The mean and standard deviation of the Park-Ang damage index were computed at the different levels of ground motion, and the
logarithmic mean of the Park and damage index was calculated by means of the following equation:

\[ m_{\ln(DI)} = \ln(m_{DI}) - \frac{1}{2} \sigma^2_{\ln(DI)} \]  

(6.8)

where:

\[ m_{\ln(DI)} = \text{logarithmic mean of the observed damage indices}, \]
\[ m_{DI} = \text{sample mean of the observed damage indices}, \]
\[ \sigma^2_{\ln(DI)} = \ln \left( 1 + \frac{\sigma^2_{DI}}{m^2_{DI}} \right), \text{ and} \]
\[ \sigma^2_{DI} = \text{sample variance of the observed damage indices}. \]

On inspection of the two databases containing the inventory of all the RC frames in Los Angeles City and the damaged frames, respectively, it was found that the second database contains information on several structures at a location, whereas the first database, in most instances, lists only a single structure for the same location. Thus, the second database contains more precise information on the different RC frames at each location. However, this database is inconsistent with the first database. To overcome this inconsistency between the two databases, a single damage factor was estimated for those sites where the second database listed more buildings than the first.

**Table 6.4:** Total number of low rise RC frame buildings at different levels of ground motion.

<table>
<thead>
<tr>
<th>Spectral Acceleration (g)</th>
<th>Number of Buildings</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>35</td>
</tr>
<tr>
<td>0.5</td>
<td>65</td>
</tr>
<tr>
<td>0.6</td>
<td>56</td>
</tr>
<tr>
<td>0.7</td>
<td>5</td>
</tr>
<tr>
<td>0.8</td>
<td>12</td>
</tr>
<tr>
<td>0.9</td>
<td>3</td>
</tr>
<tr>
<td>1.0</td>
<td>3</td>
</tr>
<tr>
<td>1.1</td>
<td>2</td>
</tr>
<tr>
<td>1.2</td>
<td>3</td>
</tr>
</tbody>
</table>
Table 6.5: Geographical locations, spectral accelerations, and the estimated Park-Ang damage indices for the low rise frames for which damage was observed.

<table>
<thead>
<tr>
<th>Latitude</th>
<th>Longitude</th>
<th>Spectral Acceleration (g)</th>
<th>Park-Ang Damage Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>34.061°N</td>
<td>118.278°W</td>
<td>0.5</td>
<td>0.03</td>
</tr>
<tr>
<td>34.057°N</td>
<td>118.252°W</td>
<td>0.5</td>
<td>0.03</td>
</tr>
<tr>
<td>34.040°N</td>
<td>118.438°W</td>
<td>0.5</td>
<td>0.03</td>
</tr>
<tr>
<td>34.053°N</td>
<td>118.468°W</td>
<td>0.6</td>
<td>0.09</td>
</tr>
<tr>
<td>34.163°N</td>
<td>118.631°W</td>
<td>0.7</td>
<td>0.20</td>
</tr>
<tr>
<td>34.155°N</td>
<td>118.449°W</td>
<td>0.7</td>
<td>0.13</td>
</tr>
<tr>
<td>34.165°N</td>
<td>118.626°W</td>
<td>0.7</td>
<td>0.24</td>
</tr>
<tr>
<td>34.154°N</td>
<td>118.465°W</td>
<td>0.8</td>
<td>0.18</td>
</tr>
<tr>
<td>34.179°N</td>
<td>118.604°W</td>
<td>0.8</td>
<td>0.07</td>
</tr>
<tr>
<td>34.172°N</td>
<td>118.603°W</td>
<td>0.8</td>
<td>0.20</td>
</tr>
<tr>
<td>34.157°N</td>
<td>118.488°W</td>
<td>0.8</td>
<td>0.06</td>
</tr>
<tr>
<td>34.173°N</td>
<td>118.561°W</td>
<td>0.8</td>
<td>0.20</td>
</tr>
<tr>
<td>34.194°N</td>
<td>118.619°W</td>
<td>0.8</td>
<td>0.12</td>
</tr>
<tr>
<td>34.179°N</td>
<td>118.466°W</td>
<td>0.8</td>
<td>0.06</td>
</tr>
<tr>
<td>34.154°N</td>
<td>118.466°W</td>
<td>0.8</td>
<td>0.25</td>
</tr>
<tr>
<td>34.182°N</td>
<td>118.501°W</td>
<td>0.9</td>
<td>0.11</td>
</tr>
<tr>
<td>34.280°N</td>
<td>118.459°W</td>
<td>1.2</td>
<td>0.26</td>
</tr>
</tbody>
</table>

