ON ADJECTIVAL COMPARATIVES

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DOCTOR OF PHILOSOPHY

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Graduate school was one of the more difficult endeavors I have yet faced. For those that supported me, if I have not acknowledged you by now, then I have failed both you and myself.
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1 | Beginnings

The syntax and semantics of adjectival comparatives like (1.1a) are complicated matters that have concerned both syntacticians and semanticists for at least 100 years.

(1.1)  
   a. Orcutt is taller than Smith (is)  
   b. The maximal degree of height Orcutt possesses is greater than the maximal degree of height Smith possesses

It is generally agreed that (1.1b) is a suitable paraphrase of the meaning of (1.1a), which suggests, both implicitly and explicitly, that any formal syntactico-semantic analysis of adjectival comparatives should make reference to (at least) the notions of **degrees**, **scales**, and **measures**.

Much attention has been paid to such concepts as they related to the semantics of, not just adjectival comparatives, but comparatives of all types, including but not limited to nominal and verbal ones. From the most general perspective, the purpose of this dissertation is to give a syntactico-semantic analysis of statements like (1.1a). That is to say, I will restrict my attention to adjectival comparatives in the hopes that the analysis I propose can be modified accordingly and generalized to account for the semantics of comparatives of all types. More to the point, the path I will pursue in this dissertation takes each of the aforementioned concepts seriously, but, as will become clearer through the course of this dissertation, moves away from standardly assumed degree-based analyses like Seuren (1973), von Stechow (1984), Heim (1985), Kennedy (1997) and Heim (2006) (to name just a few) to a more Cresswellian-one (Cresswell 1976).

This might seem odd, if only because recent work like Kennedy (1997) can too be understood in a Cresswellian-light. However, there are many aspects of Cresswell’s (1976) proposal—philosophical and formal—and in the work presented here, degrees themselves
1.1 PRIMARY MOTIVATION

will not be understood as proper objects in my semantic ontology. If degrees exist at all, they will be understood as real numbers necessary only when considering a small sub-class of comparative constructions. I will argue that, in making such a move, one gets a better and more general treatment of the syntax and semantics of adjectival comparatives. I will propose a transparent albeit Spartan semantic representation language that makes no use of various misbehaved covert operators at the level of logical form that are traditionally present in the various semantic analysis of adjectival comparatives considered here. By *misbehaved*, I simply mean operators that over-complicate, and oftentimes, get the semantic facts involving adjectival comparatives wrong. I will show that, even under the assumption of such minimal representations, my analysis gets the semantic facts involving adjectival comparatives right.

1.1 Primary motivation

The idea of a degree-less semantics for adjectival comparatives is not new and has been pursued by various authors such as Klein (1980), Larson (1988), van Rooij (2008) and van Rooij (2011) (again, to name just a few). However, one of the main philosophical motivations for the formal decisions I make is to tease apart the concepts of *order* and *measure* as they relate to the semantics of adjectival comparatives. This idea too is not new and is made clear by Sapir (1944; p. 93) in his discussion of *grading*.

The first thing to realize about grading as a psychological process is that it precedes measurement and counting. Judgments of the type “A is larger than B” or “This can contains less milk than that” are made long before it is possible to say, e.g., “A is twice as large as B” or “A has a volume of 25 cubic feet, B a volume of 20 cubic feet, therefore A is larger than B by 5 cubic feet,” or “This can contains a quart of milk, that one 3 quarts of milk, therefore the former has less milk in it.” In other words, judgments of quantity in terms of units of measure or in terms of number always presuppose, explicitly or implicitly, preliminary judgments of grading.

Understanding the concept of *grading* simply as the ability to order a set of objects from least to greatest (or greatest to least) in regard to the extent to which those objects possesses
a particular property, I take Sapir (1944) to mean this: measurement presupposes the ability
to grade; however, grading need not involve measurement.

Teasing apart the concepts of grading, or more precisely, ordering, from the concept of
measurement will play a central role in this dissertation. Degree-based approaches, to my
mind at least, seem to conflate these notions. However, as it turns out, many constructions
involving comparative adjectives—constructions that continue to give degree semanticists
problems—do not actually require us to make reference to measurements or degrees at all.
(Sometimes these terms are used interchangeably.) However, some do—a point van Rooij
(2011) argues at length. Take, for example, the difference between (1.2a) and (1.2b).

(1.2) a. Orcutt is taller than Smith or Jones
    b. Orcutt is five feet taller than Smith

I will show that, in order to capture the meaning of (1.2a), we need only assume a binary
ordering over the entity space that intuitively corresponds to the property HAVING MORE
HEIGHT (or equivalently, HAVING LESS HEIGHT). However, the example in (1.2b) explicit-
ly mentions a measurement: five feet. To understand this expression’s meaning, in some
sense at least, we, as competent speakers of English, must be able to grasp the concept of
measurement, which involves more than simply having more height. Teasing apart these
two concepts should be reflected in our semantics of adjectival comparatives.

A large part of this dissertation will be spent separating those comparative constructions
that do require reference to measures from those that do not. The point I want to make is
this: it is not necessary to define a semantics for adjectival comparatives that generalizes
to the worst-case scenario by positing semantic representation that always make reference
to degrees. A semantics of comparatives need make reference to measurements only when
they are necessary. As I will show, it is possible to construct a sort of tiered semantics
in which measurements can be built on top of, or rather plugged in, when necessary. For
the most part, degrees need not be present in our semantics; and even if we understand
measurements as being identified with degrees, they should only be present when required.
Separating measurements from the basic semantics of adjectival comparatives will prove
illuminating both from a conceptual and formal perspective in accord with Sapir’s (1944)
original intuitions.
1.2 Outlook

The recent work of authors like Heim (2006), van Rooij (2008), Beck (2010) and Alrenge and Kennedy (2013) has shown quite convincingly that problems exist for all current semantic analyses of adjectival comparatives. Empirical inadequacies of theoretical paradigms afford us (at least) two options.

- We can be evolutionary and modify pre-existing theories in attempt to broaden empirical coverage; or
- we can be revolutionary and propose an alternative paradigm.

I opt for proposing a new, degree-less semantics for adjectival comparatives. (Of course there is nothing new under the sun, and ultimately what I am proposing is an amalgamation of ideas that have been floating in the literature for (at least) 40 years.) Because I am proposing a novel paradigm, I must show (at least) two things.

- There exists a class of empirical data that current semantic analyses of adjectival comparatives cannot (obviously) account for; and
- such troublesome data and the data the previous analyses could account for can naturally be captured under my new analysis.

Chapter 2 reviews several previous analyses of adjectival comparatives. As has been pointed out by authors like von Stechow (1984), Heim (2006), van Rooij (2008) and Beck (2010) among many more, the semantics of logical expressions, including but not limited to Booleans, e.g., or and and, quantifiers, anyone, someone, and everyone, as well as negation, e.g, no and not, as they appear under the scope of than have proven to be particularly difficult.

Specifically, I will review two classes of analyses, which Schwarzschild (2008) refers to as the A-not-A and Greater-than analyses. In particular, I will focus my attention on Seuren’s (1973) and Larson’s (1988) A-not-A analyses on the one hand, and von Stechow’s (1984) and Heim’s (2006) Greater-than ones on the other. Following van Rooij (2008), I will argue that, in light of the data considered, Larson’s (1988) analysis is to be
preferred. However, I will show that Larson’s (1988) analysis still faces empirical challenges: it cannot (obviously) account for the difference in meaning between the two types of negative elements, i.e., *not* versus *no*, as they appear in (1.3).

(1.3) a. Orcutt is not taller than Smith  
    b. Orcutt is no taller than Smith

The data in (1.3) is important because it suggests that the A-not-A analysis is inadequate, suggesting to me that an alternative analysis of the semantics of adjectival comparatives should be pursued.

**Chapter 3** presents the formal framework in which I will cast my analysis, which is based in the neo-Montagovian tradition (Montague 1974b; Gallin 1975; Muskens 1995). On the semantic side of things, I choose to set my analysis in terms of Muskens’s (1995) logic TT$_2$. Although non-standard, TT$_2$ might be thought of a sort of streamlined version of Gallin’s (1975) TY$_2$ which in turn might be thought of a more user-friendly version of Montague’s (1974c) original Intensional Logic. Specifically, I choose this logic as my semantic representation language, as it allows me naturally to build off of an insight taken from Cresswell (1976), namely that the semantics of adjectival comparatives involve trans-world scales comprised of entity/world pairs.