Table 6.6: Logarithmic standard deviation (assumed deterministic) of the Park-Ang damage index for low rise frames when subjected to different levels of ground motion.

<table>
<thead>
<tr>
<th>Spectral Acceleration</th>
<th>Logarithmic Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>1.394</td>
</tr>
<tr>
<td>0.6</td>
<td>1.161</td>
</tr>
<tr>
<td>0.7</td>
<td>1.022</td>
</tr>
<tr>
<td>0.8</td>
<td>1.131</td>
</tr>
<tr>
<td>0.9</td>
<td>1.120</td>
</tr>
<tr>
<td>1.0</td>
<td>1.070</td>
</tr>
<tr>
<td>1.1</td>
<td>1.069</td>
</tr>
<tr>
<td>1.2</td>
<td>1.045</td>
</tr>
</tbody>
</table>

CHAPTER 6. Bayesian Updating of Motion-Damage Relationships
Table 6.7: Parameters of the prior distribution of the mean of the logarithm of the Park-Ang damage index for low rise frames when subjected to different levels of ground motion.

<table>
<thead>
<tr>
<th>Spectral Acceleration (g)</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>-7.474</td>
<td>0.322</td>
</tr>
<tr>
<td>0.2</td>
<td>-7.474</td>
<td>0.322</td>
</tr>
<tr>
<td>0.3</td>
<td>-12.281</td>
<td>0.322</td>
</tr>
<tr>
<td>0.4</td>
<td>-10.484</td>
<td>0.322</td>
</tr>
<tr>
<td>0.5</td>
<td>-8.039</td>
<td>0.322</td>
</tr>
<tr>
<td>0.6</td>
<td>-6.115</td>
<td>0.322</td>
</tr>
<tr>
<td>0.7</td>
<td>-4.955</td>
<td>0.322</td>
</tr>
<tr>
<td>0.8</td>
<td>-4.596</td>
<td>0.322</td>
</tr>
<tr>
<td>0.9</td>
<td>-4.249</td>
<td>0.322</td>
</tr>
<tr>
<td>1.0</td>
<td>-4.023</td>
<td>0.322</td>
</tr>
<tr>
<td>1.1</td>
<td>-3.528</td>
<td>0.322</td>
</tr>
<tr>
<td>1.2</td>
<td>-3.153</td>
<td>0.322</td>
</tr>
</tbody>
</table>

Table 6.8 also provides the parameters of the posterior distribution on the logarithmic mean of the Park-Ang damage index. The posterior parameters are calculated by using equations 6.5 and 6.6. The posterior parameters are only presented for six levels of ground motion for which observed damage data were available. These posterior parameters are used to obtain the probabilities of the different damage states. Figure 6.12 presents the probabilities of the damage states at these six levels of ground motion, along with the probabilities of the damage states at other levels of ground motion presented in Chapter 5. The fitted curves in Figure 6.12 are the same as those shown in Figure 5.4. Thus, Figure 6.12 shows that the probabilities based on the posterior distributions of the damage index are consistent with those presented in Chapter 5.

Since the posterior probabilities agree quite well with the fragility curves of Figure 5.4, the fragility curves are not revised in Figure 6.12. For cases where large differences may be observed, posterior fragility curves should be obtained by fitting curves through the posterior probabilities of the different damage states.
Table 6.8: Parameters of the posterior distribution of the mean of the logarithm of the Park-Ang damage index for low rise frames at different levels of ground motion.