On the syntactic side of things, I combine various insights from research into categorial grammar (Oehrle 1988; Morrill 1994; Jäger 2005; Morrill 2011), specifically Barker and Shan’s (2013) work on continuations, to give a straightforward analysis of the syntax of adjectival comparatives. Continuations will play an essential role in capturing certain **scopal** and **ellipsis** facts involving adjectival comparatives.

**Chapter 4** represents the first application of my analysis. I look specifically at so-called **phrasal comparatives** like (1.4).

(1.4) Orcutt is taller than Smith

The majority of this chapter will be spent showing how my analysis is able to capture the semantics of phrasal comparatives as they involve the various logical expressions and operators reviewed in chapter 2. Crucially, I will show that my analysis can capture the same range of data that both the A-not-A and Greater-than analyses can capture and more.
Along the way, I will explore some pleasant consequences of my analysis, highlighting its relative simplicity yet broad empirical coverage.

Chapter 5 builds on the ideas of chapter 4 and extends them to capture the semantics of so-called clausal comparatives like (1.5).

(1.5) Orcutt is taller than Smith is

On the face of it, the statements in (1.2) and (1.5) appear to be logically equivalent. However, upon closer investigation, the syntax and semantics of clausal comparatives seem to bring with them a new set of problems not encountered when dealing solely with phrasal comparatives.

I will argue that gradable properties, in many instances, can be envisaged as denoting trans-world scales. Combined with Barker and Shan’s (2013) work on continuations, I show that the syntax and semantics of phrasal comparatives can actually be reduced to, or seen in another light, derived from the syntax and semantics I give for clausal ones. This property of my formal analysis can be seen as an answer to a long-standing question posed by Heim (1985; p. 2), namely “whether phrasal comparatives, or at least some of them, should be analyzed as elliptical variants of clausal ones.”
2  |  A look back

2.1 Introduction

In this chapter, I will review previous work on the semantics of adjectival comparatives. Given the sheer amount of such work, it would no doubt be impossible to review it all. Instead, I choose breadth over depth and will review two classes of approaches to the semantics of adjectival comparatives, which, as mentioned in §1.2, are often referred to as the A-not-A and Greater-than analyses. These analyses are so-named because of their **logical shape**, which will become clearer through the course of this chapter.

I warn that in carving up the analysis space in this way, I have performed a massive conflation by grouping the various semantic analyses of comparatives into only two groups. The fact of the matter is, the work of the authors cited in the footnote below vary in a variety of interesting ways, both in regard to their technical assumptions and philosophical commitments. However, I choose to consider these two classes of analyses for two reasons: (i) historically, they are the most enduring—over the past 40 years, they have consistently been revised and rebooted in some form or another; and consequently, (ii) they are the most well-known, studied, and reviewed. Specifically, in this chapter, I will review certain semantic predictions of

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2.2. PRELIMINARIES

- Seuren’s (1973) original A-not-A analysis and what can be seen as Larson’s (1988) revision of it; and

- von Stechow’s (1984) original Greater-than analysis and what can be seen as Heim’s (2006) revision of it.

I use the phrase what can be seen as because to what extent either Larson (1988) or Heim (2006) would view their analyses as a revision of Seuren’s (1973) or von Stechow’s (1984) respectively, I do not know. To be clear, Larson’s (1988) analysis is actually a revision of Klein’s (1980), who, as opposed to Seuren (1973), works in a degree-less semantics. (By a degree-less semantics, I mean a semantic representation language that does not explicitly quantify over degrees.) However, the logical shape of both Seuren’s (1973) and Klein’s (1980) analysis is an A-not-A one, and consequently, Larson’s (1988) insights directly port over into Seuren’s (1973) framework. So, I see no real harm in casting things in this way.

Because it is one of the primary purposes of this dissertation to show precisely how a degree-less semantics for adjectival comparatives can be given, it might seem odd that, in this chapter at least, I am couching everything in terms of degrees. I do this for several reasons. The majority of the current work on adjectival comparatives is set in a degree-based semantics. Thus, it just makes sense to review the literature in these terms. Second, and perhaps more important, as I mentioned in chapter 1, it is necessary to show that there exist certain empirical phenomena that these analyses cannot account for, at least not obviously. That there exist such phenomena leaves open the possibility of an alternative analysis. Of course, this is not to say that a degree-based approach cannot be modified to account for the data. This is simply not the route I will pursue in this dissertation.

2.2 Preliminaries

In this chapter, I will be working in a degree-based semantics (Seuren 1973; Cresswell 1976; von Stechow 1984; Heim 1985; Bierwisch 1989; Kennedy 1997; Heim 2000; Kennedy 2001; Schwarzschild and Wilkinson 2002; Kennedy and McNally 2005; Kennedy 2007b; Lassiter 2011). Degrees themselves are generally understood as abstract units of measure that individuals possess. Intuitively, degrees are taken to represent the various sorts of
things people possess when they possess a gradable property like height or weight. So, for example, when an individual is said to possess a degree of height, the idea is that that individual actually possesses a degree of height—whatever that may be. And when an individual is said to possess a degree of cleverness that individual actually possesses a degree of cleverness—whatever that may be; and so on and so forth.

If pressed, it does not matter to me what degrees actually are in some metaphysical sense. (See Cresswell (1976) for a definition of degrees as equivalence class of individuals; and Kennedy (2001) for a definition of degrees as a convex set of points.) For the purposes of this dissertation, what matters is that degrees are the sorts of objects that are taken to comprise scales—one of the fundametal concepts of degree semantics. As such, I will take degrees to be unstructured, basic objects in my ontology.

2.2.1 Degrees and scales

It is often difficult to know exactly what people mean by the term scale in the literature. It would be impossible to go through every author’s conception of a scale, exploring in detail its ontology and mathematical properties. I imagine the structure people have in mind when they think of a scale is something resembling that in figure 2.1.

From a mathematical perspective, this structure is nothing more than an ordered set, that is to say a relational structure \((\mathbb{D}, >)\), where \(\mathbb{D}\) is a (non-empty) set, usually understood as being composed of degrees, and the relation symbol ‘\(>\)’ is understood as denoting a binary relation over \(\mathbb{D}\) that itself satisfies certain axioms, or rather has certain mathematical properties. At a minimum, it is reasonable to assume a scale is a strict order as given by definition 1.
2.2. PRELIMINARIES

Definition 1 (Strict order). A relation $R$ over a set $X$ is said to be a **strict order** just in case for all $x, y, z \in X$

1. $\neg (xRx)$ \textit{Irreflexivity}

2. $(xRy) \land (yRz) \Rightarrow xRz$ \textit{Transitivity}

Irreflexivity and transitivity jointly imply that $R$ is **asymmetric**. Specifically,

3. $xRy \Rightarrow \neg (yRx)$ \textit{Asymmetry}

Some examples of strict orders are shown in figure 2.2. Note that, in some strict orders, not every element is related to every other element. This is made clear in the contrast between figure 2.2(a) and figure 2.2(b).

When formalizing the notion of a **scale**, we would like to rule out mathematical structures like figure 2.2(a). So, I enforce the **trichotomy condition** and consider only the class of **strict total orders**, also referred to as **linear orders**. This is formalized in definition 2.

Definition 2 (Strict total order/linear order). A relation $R$ over a set $X$ is said to be a **strict total order**, or rather **linear order**, just in case $R$ is a strict order per definition 1 and for all $x, y \in X$

1. $(xRy) \lor (yRx) \lor (x = y)$ \textit{Trichotomy}

where the binary relation symbol ‘$=$’ is understood as **metaphysical equality**.

An example of a linear order is given in figure 2.2(b).
2.2. PRELIMINARIES

2.2.2 A language of degrees

Having defined the concept of a scale, I now turn to giving a representation language will serve as the basis for formalizing all of the A-not-A and Greater-than analyses I consider in this chapter. On the semantic side of things, I will use the following stock of variables.

- \( x, y, z, x', y', z' \ldots \) are variables of type \( e \) for entity;
- \( d, d' \ldots \) are variables of type \( d \) for degree;\(^2\)
- \( D, D' \ldots \) are variables of type \((d, t)\);
- \( P, P' \ldots \) are variables of type \((e, t)\);
- \( Q, Q' \ldots \) are variables of type \(((e, t), t)\); and
- \( D, D' \ldots \) are variables of type \((e, (d, t))\).