<table>
<thead>
<tr>
<th>Spectral Acceleration (g)</th>
<th>Sample Mean</th>
<th>Posterior Mean</th>
<th>Posterior Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>-8.176</td>
<td>-8.153</td>
<td>0.133</td>
</tr>
<tr>
<td>0.6</td>
<td>-8.504</td>
<td>-8.150</td>
<td>0.124</td>
</tr>
<tr>
<td>0.7</td>
<td>-2.502</td>
<td>-3.976</td>
<td>0.249</td>
</tr>
<tr>
<td>0.8</td>
<td>-2.688</td>
<td>-3.358</td>
<td>0.191</td>
</tr>
<tr>
<td>0.9</td>
<td>-4.020</td>
<td>-4.172</td>
<td>0.262</td>
</tr>
<tr>
<td>1.2</td>
<td>-3.072</td>
<td>-3.118</td>
<td>0.242</td>
</tr>
</tbody>
</table>

Figure 6.12: Updated fragility curves for low rise frames.
6.5 Confidence Bounds on the Fragility Curves

The fragility curves for the three classes of frames presented in Chapter 5 were based on the maximum likelihood value of the median of the Park-Ang damage index at different levels of ground motion. However, there is an uncertainty associated with the median damage index as presented in Section 6.2.4. This uncertainty is taken into account by establishing confidence bounds on the fragility curves. The Bayesian analysis is used to update the median as well as the uncertainty on the median at different levels of ground motion. The Bayesian technique was applied to low rise frames at six levels of ground motion, the results of which are presented in Table 6.8.

The uncertainty on the median of the Park-Ang damage index is used to establish the 90% confidence bounds on the fragility curves for the different damage states. The median is assumed to have a coefficient of variation of 0.33. A lognormal distribution on the median is used to establish the 5% and the 95% fractiles. These fractiles are then used to form the 90% bounds. Figures 6.1 through 6.3 show that the coefficient of variation of the damage index is almost constant. Therefore, the same logarithmic standard deviation of the Park-Ang damage index as obtained from the simulations is used to establish the bounding fragility curves. For low rise frames, the updated logarithmic standard deviation is used to establish the bounds. Figures 6.13 through 6.18 show the maximum likelihood and the bounding fragility curves for the three classes of frames. The 90% confidence bounds on the fragility curves are obtained by arbitrarily fitting lognormal distribution functions to the discrete results, similar to the way fragility curves were obtained in Chapter 5. The median and the logarithmic standard deviation, $\beta$, of these lognormal curves are presented in Tables 6.9 through 6.11. The maximum likelihood fragility curves are referred to as the median fragility curves in Figures 6.13 through 6.18 and in Tables 6.9 through 6.11.

### Table 6.9: Median and logarithmic standard deviations of the median and the bounding fragility curves for low rise frames.

<table>
<thead>
<tr>
<th>Damage State</th>
<th>5% Bounds</th>
<th></th>
<th>Median Curves</th>
<th></th>
<th>95% Bounds</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Median</td>
<td>$\beta$</td>
<td>Median</td>
<td>$\beta$</td>
<td>Median</td>
<td>$\beta$</td>
</tr>
<tr>
<td>Minor</td>
<td>1.17g</td>
<td>0.40</td>
<td>1.48g</td>
<td>0.40</td>
<td>1.82g</td>
<td>0.45</td>
</tr>
<tr>
<td>Moderate</td>
<td>1.55g</td>
<td>0.40</td>
<td>1.93g</td>
<td>0.44</td>
<td>2.48g</td>
<td>0.50</td>
</tr>
<tr>
<td>Severe</td>
<td>2.34g</td>
<td>0.50</td>
<td>3.07g</td>
<td>0.50</td>
<td>4.22g</td>
<td>0.55</td>
</tr>
<tr>
<td>Collapse</td>
<td>3.35g</td>
<td>0.52</td>
<td>4.81g</td>
<td>0.60</td>
<td>6.69g</td>
<td>0.60</td>
</tr>
</tbody>
</table>
Table 6.10: Median and logarithmic standard deviations of the median and the bounding fragility curves for mid rise frames.