I assume the set of degrees \( D \) forms a strict total order under the binary relation represented by the non-logical constant ‘\( H'\) of semantic type \((d, (d, t))\). This relation will be read greater degree of height. Moreover, I will assume the existence of the non-logical constant ‘\( \text{tall} \)' of semantic type \((e, (d, t))\), which itself will be the logical translation of the English expression tall. The logical expression ‘\( \text{tall} \)' will denote a binary possession relation between individuals and degrees of height. Finally, I assume that the possession of degrees of height is downward closed. I do this by adopting the axiom in (2.1).

\[(2.1) \quad (d >_H d') \land (\text{tall} (x) (d)) \rightarrow \text{tall} (x) (d')\]

for all \( x \in D_e \) and \( d, d' \in D_d \)

The statement in (2.1) reads if an individual possesses a degree of height, then for all degrees of height less than it, that individual possesses those degrees of height as well. Oftentimes, the relevant degree for comparisons is an individual’s maximal degree of height, which itself is derived by the function/operator \( \text{max} \), as given by (2.2).

\(^2\)Here, \( d \) is doing double duty as both a type and an object-language variable. I trust the reader’s ability to differentiate the two.
2.2. PRELIMINARIES

The value of $\max$ is a function that takes a set of degrees and returns the maximal degree of that set under the relation denoted by the non-logical constant ‘$>_H$’. Putting everything together, the degree-world that I have in mind is depicted pictorially in figure 2.3. Here, the sets represent the sets of degrees of height the individuals Orcutt, Smith, and Jones possess.

2.2.3 The A-not-A analysis

I now have everything at my disposal to explicate the A-not-A analysis and the Greater-than analysis in detail. Seuren (1973, 1984) analyzes the meaning of a sentence involving the comparative adjective taller as in (2.3).

(2.3) Orcutt is taller than Smith (is) $\sim \exists d (\text{tall}(o)(d) \land \neg (\text{tall}(s)(d)))$

The logical translation in (2.3) is true just in case there exists a degree of height that Orcutt possesses but Smith does not. Given that the set of degrees is downward closed (see, again, (2.1)), it follows that every degree of height that Smith possesses, Orcutt does and more.
At first glance, the A-not-A analysis might appear odd, as it posits negation that itself does not appear in the sentence’s natural language form. However, it is often useful to translate natural expressions into a pseudo-logico-philosophical language, because such a language (often) traffics in concepts that we, as analysts, have more robust intuitions about. Moreover, it (often) allows the analyst to do away with (often) misleading or confusing aspects of, say, ordinary English. (Paraphrasal semantic analyses were very much at the heart of Quine’s (1960) philosophical program.) To better garner the philosophical intuitions behind the A-not-A analysis, consider the way in which Schwarzschild (2008; p. 309) puts it.

Suppose we have two objects with monetary value, A and B, and some monetary threshold. Suppose that A meets or exceeds the threshold and B does not. It follows that A is more expensive than B is. Conversely, if it is true that A is more expensive than B, it follows that there must be some monetary threshold that A meets or exceeds that B does not.

The idea of a threshold is cashed out in different terms by different authors. Kamp (1975), for example, takes thresholds to be precisifications, whereas Klein (1980) understands them to be degree modifiers (but see also van Rooij (2008, 2010b, 2011) for an extension of Klein’s (1980) basic idea). Ultimately, it does not matter what thresholds are. For the purposes of this chapter, I will simply assume that they are degrees.

2.2.4 The Greater-than analysis

The Greater-than analysis, on the other hand, accords better with my pre-theoretical intuitions as to what exactly an adjectival comparative means. The most well known Greater-than analysis is von Stechow’s (1984), and it is given in (2.4).

\[(2.4)\]

\[
\text{Orcutt is taller than Smith (is) } \sim \\
\max (\lambda d (\text{tall} (o) (d))) \succ_H \max (\lambda d' (\text{tall} (s) (d'))) 
\]

The statement in (2.4) is true just in case the maximal degree of height Orcutt possesses is greater than the maximal degree of height Smith possesses.
However (un)natural the A-not-A and Greater-than analyses might feel, interestingly enough, a little bit of reflection will indicate that, given my assumptions about the nature of degrees and scales (see, again, §2.2.1 and §2.2.2), the analyses in (2.3) and (2.4) are logically equivalent. This is just to say that, from an empirical perspective at least, although their logical shape differ, they equivalently capture the truth-conditions of statements like *Orcutt is taller than Smith*. That the statements are equivalent is proven in theorem (1).

**Theorem 1.** The statements in (2.3) and (2.4) are logically equivalent.

**Proof.** I prove the left-to-right direction (⇒) of the equivalence, leaving the right-to-left (⇐) direction to the reader, as it is proven in a similar fashion.

- ⇒ Let $d$ be a witness. By assumption, Orcutt possesses $d$, but Smith does not. Because possession of degrees is downward closed (see, again, definition (2.1)), $d$ is greater than all degrees $d'$ Smith possess. If $d$ is the maximal degree that Orcutt possesses, then the proof is completed. If not, then Orcutt’s maximal degree of height can be represented by $d''$ such that $d'' > H d > H d'$. So, the maximal degree height Orcutt possessess is greater than the one Smith does.

- ⇐ Analogous to the above.

Given that both analyses are logically equivalent, if we restrict ourselves to simple cases like *Orcutt is taller than Smith* and neglect more complex semantic data, it is impossible to determine which analysis is (or is not) materially adequate in the sense that it can capture the data. I turn, now, and consider a wider range of data to see if the two analyses can be teased apart in terms of their ability to capture the sort of inferences patterns speakers of English judge to be intuitively valid.

### 2.3 Boolean or

In this section, I will investigate how the various analyses considered so far deal with Boolean *or* as it appears under the scope of *than.*
2.3. BOOLEAN OR

Both Cresswell (1976) and von Stechow (1984), claim that the following inference in (2.5) involving the connective or is valid.

(2.5)  

a. Orcutt is taller than Smith or Jones

b. \(\iff\) Orcutt is taller than Smith is and Orcutt is taller than Jones is

The idea is that a colloquial speaker of English will judge the conclusion in (2.5b) as (logically) following from (or entailed by) the premise in (2.5a). This inference can be schematized as in (2.6).

(2.6)

\[
\frac{\text{than}(A \lor B)}{\text{than}(A) \land \text{than}(B)}
\]

Cresswell (1976) goes so far as to say that under its ordinary reading, or, as it appears in (2.5a), has a strengthened, conjunctive reading. Under this reading, this statement is compatible with only a situation like the one shown in figure 2.4.

The idea of natural language inference will play a central role in this dissertation. To quote Barwise (1981; p. 372)

[the conclusion in (2.5b)] is judged valid because of what the sentences mean, not because it can be translated into a formal argument in this or that formal

---

3I use the terms following (or derived from) and entailed by interchangeably with the knowledge that, from a logical perspective at least, one is a proof-theoretic notion while the other is a model-theoretic one.
system of logical inference. Formal accounts of logic have to square with such judgments, not vice versa.

What I take Barwise (1981) to be getting at is this: those inferences which a competent speaker of, say, English judge to be (in)valid are taken as a primary data source inasmuch as grammaticality judgements are in syntax. It is the job of the semanticist to come up with a semantic theory that derives those inferences which are judged to be valid by some speaker of English and does not derive those inferences which are judged to be invalid. In the same way Chomsky, for example, wanted a syntactic theory, i.e., a grammar, to generate all and only the grammatical sentences of a particular natural language like English; and Davidson wanted the true, interpretative (bi-conditionals) of a semantic meta-language, the semantic tradition, should, to my mind at least, be seen as one that looks to develop logical fragments with theories that derive all and only the valid inferences of a language like English.

No doubt such an enterprise is complicated by the fact that it necessitates a formal theory of validity which carefully distinguishes between semantic and pragmatic validities. So, for the remainder of this dissertation, I will rely on a variety of natural language inferences—teasing apart the semantic ones from the pragmatic ones—as a guide in constructing my semantic theory.

The A-not-A analysis

Both the A-not-A and Greater-than analyses can capture the inference in (2.6). To see this, first consider the A-not-A analysis of this inference, which is shown in (2.7).

(2.7)

Orcutt is taller than Smith or Jones \( \sim \)

\[ \exists d (\text{tall}(o)(d) \land \neg (\text{tall}(s)(d) \lor \text{tall}(j)(d))) \]

\textit{A-not-A analysis: conjunctive reading of disjunction}

By De Morgan’s laws, the equivalence in (2.8).
(2.8)

\[ \exists d \left( \text{tall} \left( o \right) \left( d \right) \land \neg \left( \text{tall} \left( s \right) \left( d \right) \lor \text{tall} \left( j \right) \left( d \right) \right) \right) \iff \exists d \left( \text{tall} \left( o \right) \left( d \right) \land \left( \neg \text{tall} \left( s \right) \left( d \right) \land \neg \text{tall} \left( j \right) \left( d \right) \right) \right) \]

The statements in (2.8) are true just in case Orcutt possesses a degree of height that neither Smith possesses nor Jones possesses. Given that the set of degrees of height an individual possesses is downward closed (see, again, (2.1)), it follows that the maximal degree of height that Orcutt possesses is greater than the maximal degree of height Smith possesses and the maximal degree of height Jones possesses.

**The Greater-than analysis**

The Greater-than analysis also correctly predicts the conjunctive interpretation of disjunction. Proponents of this analysis capture it, as shown in (2.9).

(2.9)

\[ \text{Orcutt is taller than Smith or Jones } \sim \quad \max(\lambda d \left( \text{tall} \left( o \right) \left( d \right) \right)) >_H \max(\lambda d' \left( \text{tall} \left( s \right) \left( d' \right) \lor \text{tall} \left( j \right) \left( d' \right) \right)) \]

*Greater-than analysis: conjunctive reading of disjunction*

Importantly, the \( \lambda \)-statement in (2.9) is equivalent to its set-theoretic rendition in (2.10).

(2.10)

\[ \max(\lambda d \left( \text{tall} \left( o \right) \left( d \right) \right)) >_H \max(\lambda d' \left( \text{tall} \left( s \right) \left( d' \right) \lor \text{tall} \left( j \right) \left( d' \right) \right)) \iff \max(\left\{ d \in D_d \mid \text{tall} \left( o \right) \left( d \right) \right\}) >_H \max(\left\{ d' \in D_d \mid \text{tall} \left( s \right) \left( d' \right) \right\} \cup \left\{ d' \in D_d \mid \text{tall} \left( j \right) \left( d' \right) \right\}) \]

Observe that the statements in (2.10) are true just in case the maximal degree of height Orcutt possesses is greater than the maximal degree of height of the union of the set of degrees of height Smith possesses and the set of degrees of height Jones possesses. The maximal
2.3. BOOLEAN OR

Orcutt's max height

Smith's max height

Jones' max height

\( f \)

Figure 2.5: The function \( f \) maps a set of degrees to a (subset) of degrees

degree of this derived set is the degree of height the taller of the two men possesses.

To better understand this point, observe that \( \lambda \)-abstracting over the free variable ‘\( d \)’ in the statement ‘\( \text{tall} (s)(d) \lor \text{tall} (j)(d) \)’ can be understood set-theoretically in terms of the function \( f \) depicted in figure 2.5. There, \( f \) represents a mapping from the set of degrees of height to the union of the set of degrees of height Smith possesses and the set of degrees of height Jones possesses.

2.3.2 The disjunctive interpretation of disjunction

Notice, however, that there exists a reading in which, under the scope of \textit{than}, \textit{or} is interpreted disjunctively. This is shown by the example in (2.11).

(2.11) Orcutt is taller than Smith or Jones, but I don’t know who

If interpreted conjunctively, then (2.11) would be odd, as it would contradict the fact that speaker is committed to the truth of the statements \textit{Orcutt is taller than Smith} \textit{is} and \textit{Orcutt is taller than Jones} \textit{is}. However, (2.11) is perfectly felicitous, showing that the conjunctive interpretation of disjunction is cancelable—possibly suggesting that the conjunctive interpretation of disjunction is a pragmatic phenomenon.\footnote{This idea is explored in detail in §2.7 and §2.8 of this chapter.} I conclude, then, that in (2.11), \textit{or} has its standardly assumed disjunctive reading; so, the inference in (2.12) is also valid.
2.3. BOOLEAN OR

Figure 2.6: Possible situations of Orcutt’s, Smith’s, and Jones’ heights consistent with (2.12a) under the reading in (2.12b)

(2.12)  
\[ \text{a. Orcutt is taller than Smith or Jones} \]
\[ \text{b. } \iff \text{Orcutt is taller than Smith is or Orcutt is taller than Jones is} \]

In quasi-formal notation, the inference in (2.12) can be schematized, as in (2.13).

(2.13)  
\[ \frac{\text{than} (A \lor B)}{\text{than} (A) \lor \text{than} (B)} \]

On its disjunctive reading, the statement in (2.12a) is compatible with situations like the ones depicted in figure 2.6.

Without further assumptions, say, some form of movement at logical form, neither the A-not-A analysis nor the Greater-than one can capture the weakened, disjunctive reading of \textit{or}. This is easily attributable to the relative scope of negation (\(\neg\)) and \textit{max}, as shown in (2.14) and (2.15) respectively.
2.3. BOOLEAN OR

<table>
<thead>
<tr>
<th>Lexeme</th>
<th>Type</th>
<th>Representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orcutt</td>
<td>(e,t)</td>
<td>λP(P(o))</td>
</tr>
<tr>
<td>tall</td>
<td>(e,(d,t))</td>
<td>tall</td>
</tr>
<tr>
<td>er</td>
<td>(e,(d,t)),((e,t),(t)),((e,t),(t)))</td>
<td>λRλQ∃d(Q(λx(∃(x(d))))∧Q(λx(¬(∃(x(d)))))))</td>
</tr>
<tr>
<td>Smith</td>
<td>(e,t)</td>
<td>λP(P(o))</td>
</tr>
</tbody>
</table>

Table 2.1: A sample lexicon for a Larsonian-style analysis

(2.14) \[ \exists d \left( \text{tall} (o)(d) \land \neg \left( \text{tall} (s)(d) \lor \text{tall} (j)(d) \right) \right) \]

A-not-A analysis: scope of negation

(2.15) \[ \max (\lambda d (\text{tall} (o)(d))) > H \max \left( \lambda d (\text{tall} (s)(d) \lor \text{tall} (j)(d)) \right) \]

Greater-than analysis: scope of the max-operator

In the case of (2.14), negation scopes over disjunction, which by De Morgan’s, forces a conjunctive interpretation of disjunction. In the case of (2.15), max scopes over disjunction, again forcing a conjunctive interpretation. So, both the original A-not-A and Greater-than analyses face the problem of meaning under-generation.

2.3.3 Larson’s revised A-not-A analysis

In order to capture the inference pattern in (2.13), Larson (1988) proposes a revised A-not-A analysis. His solution is simple: force negation to take obligatory scope under disjunction as opposed to over it. This is accomplished not by some form of movement at, say, logical form, but rather directly through his lexical entry for the comparative morpheme -er.

To better understand this point, I have provided a sample lexicon for a Larsonian-style analysis of the semantics of adjectival comparatives. It is shown in table 2.1. (See, again, §2.2.2 for variable types.) Assuming a syncategorematic rule for disjunction, which allows for the coordination of two expressions of the same syntactic type, (2.16a) shows the semantic representation Larson’s (1988) analysis predicts for the inference in (2.13). I give a partial derivation of (2.16a) in (2.16b), mixing aspects of the syntactic derivation with its
2.3. BOOLEAN OR

(a) Situation 1  
(b) Situation 2  
(c) Situation 3

Figure 2.7: Possible situations of Orcutt’s, Smith’s, and Jones’ heights consistent with (2.16a)

(corresponding semantic composition in order to aid in readability.)

(2.16)  

\[
\exists d \left( \text{tall}(o)(d) \land \neg \text{tall}(s)(d) \lor \neg \text{tall}(j)(d) \right)
\]

\[
\exists d \left( \text{tall}(o)(d) \land \neg \text{tall}(s)(d) \lor \neg \text{tall}(j)(d) \right)
\]

\[
\lambda Q \exists d (Q'(\lambda x (\text{tall}(x)(d))) \land \neg (\text{tall}(s)(d)) \lor \neg (\text{tall}(j)(d)))
\]

\[
\lambda Q \lambda Q' \exists d (Q'(\lambda x (\text{tall}(x)(d))) \land Q(\lambda x (\neg (\text{tall}(x)(d))))))
\]

\[
\lambda P (P(o)) \land \lambda P'(P(o))
\]

\[
\lambda P (P(s) \lor P(j))
\]

\[
\lambda Q \lambda Q' \exists d (Q'(\lambda x (\text{tall}(x)(d))) \land Q(\lambda x (\neg (\text{tall}(x)(d))))))
\]

Larson’s (1988) analysis: disjunctive reading of disjunction

From a model-theoretic perspective, the statement in (2.16a) is true just in case Orcutt possesses a degree of height that Smith does not possess or that Jones does not possesses. Again, given that possession of degrees is downward closed (see, again, (2.1)), this guarantees that Orcutt’s maximal degree of height will be greater than either Smith’s or Jones’—perhaps both. The set of situations consistent with this semantic representation are represented graphically in figure 2.7.
To see more clearly why Larson’s (1988) analysis can capture the disjunctive interpretation of disjunction whereas the original A-not-A analysis cannot, compare the relative scope of negation in (2.17) and (2.18).

\[
(2.17) \quad \exists d \left( \text{tall}(o)(d) \land \neg \left( \underbrace{\text{tall}(s)(d) \lor \text{tall}(j)(d)}_{\text{scope of negation}} \right) \right)
\]

\textit{Original A-not-A analysis: scope of negation}

\[
(2.18) \quad \exists d \left( \text{tall}(o)(d) \land \left( \underbrace{\neg \text{tall}(s)(d)}_{\text{scope of negation}} \lor \underbrace{\neg \text{tall}(j)(d)}_{\text{scope of negation}} \right) \right)
\]

\textit{Larson’s (1988) analysis: scope of negation}

As shown in the above, Larson’s (1988) analysis requires disjunction to take obligatory scope over negation, whereas the original A-not-A analysis requires disjunction to scope under negation. However, this means that Larson’s (1988) is only able to capture the inference in (2.13) at the expense of the one in (2.6). So, he also runs into the problem of meaning under-generation, but for the exact opposite reason the original A-not-A and Greater-than analyses do.