<table>
<thead>
<tr>
<th>Damage State</th>
<th>5% Bounds</th>
<th>Median Curves</th>
<th>95% Bounds</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Median</td>
<td>β</td>
<td>Median</td>
</tr>
<tr>
<td>Minor</td>
<td>0.61g</td>
<td>0.32</td>
<td>0.78g</td>
</tr>
<tr>
<td>Moderate</td>
<td>0.86g</td>
<td>0.32</td>
<td>1.11g</td>
</tr>
<tr>
<td>Severe</td>
<td>1.45g</td>
<td>0.37</td>
<td>2.08g</td>
</tr>
<tr>
<td>Collapse</td>
<td>2.34g</td>
<td>0.36</td>
<td>3.46g</td>
</tr>
</tbody>
</table>

Table 6.11: Median and logarithmic standard deviations of the median and the bounding fragility curves for high rise frames.

<table>
<thead>
<tr>
<th>Damage State</th>
<th>5% Bounds</th>
<th>Median Curves</th>
<th>95% Bounds</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Median</td>
<td>β</td>
<td>Median</td>
</tr>
<tr>
<td>Minor</td>
<td>0.43g</td>
<td>0.20</td>
<td>0.54g</td>
</tr>
<tr>
<td>Moderate</td>
<td>0.62g</td>
<td>0.21</td>
<td>0.79g</td>
</tr>
<tr>
<td>Severe</td>
<td>1.07g</td>
<td>0.25</td>
<td>1.50g</td>
</tr>
<tr>
<td>Collapse</td>
<td>1.67g</td>
<td>0.32</td>
<td>2.39g</td>
</tr>
</tbody>
</table>

Figure 6.13: 90% confidence bounds and the median fragility curves for Minor and Severe damage states for low rise frames.
Figure 6.14: 90% confidence bounds and the median fragility curves for Moderate and Collapse damage states for low rise frames.

Figure 6.15: 90% confidence bounds and the median fragility curves for Minor and Severe damage states for mid rise frames.
Figure 6.16: 90% confidence bounds and the median fragility curves for Moderate and Collapse damage states for mid rise frames.

Figure 6.17: 90% confidence bounds and the median fragility curves for Minor and Severe damage states for high rise frames.
Figure 6.18: 90% confidence bounds and the median fragility curves for Moderate and Collapse damage states for high rise frames.

6.6 Summary

The first part of this chapter presented the concepts of randomness and uncertainty as applied to the response of RC frames. The randomness and uncertainty associated with the parameters of RC frames were discussed. In contrast to Chapter 5 which only considered randomness in ground motion and structural parameters in arriving at the fragility curves, this chapter incorporated uncertainty in the fragility curves.

The application of the Bayesian analysis to motion-damage relationships was also presented in this chapter. In this study, only the median of the Park-Ang damage index was considered as a random variable, the logarithmic standard deviation being assumed to be a constant. It is possible to extend the technique presented in this chapter to include the case where both the median and the logarithmic standard deviation are assumed as random variables. The basic approach will remain the same.

The Bayesian technique was then illustrated for low rise RC frames in Los Angeles City subjected to ground shaking during the Northridge earthquake. Due to insufficient amount of data for the mid rise and the high rise frames, the application of the technique
was not attempted for these two classes of frames. For low rise frames, the analytical fragility curves presented in Chapter 5 provide the best estimates of the updated probabilities.

The uncertainty in the median Park-Ang damage index is used to establish the confidence bounds on the fragility curves. For low rise frames, the uncertainty obtained after Bayesian analysis is used to establish the confidence bounds at those levels of ground motion at which Bayesian analysis is performed.
CHAPTER 7

CONCLUSIONS AND FUTURE WORK

This research presented a general method for developing relationships between earthquake ground motion and damage. The motion-damage relationships were presented as fragility curves and damage probability matrices (DPMs). Only structural damage was considered in this study.