However, as I mentioned in §2.3.2, one can argue that the conjunctive interpretation of disjunction, as exhibited by the inference in (2.6), is a pragmatic phenomenon. We should only expect our semantics to deliver the normal, disjunctive interpretation of disjunction. Under this assumption, Larson’s (1988) analysis does not under-generate meanings at all, but rather, perfectly fits the data.

2.3.4 Heim’s revised Greater-than analysis

Heim (2006) proposes a revision to the Greater-than analysis that can be extended to capture both of the inferences in (2.6) and (2.13).\(^5\) Specifically, her analysis is based on positing a covert operator \(\Pi\) at the level of syntax and logical form. She does not actually

\(^5\)This means that Heim (2006) commits herself to the fact that the strengthened, conjunctive interpretation of \(\text{or}\), as this expression appears under the scope of \(\text{than}\), is in fact a semantic phenomenon such that the semantic component of the grammar should account for it.
2.3. BOOLEAN OR

<table>
<thead>
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</thead>
<tbody>
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<td>o</td>
</tr>
<tr>
<td>tall</td>
<td>(e, (d, s))</td>
<td>(\lambda x \lambda d (\text{tall}(x)(d)))</td>
</tr>
<tr>
<td>er</td>
<td>(d, (d, t))</td>
<td>(\lambda d \lambda d' (d &gt;_H d'))</td>
</tr>
<tr>
<td>II</td>
<td>((d, t), ((d, t), t))</td>
<td>(\lambda D \lambda D' (D(\text{max}(D'))))</td>
</tr>
<tr>
<td>Smith</td>
<td>e</td>
<td>s</td>
</tr>
</tbody>
</table>

Table 2.2: A sample lexicon for a Heimian-style analysis

give an explicit analysis of comparative constructions involving or under the scope of than. However, given standard assumptions about the semantics of natural language coordination, relativized of course to a degree-based setting, it is not hard to bring out her system’s predictions.

To be clear, I provide a sample lexicon necessary to get her proposal off the ground in table 2.2 and, again, tacitly assume a single syncategorematic rule for coordinating expressions of the same syntactic type. (See, again, §2.2.2 for variable types; and §4.4.1 for an example of such a syncategorematic rule.)

I begin, first, by showing how Heim (2006) is able to capture the inference in (2.6). In (2.19a), I give the semantic representation she predicts for this reading, showing a partial derivation of it in (2.19b).\(^6\)

\(^6\)Observe that Heim’s (2006) semantic representation in (2.19a) is the same as the original Greater-than analysis shown in (2.9).
2.3. BOOLEAN OR

\[(2.19)\]

a. Orcutt is taller than Smith or Jones \(\sim \Rightarrow\)

\[\max (\lambda d (\text{tall}(o)(d))) >_H \max (\lambda d' (\text{tall}(s)(d') \lor \text{tall}(j)(d')))\]

b. \(\lambda_d D(D(\max (\lambda_d (\text{tall}(o)(d))))))\)

\(\lambda_3 D(D(\max (\lambda_d (\text{tall}(o)(d))))))\)

Heim’s (2006) analysis: conjunctive reading of disjunction

A few things about the derivation in (2.19b). Heim (2006) defends the existence of the \(\Pi\) operator on morpho-syntactic grounds: “\(\Pi\) is generated in the degree-argument position of an adjective, where it combines with whatever is traditionally generated in that slot.” (Heim 2006; p. 14). (But see also Beck (2010) and Aloni and Roelofsen (2011) for a more detailed explanation of Heim’s (2006) analysis. Also see Larson (1988) and Schwarzschild and Wilkinson (2002) from which parts of Heim’s (2006) syntactic analysis is based on.)

For the purposes of this discussion, whether or not the existence of this operator can be defended on such grounds is irrelevant. What is important is that the semantic representation in (2.19a) is equivalent to the ones in (2.10). The situation the value of this representation is consistent with is shown pictorially in figure 2.8. In this way, Heim (2006) is able to derive the strengthened, conjunctive reading of disjunction.

Now for the inference in (2.13). The semantic representation of this inference is given in (2.20a), and a partial derivation is given in (2.20b).
Figure 2.8: Possible situations of Orcutt’s, Smith’s, and Jones’ heights consistent with (2.19a)

(2.20)  a.

Orcutt is taller than Smith or Jones →

\[ \max(\lambda d(tall(o)(d))) >_H \max(\lambda d'(tall(s)(d'))) \lor \max(\lambda d(tall(o)(d))) >_H \max(\lambda d'(tall(j)(d'))) \]

b.

\[ \max(\lambda d(tall(o)(d))) >_H \max(\lambda d'(tall(s)(d'))) \lor \max(\lambda d(tall(o)(d))) >_H \max(\lambda d'(tall(s)(d'))) \]

Heim’s (2006) analysis: disjunctive reading of disjunction

The logical statement in (2.20a) is true just in case Orcutt’s maximal degree of height is greater than Smith’s or it is greater than Jones’—perhaps both. The sorts of situations the value of this representation is consistent with are shown in figure 2.9.

In sum, the difference in meaning between (2.19a) and (2.20b) is gotten by letting the operator Π scope both above and below disjunction, as it allows for \( \text{max} \) to take varying scope. The varying scope of the latter operator are shown in (2.21) and (2.22).
2.3. BOOLEAN OR

Figure 2.9: Possible situations of Orcutt’s, Smith’s, and Jones’ heights consistent with (2.20a)

\[
\begin{align*}
(2.21) \quad \text{max}(\lambda d (\text{tall}(o)(d))) >_H \text{max} \left( \lambda d' \left( \begin{array}{c}
\text{tall}(s)(d') \\
\text{tall}(j)(d')
\end{array} \right) \right) \\
\text{scope of the max-operator}
\end{align*}
\]

Heim’s (2006) analysis: scope of the max-operator

\[
\begin{align*}
(2.22) \quad \text{max}(\lambda d (\text{tall}(o)(d))) >_H \text{max} \left( \lambda d' \begin{array}{c}
\text{tall}(s)(d') \end{array} \right) \vee \\
\text{scope of the max-operator}
\end{align*}
\]

Heim’s (2006) analysis: scope of the max-operator

Because of the flexibility of applicability of this operator (Π) at the level of logical form, Heim’s (2006) analysis presumably fares better than Larson’s (1988) in that she is able to capture both of the inferences in (2.6) and (2.13). However, this is only the case if we accept that both readings should be represented semantically, not pragmatically.
2.4 BOOLEAN AND

In this section, I will investigate how the various analyses considered deal with the Boolean and as it appears under the scope of than.

2.4.1 The conjunctive interpretation of conjunction

To round out the dataset, observe now that the inferences in (2.23) and (2.24), involving and under the scope of than, behave only in their expected directions.

(2.23)  
\begin{align*}
& a. \text{ Orcutt is taller than Smith and Jones} \\
& b. \implies \text{ Orcutt is taller than Smith is or Orcutt is taller than Jones is}
\end{align*}

(2.24)  
\begin{align*}
& a. \text{ Orcutt is taller than Smith and Jones} \\
& b. \iff \text{ Orcutt is taller than Smith is and Orcutt is taller than Jones is}
\end{align*}

Under the scope of than, Boolean and has only its normal conjunctive reading, and is schematized in (2.25).

(2.25)

$$
\frac{\text{than} (A \land B)}{\text{than} (A) \land \text{than} (B)}
$$

Specifically, the statements in (2.23a) and (2.24a) are compatible only with situations like the one in figure 2.10.
2.4. BOOLEAN AND

| |= | \( \lor \) | \( \land \) |
|---|---|---|
| or | ✓ | ✓ |
| and | x | ✓ |

Table 2.3: Entailment patterns of the natural language booleans *or* and *and* as they appear under the scope of *than*.

Evidently, there exists an asymmetry between the valid inferences *or* licenses and the ones *and* does. This is summarized succinctly in table 2.3. The rows of this table can be read as corresponding to the natural language expressions *or* and *and* as they appear under the scope of *than* in an adjectival comparative; and the logical operators disjunction (\( \lor \)) and conjunction (\( \land \)) can be read as the readings those expressions license. Note that the *or/\( \land \)* interpretation is a possible pragmatic one.

**The A-not-A analysis**

As it turns out, neither the A-not-A analysis nor the Greater-than analysis can capture the inference in (2.25): they both predict that the sentence has a weakened, disjunctive meaning. Beginning with the A-not-A analysis, I have depicted its analysis of the inference in (2.25) in (2.26).

(2.26)

Orcutt is taller than Smith and Jones \( \sim \)

\[ \exists d (\text{tall}(o)(d) \land \neg(\text{tall}(s)(d) \land \text{tall}(j)(d))) \]

*A-not-A analysis: non-existent disjunctive reading of conjunction*

The logical statement in (2.26) is equivalent to the one in (2.27).
\[ \exists d (\text{tall}(o)(d) \land \neg (\text{tall}(s)(d) \land \text{tall}(j)(d))) \iff \exists d (\text{tall}(o)(d) \land (\neg \text{tall}(s)(d) \lor \neg \text{tall}(j)(d))) \]

The representation in (2.27) is true just in case there is a degree of height that Orcutt possesses that Smith does not possess or Jones does not possess.

**The Greater-than analysis**

The Greater-than analysis makes a similar prediction and is shown in (2.28).

\[ \text{Orcutt is taller than Smith and Jones} \sim \]

\[ \max (\lambda d (\text{tall}(o)(d))) >_H \max (\lambda d' (\text{tall}(s)(d') \land \text{tall}(j)(d')))) \]

Greater-than analysis: non-existent disjunctive reading of conjunction

Note that the representation in (2.28) is equivalent to the set-theoretic one in (2.29).

\[ \max (\lambda d (\text{tall}(o)(d))) >_H \max (\lambda d' (\text{tall}(s)(d') \land \text{tall}(j)(d')))) \iff \]

\[ \max (\{ d \in D_d \mid \text{tall}(o)(d) \}) >_H \max (\{ d' \in D_{d'} \mid \text{tall}(s)(d') \} \cap \{ d' \in D_{d'} \mid \text{tall}(j)(d') \}) \]

The statement in (2.29) is true just in case Orcutt is taller than Smith is or Orcutt is taller than Jones is. Specifically, the statement is true just in case the maximal degree of height Orcutt possesses is greater than the maximal degree of height of the intersection of the set of degrees of height Smith possesses and the set of degrees of height Jones possesses. Importantly, the maximal degree of this derived set is the degree of height the shorter of the two possesses.
To better understand this point, observe that \( \lambda \)-abstracting over the free variable \( 'd' \) in the statement \( \text{tall}(s)(d) \land \text{tall}(j)(d) \) can be understood set-theoretically in terms of the function \( f \) depicted in figure 2.11. There, \( f \) depicts a mapping from the set of degrees of height to the intersection of the set of degrees of height Smith possesses and the set of degrees of height Jones possesses. Both semantic representations in (2.26) and (2.28) are compatible with the situations depicted in figure 2.12.

Again, it is clear why both the A-not-A and Greater-than analyses fail to capture the meaning of \( \text{and} \) as it appears under the scope of \( \text{than} \): in the case of the former analysis, negation scopes over conjunction, which by De Morgan’s laws, forces a disjunctive interpretation of \( \text{and} \); whereas in the case of the latter, \( \text{max} \) scopes over conjunction, again, causing a disjunctive interpretation of \( \text{and} \). This is shown clearly in (2.30) and (2.31) respectively.

\[
(2.30) \quad \exists d \left( \text{tall}(o)(d) \land \neg \left( \text{tall}(s)(d) \land \text{tall}(j)(d) \right) \right)
\]

A-not-A analysis: scope of negation

\[
(2.31) \quad \max(\lambda d \text{(tall}(o)(d))) > \max_{H} \left( \lambda d \text{(tall}(s)(d) \land \text{tall}(j)(d)) \right)
\]

\text{scope of the max-operator}
2.4. BOOLEAN AND

Greater-than analysis: scope of the max-operator

In this case, both the A-not-A and Greater-than analyses face the problem of both meaning under-generation in the case or, but also over-generation in the case of and: they fail to derive a meaning that does exist—the disjunctive interpretation of disjunction—but predict one that does not—the disjunctive interpretation of conjunction.

2.4.2 Larson’s revised A-not-A analysis

Larson’s (1988) A-not-A analysis can capture the normal conjunctive reading of conjunction. He is able to do this for precisely the same reasons it was captured the weakened reading of or: for him, negation obligatorily scopes under conjunction, as opposed to over it. The semantic representation of the inference in (2.25) his analysis gets is shown in (2.32).

(2.32)

\[
\exists d (\text{tall} (o) (d) \land (\neg \text{tall} (s) (d) \land \neg \text{tall} (j) (d)))
\]

Larson’s (1988) analysis: conjunctive reading of conjunction

The representation in (2.32) is true just in case there exists a degree of height that Orcutt possess that neither Smith nor Jones possess. Specifically, this representation is consistent
with the situation depicted in figure 2.13.

To see more clearly why Larson’s (1988) analysis gets the facts right and the original A-not-A analysis does not, compare the relative scope of negation in (2.33) and (2.34).

\[(2.33) \; \exists d \left( \text{tall}(o)(d) \land \neg \left( \underbrace{\text{tall}(s)(d) \land \text{tall}(j)(d)}_{\text{scope of negation}} \right) \right) \]

**Original A-not-A analysis: scope of negation**

\[(2.34) \; \exists d \left( \text{tall}(o)(d) \land \neg \left( \underbrace{\neg \text{tall}(s)(d) \land \neg \text{tall}(j)(d)}_{\text{scope of negation}} \right) \right) \]

**Larson’s (1988) analysis: scope of negation**

Because Larson’s (1988) analysis does not allow for negation to scope over conjunction, it cannot be pushed through by De Morgan’s laws. Thus, his analysis perfectly fits the data.

2.4.3 Heim’s revised Greater-than analysis

Heim’s (2006) revised Greater-than analysis predicts that *and* is ambiguous between a weakened, disjunctive interpretation, as well as a conjunctive one. To see this, one must assume a syncategorematic rule for coordinating syntactic expressions of the same type with Boolean *and*. Both representations and the situations they are consistent with are shown
2.4. BOOLEAN AND

Figure 2.14: Possible situations of Orcutt’s, Smith’s, and Jones’ heights consistent with (2.35)

below.\footnote{Observe that Heim’s (2006) semantic representation in (2.35) is the same as the original Greater-than analysis shown in (2.28).}

(2.35)

Orcutt is taller than Smith and Jones \(\leadsto\)

\[
\max(\lambda d^{\text{tall}}(o)(d))) >_H \max(\lambda d'(s)(d') \land \text{tall}(j)(d'))
\]

\textit{Heim’s (2006) analysis: non-existent disjunctive reading of conjunction}

(2.36)

Orcutt is taller than Smith and Jones \(\leadsto\)

\[
\max(\lambda d^{\text{tall}}(o)(d))) >_H \max(\lambda d'(s)(d')) \land \\
\max(\lambda d^{\text{tall}}(o)(d))) >_H \max(\lambda d'(s)(d'))
\]

\textit{Heim’s (2006) analysis: conjunctive reading of conjunction}

For the same reasons Heim (2006) is able to capture the conjunctive and disjunction interpretations of \textit{or}, she over-generates meanings in the case of \textit{and}. This is because the operator \(\Pi\) is able to scope both above and below conjunction. As a result, \(\max\) is able to scope both above and below conjunction.
2.4. BOOLEAN AND

The relative strengths and weaknesses of the four semantic analyses of adjectival comparatives are shown in table 2.4. In sum,

- both the original A-not-A and Greater-than analyses under-generate meanings with respect to Boolean *or*; and both under- and over-generate with respect to *and*.

- Larson’s (1988) revised A-not-A analysis under-generates meanings with respect to Boolean *or*; but perfectly fits when it comes to *and*.

- Heim’s (2006) revised Greater-than analysis perfectly fits the data with respect to Boolean *or*; but over-generates meanings when it comes to *and*.

I think it’s clear that both Larson’s (1988) and Heim’s (2006) revised A-not-A and Greater-than analyses mark improvements over the respective analyses they are extensions of. Stepping back, I think it’s clear what one would want out of an ideal A-not-A and Greater-than analysis.