7.1 Conclusions

This study proposed a general method for developing fragility curves and DPMs. The following are the main characteristics of this methodology.

- Unlike previous approaches, this methodology uses a systematic approach for developing motion-damage relationships and does not rely either on heuristics or on empirical damage data.
- The method utilizes several critical ground motion characteristics and reflects the nonlinear behavior of the structure when subjected to earthquake ground motions.
- Structural damage at different ground motion levels is evaluated using the Monte Carlo simulation approach.
- The Latin hypercube technique is used to increase the efficiency of the Monte Carlo simulation.
- The methodology is applicable to a wide range of structural classes. In this study, the methodology is demonstrated for reinforced concrete (RC) frames.
- The Bayesian technique is presented that will enable periodic updating of fragilities as damage data become available from future earthquake events.

The major components of the proposed methodology consist of (a) characterization of the potential ground motions, (b) characterization of the nonlinear response of the structure when subjected to extreme dynamic loads, (c) application of the methodology to RC frames, (d) sensitivity study for different structural attributes, and (e) Bayesian technique to update the motion-damage relationships. The conclusions for each of these components are presented below.
Ground Motion Characterization

The methodology developed in this research can be used with any ground motion parameter. However, root mean square (RMS) acceleration and spectral acceleration, $S_a$, for a specified structural period range are used to develop the methodology. The following conclusions are drawn regarding the ground motion characterization for fragility formulation.

- A correlation coefficient of about 0.7 was observed between the Trifunac and Brady strong motion duration and RMS acceleration.
- Appropriate period bands were defined for the computation of average spectral acceleration to characterize ground motion for RC frames. These period bands are 0.1-0.5 seconds for low rise frames, 0.5-0.9 seconds for mid rise frames, and 0.9-2.5 seconds for high rise frames. These period bands are consistent with the behavior of frames within the three structural classes.
- A poor correlation was observed between the Trifunac and Brady strong motion duration and the average spectral acceleration in the three period bands corresponding to the three classes of RC frames. Therefore, a lognormal distribution of strong motion duration, independent of spectral acceleration, with a median of 10.92 seconds and a logarithmic standard deviation of 0.33 was obtained.
- Conditional distributions for the average spectral acceleration in the three period bands for a given MMI were developed for the first time. The parameters of these conditional lognormal distributions were obtained from regression analyses.
- Lognormal distributions of the dynamic amplification factors (DAFs) were verified by the Kolmogorov-Smirnov analysis at the 5% significance level. These distributions can be used in other applications.
- In order to achieve nonstationarity in both amplitude and frequency content in the simulated ground motion, the nonstationary ARMA(2,1) model was utilized. These two nonstationarities are important for evaluating nonlinear response of structures.
- Parameters of the nonstationary ARMA(2,1) model were estimated for an ensemble of time histories. These parameters can be used in general for simulation in other applications.
- The mean spectral shape of the time histories simulated using the nonstationary ARMA(2,1) model is slightly different from the mean spectral shape of the recorded ground motions. Due to this difference, the damage estimates for the mid rise frames are larger when the frames are subjected to the simulated time histories than when the

CHAPTER 7. Conclusions and Future Work 165
frames are subjected to recorded ground motions. On the contrary, the damage
estimates for the high rise frames are smaller when the frames are subjected to
simulated ground motions than when the frames are subjected to recorded ground
motions. However, the differences in the response of the frames are small compared
to the uncertainties in the ground motion and structural parameters like stiffness and
strength of the members.

**Nonlinear Dynamic Analysis**

The methodology involves the nonlinear dynamic analysis of structures subjected to an
ensemble of ground motion time histories. The following conclusions are drawn regarding
the nonlinear behavior of structures.

- Several models for performing nonlinear dynamic analyses were compared in order to
establish the bounds within which the response of the structures may lie. Nonlinear
response from DRAIN-2DX using three elements for each member is similar to that
produced from IDARC2D and CU-DYNAMIX. The differences in the response
predicted by the programs are likely due to the modeling of strain-hardening and the
formulation of the basic inelastic model in the programs.