- To capture the strengthened, conjunctive interpretation of *or*, negation (¬) and *max* must be able to scope under disjunction (∨); and to capture the normal, disjunctive interpretation of *or*, negation and *max* must scope over disjunction (∨).

- To capture the normal, conjunctive interpretation of *and*, negation (¬) and *max* must be able to scope over conjunction (∧) and never under it.

One might argue that Heim’s (2006) analysis is to be preferred to Larson’s (1988) because to over-generate meanings is better than to under-generate them. This is because it
2.5. *THE QUANTIFIER ANYONE*

<table>
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<td>Greater-than analysis</td>
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<tr>
<td>Heim’s (2006) revised</td>
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Table 2.4: The various entailment patterns the various analyses considered are able to capture

is now possible to begin to engage in a meaning-blocking game so-to-speak by positing constraints, say, at logical form that prevent Π from scoping over conjunction. However, as Alrenga and Kennedy (2013; p. 43) point out, “it is not clear that this can be done in a non-ad hoc manner, given that the proposed operator is specific to comparatives (and perhaps, other degree constructions).” From a pragmatic perspective, it may also be argued that it is to be preferred of a semantic analysis to derive the weak(er) interpretations of statements and rely on general pragmatic inferential devices to derive their strengthened interpretations, in which case, Larson’s (1988) analysis captures all of the relevant inference patterns and should be preferred.

2.5 The quantifier *anyone*

In this section, I will investigate how the various analyses considered deal with the quantifier *anyone* as it appears under the scope of *than*. von Stechow (1984) claims that the following inferences in (2.37) involving the quantifier *anyone* are valid.

(2.37) a. Orcutt is taller than anyone else
    b. ⇔ Orcutt is taller than everyone else
    c. ⇔ For everyone\(_1\) other than Orcutt, Orcutt is taller than them\(_1\)
2.5. THE QUANTIFIER ANYONE

Under the assumption that *anyone* is to be analyzed as the existential quantifier ($\exists$), under the scope of *than*, it carries with it a strengthened, universal meaning. The statement in (2.37a) is compatible only with situations like the one depicted in figure 2.16.

In the meta-language, interpreting the semantics of existential quantifier as disjunction ($\lor$) and the universal as conjunction ($\land$), the inference in (2.38) seems to be valid.

\[
\text{than} (A_1 \lor \ldots \lor A_n) \\
\text{than} (A_1 \land \ldots \land \text{than} (A_n))
\]

where $n < \omega$

There is an obvious parallel between the inference in (2.6) and (2.38). Because the semantics of the existential quantifier can be reduced to the semantics of disjunction, it would make sense that this inference schema patterns like the one in (2.6). The question becomes: does this quantifier have its standardly assumed existential meaning? The strengthened interpretation of *anyone* is not obviously cancelable in the way in which the strengthened interpretation of *or* is.

\[
\text{#Orcutt is taller than anyone, but I don’t know who}
\]

Contrasted with (2.11), the data in (2.39) demonstrates that the behavior of *or* and *anyone* under the scope of *than* is not completely analogous.

---

8 Analyzing *anyone* as the existential ($\exists$) is a controversial assumption and one that is discussed in detail in §2.7. Some authors have proposed that *anyone* be given universal semantics.
2.5. THE QUANTIFIER ANYONE

2.5.1 The A-not-A analysis

That being said, both the original A-not-A and Greater-than analyses can capture the strengthened, universal interpretation of the existential. To see this, consider, first, the A-not-A analysis.

(2.40)

Orcutt is taller than anyone else \( \sim \)

\[ \exists d (\text{tall}(o)(d) \land \neg \exists x (x \neq o \land \text{tall}(x)(d))) \]

A-not-A analysis: universal reading of the existential

The semantic representation in (2.40) is logically equivalent to the one in (2.41).

(2.41)

\[ \exists d (\text{tall}(o)(d) \land \neg \exists x (x \neq o \land \text{tall}(x)(d))) \Leftrightarrow \exists d (\text{tall}(o)(d) \land \forall x (x \neq o \rightarrow \neg (\text{tall}(x)(d)))) \]

The logical statement in (2.41) is true just in case there exists a degree of height that Orcutt possesses that no one else but he (=Orcutt) does.

2.5.2 The Greater-than analysis

Similarly, the Greater-than approach is able to capture the apparent validity of (2.38) in the following way.

(2.42)

Orcutt is taller than anyone else \( \sim \)

\[ \max (\lambda d (\text{tall}(o)(d))) \succ_H \max (\lambda d' (\exists x (x \neq o \land \text{tall}(x)(d')))) \]

Greater-than analysis: universal reading of the existential

Importantly, the \( \lambda \)-statement in (2.42) is equivalent to its set-theoretic rendition in (2.43).
2.5. THE QUANTIFIER ANYONE

\[ \max(\lambda d (\text{tall}(o)(d))) > H \max(\lambda d' (\exists x (x \neq o \land \text{tall}(x)(d')))) \iff \]
\[ \max(\{d \in D_d | \text{tall}(o)(d)\}) > H \max(\bigcup_{x \neq o} \{d' \in D_d | \text{tall}(x)(d')\}) \]

The statement in (2.43) is true just in case the maximal degree of height Orcutt possesses is greater than the maximal degree of height of the union of the sets of degrees of height that all individuals but Orcutt possess. This union represents the degrees of height the tallest person other than Orcutt possesses. (See, again, figure 2.5 to harness intuitions regarding the behavior of max in this context.) So, just like the A-not-A analysis, the proponents of the Greater-than approach can account for (2.38).

The reason both analyses are successful is clear: in the case of the A-not-A one, negation out-scopes the existential quantifier and can subsequently be pushed through by De Morgan’s laws. In the case of the Greater-than one, max too out-scopes the existential. This is shown in (2.44) and (2.45) respectively.

\[ \exists d \left( \text{tall}(o)(d) \land \neg \left( \exists x (x \neq o \land \text{tall}(x)(d)) \right) \right) \]
\[ \text{A-not-A analysis: scope of negation} \]

\[ \max(\lambda d (\text{tall}(o)(d))) > H \max \left( \lambda d' (\exists x (x \neq o \land \text{tall}(x)(d'))) \right) \]
\[ \text{Greater-than analysis: scope of max-operator} \]

However, it is precisely because of the relative scope of negation and max that both the original A-not-A and Greater-than analyses face immediate problems when confronted with the quantifiers someone and everyone.
2.6. THE QUANTIFIERS SOMEONE AND EVERYONE

In this section, I will show that, under the scope of *than*, the quantifiers *someone* and *everyone* behave in their expected way from the perspective of natural language inference. Establishing the pre-theoretical (in)validities of these expressions is important, as they will serve as a baseline in which to compare the quantifier *anyone*. Such a baseline is necessary simply because, as mentioned previously, there is debate in the literature as whether to identify the semantics of *anyone* with the existential ($\exists$) or universal ($\forall$) quantifier. To begin with, observe the (in)validity of the inferences in (2.46).

(2.46) a. Orcutt is taller than someone else
    b. $\not\equiv\Rightarrow$ Orcutt is taller than everyone else
    c. $\not\equiv\Rightarrow$ For everyone$_1$ other than Orcutt, Orcutt is taller than them$_1$

Under the scope of *than*, *someone* has only its normally assumed existential meaning. Specifically, this statement is compatible with situations like the ones depicted in figure 2.17. As Christopher Potts points out to me, the right-to-left entailment ($\Leftarrow$) of (2.46a) to (2.46b) goes through under the assumption that *else* carries with it an existence presupposition. However, because the right-to-left ($\Rightarrow$) direction does not hold in general, the expression *everyone* and *someone* cannot be said to be logically equivalent as they appear under the scope of *than*. And this is the important point. This inference pattern in (2.46) is schematized in (2.47).
2.6. THE QUANTIFIERS SOMEONE AND EVERYONE

(2.47)\[
\frac{\text{than } (A_1 \lor \ldots \lor A_n)}{\text{than } (A_1) \lor \ldots \lor \text{than } (A_n)}
\]

where $n < \omega$

However, the schema in (2.47) is not representative of all indefinites under the scope of \textit{than}. Authors like von Stechow (1984), Beck (2010) and Lassiter (2010) (among many more) have all argued that, under the scope of \textit{than}, (some) indefinites have both an existential and quasi-universal interpretation. To see this, consider the inferences in (2.48) and (2.49) respectively.