- The advantages and disadvantages of these three analysis programs are as follows:
  - DRAIN-2DX is computationally a very stable program and is widely used for
    nonlinear analyses of structures. It is computationally very efficient as it uses an
    event-to-event strategy. However, it uses only a bilinear hysteretic model with no
deterioration in the hysteretic behavior.
  - IDARC2D can incorporate stiffness degradation, strength deterioration, and
    pinching in the hysteretic model. It is capable of modeling distributed plasticity.
    However, it is numerically unstable in some analyses.
  - CU-DYNAMIX is capable of performing two- and three-dimensional analyses. It
    models distributed plasticity as well as stiffness degradation. It can analyze only
    bisymmetric sections and can be used only in an interactive mode.
  - Modeling each member using three elements in DRAIN-2DX results in a decrease of
    the drift ratio and the Park-Ang damage index compared to the case when each
    member is modeled by a single element.
Motion-Damage Relationships for RC Frames

The methodology was demonstrated by application to RC frames. The following conclusions are drawn from the motion-damage relationships for RC frames.

- New fragility curves for RC frames were developed that can be used for damage assessment and retrofit decision making. These curves are consistent with ground motion and structural parameter uncertainties and do not depend on heuristics.
- At a given ground motion level, the lognormal distributions are excellent representation of the empirical distributions of the Park-Ang damage index based on the Kolmogorov-Smirnov test validated at the 5% significance level.
- A formulation was developed for estimating the DPMs from the fragility curves. This formulation was applied to obtain DPMs for RC frames.
- ATC-13 DPMs show significant probabilities only for the Minor and Moderate damage states for MMI in the VI to X range. Furthermore, the negligible probability of collapse of the frames at MMI values of XI and XII for the ATC-13 DPMs appear rather unrealistic, particularly in view of the performance of concrete frame structures in recent large earthquakes. The DPMs developed in this study show significant non-zero probabilities for more damage states at a given level of MMI.

Sensitivity Analyses

Sensitivity analyses were carried to study the influence of different structural attributes on the nonlinear dynamic response of structures. The structural attributes included the number of bays in a structure, the second-order effects, and the site conditions. The following conclusions are drawn from the sensitivity analyses.

- The number of bays did not have a significant influence on the nonlinear response of structures in terms of drift ratios, dissipated hysteretic energies, and the Park-Ang damage index.
- The second-order effects were negligible for lower levels of ground motion. However, second-order effects become increasingly important at higher levels of ground motion as the building height increases.
- Significant differences in the nonlinear response of structures were observed for different site condition. Due to the reduction in seismic demand as the period of the
low rise frame elongates, the rock ground motions lead to a considerable reduction in the Park-Ang damage index compared to the firm site ground motions. However, due to higher mode effects for the mid rise and the high rise frames, rock ground motions lead to an increase in the Park-Ang damage index compared to the firm site ground motions.

- For the three sample frames, the deformation and the energy terms of the Park-Ang damage index contribute almost equally to the damage index at different levels of ground motion.
- The overall drift ratios and the Park-Ang damage index at different levels of ground motion are almost perfectly correlated for the three sample frames. A correlation coefficient greater than 0.99 was observed for the three frames.

**Bayesian Analysis**

The following are the summaries and conclusions of the Bayesian analysis presented in this study.

- The technique presented in this research is the first systematic approach to incorporate damage data with synthetic or heuristic motion-damage relationships.
- The uncertainty due to the use of a limited number of representative structures in the development of synthetic fragility curves is reduced by incorporating observed damage data into the fragility curves.
- In the Bayesian analysis, only the median damage index was assumed as a random variable as the coefficient of variation of the Park-Ang damage index at higher levels of ground motion converges to a value of about 1.1 for the low rise frame, 0.5 for the mid rise and the high rise frame. The reduction in the coefficient of variation for the mid rise and the high rise frames is due to the use of a high-pass, bi-lateral Butterworth filter to remove the low frequency components.
- The methodology is demonstrated with sample data on buildings damaged during the Northridge earthquake. A total of 144 ductile, low rise reinforced concrete frame buildings subjected to different levels of ground motion were used in the Bayesian analysis. Out of these, seventeen buildings sustained different degrees of damage.
- The synthetic fragility curves were found to provide the best estimates of the updated probabilities of the different damage states. The updated probabilities lie both above and below the synthetic fragility curves.
• Confidence bounds on the fragility curves were established. The confidence bounds are wide because they account for the uncertainty in the stiffness and strength of the different members and the uncertainty in damping and mass. The confidence bounds also represent the variation in the behavior of the structures belonging to a class of RC frames.

• The parameters of the updated fragility curves can be used as the prior estimates in the Bayesian analysis as more data become available.

7.2 Future Work

This study examined the vulnerability of reinforced concrete frames to earthquake ground shaking. In general, these buildings may be subjected to other earthquake hazards such as liquefaction and landslides. An approach needs to be developed which can estimate the vulnerability of buildings under all hazards due to earthquakes. Furthermore, this research was limited to the evaluation of structural damage. It is known that nonstructural and contents damage can be a significant portion of the total loss. Therefore, a rational approach for evaluating damage to nonstructural components and building contents is needed.

Further research is needed in the simulation of ground motion. A better simulation model which captures the nonstationarity in amplitude and frequency content, along with the characteristics of travel path, distance, and local soil parameters needs to be investigated.

This research relied on previous relationships between economic loss and damage states. These relationships were used to estimate the damage index of buildings subjected to ground motion during the Northridge earthquake. An estimate of the damage index was needed to perform the Bayesian analysis. A more rational relationship between economic loss and damage states is needed. Such an approach was attempted in Chapter 3 where the damage states were defined in terms of crack width. Crack width is a parameter which may be easily related to economic loss. However, crack width can only be used for the Minor and Moderate damage states. Furthermore, the expression for crack width needs to be calibrated or a better expression should be obtained.
In general, a region will consist of buildings belonging to different structural classes, for example, wood, unreinforced masonry, and steel frames. Thus for regional damage evaluation, it is desirable to obtain a consistent set of fragility curves for all structural classes. Such a set is not currently available. This study arrives only at a consistent set of fragility curves for reinforced concrete frames.

Furthermore, the effect of structural characteristics not included in this study also need to be investigated. For example, three-dimensional, non-linear dynamic analyses need to be performed in order to study the effect of plan irregularity on the damage to the structure. For example, the six story Barrington Building is L-shaped and suffered damage during the Northridge earthquake. This building consists of a perimeter frame and irregularly placed shear walls. Mitchell et al. (1995) suggested that torsional effects increased the shear taken by the exterior frames. Furthermore, the column at the southwest corner, farthest from the shear walls, suffered the most damage. Further details on this example as well as the other examples in the remainder of this section can be found in Mitchell et al. (1995) and Holmes and Somers (1996).

The effect of elevation irregularities should also be investigated. Furthermore, in this study, it is assumed that flexural behavior alone causes damage in reinforced concrete members. In reality, different conditions may prevail. For example, in the seven story Saint John's Hospital located in Santa Monica, the second story experienced considerable damage. This story had larger openings compared to other stories, creating an irregularity in stiffness over the height of the building. Thus, the second story experienced damage due to being a weak story. Interestingly, the cracked columns in this story were shorter than the uncracked columns. The shear deformations became important in the short columns leading to shear cracking.

Shear effects were also found to cause damage in Champaign Tower, a 15 story reinforced concrete building located in Santa Monica. This building consists of nonductile moment frames in the longitudinal direction. In this direction, the presence of balcony parapets led to a shortening of the column spans which resulted in their developing diagonal shear cracks in the lower stories.

Another important issue often ignored in analysis is the connection between different members of a reinforced concrete frame. It is assumed that the frame’s joints are rigid and do not undergo deformations. However, they may be inadequate in some cases and lead
to failure of the structure. For example, the Kaiser Permanente building suffered severe
damage due to inadequate connections. The beam-column joints had insufficient amount
of confinement steel which led to the complete shattering of these joints.

The influence of soil-structure effects is not fully explored in this study. To determine
the sensitivity of response to site conditions, the effect of soil-structure interaction is
ignored in this research. Soil-structure effects may be important in some cases, especially
if the building is located on softer soil.

A considerable amount of computational effort is required to perform non-linear
dynamic analyses for estimating damage. Procedures for obtaining fragility curves with
reduced computational effort also need to be investigated.
APPENDIX A

COMPUTATION OF MOMENT AND CURVATURE FOR AN EXAMPLE RC SECTION

The example section is assumed to have overall dimensions of 14" x 18". It has four No. 9 longitudinal bars at the bottom and two No. 9 longitudinal bars at the top. The beam has No. 3 closed stirrups at 4" centers. The cover to the hoops is 1.5". The details of the section are shown in Figure A.1. The reinforcing steel has a trilinear stress-strain relationship shown in Figure A.2. The steel is assumed to have a yield strength, $f_y$, of 60 ksi, a modulus of elasticity, $E$, of 29,000 ksi, and a strain hardening modulus, $E_s$, of $E/60$. The strain at the commencement of strain hardening, $e_{sh}$, is assumed to be 0.03. The concrete is assumed to have a cylinder strength of 4 ksi. The Kent and Park stress-strain relationship discussed in Chapter 4 is used for concrete.

![Figure A.1: Details of the example section.](image)

Based on the spacing of stirrups and the section dimensions, the ratio of the volume of transverse reinforcement to volume of concrete core, $\rho_s$, is calculated as:

$$\rho_s = \frac{0.11 \times 2(10.63 + 14.63)}{11 \times 15 \times 4} = 0.0084$$

(A.1)

The slope of the descending portion of the Kent and Park stress-strain relationship, $Z$, is calculated by the following expression as defined in Chapter 4:
\[ Z = \frac{0.5}{\frac{3 + 0.002f'_c}{f'_c - 1000} + \frac{3}{4} \rho_s \sqrt{\frac{b'^*}{s_h} - 0.002}} \]  \hspace{1cm} (A.2)

where:

\[ f'_c = \text{compressive strength of concrete}, \]
\[ b'^* = \text{width of confined core measured to outside of hoops, and} \]
\[ s_h = \text{spacing of hoops}. \]

\[ f_c' = 1.4 f'_c \]

\[ E_s = E/60 \]

**Figure A.2:** Stress-strain relationship for reinforcing steel.

Thus,

\[ Z = \frac{0.5}{(3+8)/3000 + \frac{3}{4} \times 0.0084 \sqrt{11/4} - 0.002} = 41 \]  \hspace{1cm} (A.3)

The calculations of the moments and curvatures corresponding to the yield and ultimate conditions are presented below.
Yield Condition

The yield moment and curvature are the respective values when the tensile (bottom) steel first reaches its yield strain. The following values are computed when the tensile steel yields:

Depth of neutral axis: 5.98 in
Compressive strain in extreme concrete fiber: 0.0013
Yield Moment: 3186 k-in
Yield Curvature: 0.0002 per in

Ultimate Condition

The ultimate moment and curvature are the respective values when either the compressive strain in concrete reaches its ultimate value or when the tensile steel reaches its ultimate strength. For this example section, the ultimate condition is governed by the concrete reaching its ultimate compressive strain. As implemented in IDARC2D (Kunnath and Reinhorn, 1994), the ultimate strain in concrete is defined as the strain corresponding to a stress of 20% of the compressive strength of concrete on the falling branch of the Kent and Park stress-strain relationship. This value is close to the value obtained by using expression 4.7 suggested by Paulay and Priestley (1992). It is assumed that the concrete outside the hoops does not spall off. The following values are computed when the ultimate condition is reached:

Depth of neutral axis: 4.67 in
Compressive strain in extreme concrete fiber: 0.0215
Ultimate Moment: 3603 k-in
Ultimate Curvature: 0.0046 per in
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