(2.48) \begin{align*}
\text{a. Orcutt is taller than a man} \\
\text{b. } \Leftrightarrow \text{ Orcutt is taller than some man}
\end{align*}

\textit{Existential interpretation of the existential}

(2.49) \begin{align*}
\text{a. Orcutt is taller than a man} \\
\text{b. } \Leftrightarrow \text{ Orcutt is taller than any man}
\end{align*}

\textit{Universal interpretation of the existential}

Beck (2010) points out that the intuitions surrounding the inference in (2.49b) are sensitive. And that is why I choose the term \textit{quasi-universal} to describe this particular interpretation. There seems to be a subtle meaning difference between \textit{any man} and, say \textit{every man}, assuming the latter were to be substituted for the former in (2.49b). That being said, such an interpretation has been observed in the wild so-to-speak by Lassiter (2010; (32)).

(2.50) \text{I made the Yankee hat more famous than a Yankee can} \hspace{1cm} (\text{Jay-Z, Empire State of Mind}, \text{The Blueprint 2, Roc Nation, 2009})

According to Lassiter (2010; p. 48), “From the context of this song, the artist is clearly not saying that he made the Yankees hat more famous than some particular Yankee (baseball player) can, or more than a typical Yankee can, but more than any Yankee can.”

There are seemingly a lot of issues surrounding the quasi-universal interpretation of (2.49), possibly relating to scope, genericity, and free-choice. I do not claim to have an explanation, nor will I provide a formal analysis of the ambiguity certain indefinites under
the scope of *than* allow for. It may very well be the case that *someone* is the limiting case and that most indefinites behave like *a* and are ambiguous between an existential and quasi-universal reading. I do not have a solution to the semantic asymmetry that exists between *someone* and *a* as they appear under the scope of *than*. For now, I want to merely draw an analogy between the behavior of *a* as it appears in comparative constructions and another puzzling class of English constructions. I hope such an analogy will shed light on the behavior of indefinites as they appear in certain syntactico-semantic contexts of English. If I am on the right track, then every formal theory of the semantics of comparatives must be rethought as they relate to indefinites, including the one I propose in this dissertation.

Dating back to (at least) Quine (1960) (but see also Montague (1974c), Zimmermann (1993), Larson et al. (1997), Forbes (2000), Moltmann (2008) and Forbes (2010)), authors have drawn a distinction between extensional transitive verbs and intentional transitive ones like the ones shown in (2.51) and (2.52) respectively.

(2.51) Orcutt finds a unicorn

*Extensional verb*

(2.52) Orcutt seeks a unicorn

*Intensional verb*

Intensional transitive verbs like *seek* are thought to have (at least) three properties that distinguish them from extensional ones.

(2.53) a. Intensional transitive verbs do not preserve truth under the substitution of identicals.

b. Intensional transitive verbs are ambiguous between *de dicto* and *de re* readings, i.e., they are referentially opaque.

c. Intensional transitive verbs do not have existential commitment.

Limiting my attention simply to the property described in (2.53b), the statement *Orcutt seeks a unicorn* allows for both a specific (*de dicto*) and non-specific (*de re*) reading, as shown in (2.54) and (2.55) respectively.
2.6. **THE QUANTIFIERS SOMEONE AND EVERYONE**

(2.54)  
   a. Orcutt seeks a unicorn  
   b. \( \Rightarrow \) Orcutt seeks a particular unicorn

*De dicto (specific reading)*

(2.55)  
   a. Orcutt seeks a unicorn  
   b. \( \Rightarrow \) Orcutt seeks any object satisfying the description *unicorn*

*De re (non-specific reading)*

The idea is that intensional transitive verbs create what is sometimes referred to as a referentially opaque context in their object position. Such a context allows for multiple readings of an indefinite like *a* as it appears in *a unicorn*. As far as I can tell, the difference in interpretations between (2.48) and (2.49) parallels the one in (2.54) and (2.55). This possibly suggests that *than* too creates a referentially opaque context. I will have little more to say about this problem other than referential opacity is a complex and not particularly well-understood phenomenon. But if *than* creates such a context, this would be, as far as I know at least, a novel discovery.\(^9\) For now, I will focus my attention specifically on the quantifier *someone* and refer the reader to both Beck (2010) and Lassiter (2011) for alternative explanations for the puzzling behavior of indefinites under the scope of *than*.

Let me turn, now, to the invalidity of the inference in (2.56). This example shows us that, under the scope of *than*, the quantifier *everyone* has only its normally assumed universal meaning.

\(^9\)I imagine any solution will follow in the spirit of Montague (1974c) who argues that intensional transitive verbs like *seek* should be understood as a binary relation between individuals and properties of (first-order) properties, i.e., generalized quantifier meanings, as in (i).

(i) \( \text{seek} \sim \lambda Q \lambda x \left( \text{seek} (x) (Q) \right) \)

where \( Q \) ranges over properties of (first-order) properties.

Montague (1974c) is able to capture the inference in (2.55) by analyzing the semantics of (2.55a) as a binary relation between Orcutt and the existential sublimation corresponding to a unicorn. That is to say the set of first order properties any unicorn possesses. Returning to the semantics of adjectival comparatives, it very well may be the case that the correct semantic is to treat the object referred to under the scope of *than* as something abstract like a property of first-order properties.
2.6. THE QUANTIFIERS SOMEONE AND EVERYONE

Figure 2.18: Possible situations of Orcutt’s, Smith’s, and Jones’ heights consistent with (2.56a) under its normal universal interpretation

(2.56) a. Orcutt is taller than everyone else
b. $\not= \not\Rightarrow$ Orcutt is taller than someone else
c. $\not= \not\Rightarrow$ For someone\textsubscript{1} other than Orcutt, Orcutt is taller than them\textsubscript{1}

The statement in (2.56a) is compatible only with situations like the one shown in figure 2.18. Again, the inference in (2.56) can be schematized quasi-formally as I have done in (2.57).

(2.57) \[
\frac{\text{than} \, (A_1 \land \ldots \land A_n)}{\text{than} \, (A_1) \land \ldots \land \text{than} \, (A_n)}
\]

where $n < \omega$

Just like in the case of the and, under the scope of than, the quantifiers someone and everyone have only their normally assumed interpretations. Similar to or and and, there seems to be an asymmetry between the sorts of entailment patterns anyone licenses on the one hand, and someone and everyone do on the other. The distribution of these various entailment patterns is shown in table 2.5. Again, the rows of this table can be read as corresponding to the natural language expressions anyone, someone, and everyone, as they appear under the scope of than; and the logical operators universal ($\forall$) and exists ($\exists$) can be read as the readings those expressions license.

The above data is confusing in the following precise sense: if disjunctive meanings in the than-phase of an adjectival comparative give rise to conjunctive ones, then comparatives involving someone should also have a strengthened, conjunctive reading, as the existential quantifier is interpreted disjunctively in the meta-language.
2.6. THE QUANTIFIERS SOMEONE AND EVERYONE

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<td>x</td>
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<tr>
<td>everyone</td>
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</tr>
</tbody>
</table>

Table 2.5: Entailment patterns of the natural language quantifiers *anyone, someone,* and *everyone* as they appear under the scope of than

The universal interpretation of the existential

Neither the A-not-A nor Greater-than analysis can account for the normal interpretations of *someone* or *everyone* under the scope of *than*. In fact, they both predict the opposite—*someone* is to be interpreted universally and *everyone* is to be interpreted existentially. Taking, first, the A-not-A analysis, the predicted semantic representation of a comparative involving *someone* is given in (2.58).

(2.58)

\[
\text{Orcutt is taller than someone else} \sim \exists d (\text{tall}(o)(d) \land \neg (\exists x (x \neq o \land \text{tall}(x)(d))))
\]

A-not-A analysis: non-existent universal reading of the existential

As was shown above, by De Morgan’s laws, (2.58) is equivalent to (2.41). The value of this semantic representation yields the strengthened, universal interpretation of the existential quantifier. The Greater-than analysis makes similar predictions, as shown in (2.59).

(2.59)

\[
\text{Orcutt is taller than someone else} \sim 
\max (\lambda d (\text{tall}(o)(d))) >_H \max (\lambda d' (\exists x (x \neq o \land \text{tall}(x)(d'))))
\]

Greater-than analysis: non-existent universal reading of the existential
2.6. THE QUANTIFIERS SOMEONE AND EVERYONE

Again, (2.59) is equivalent to the set-theoretic statement in (2.43), which, as was shown, gives the strengthened, universal interpretation of the existential quantifier. To be clear, both the representations in (2.58) and (2.59) are compatible with situations like the one shown in figure 2.19. But of course the universal reading of someone does not exist. (Note, however, that both the A-not-A analysis and the Greater-than one can seemingly account for the quasi-universal interpretation of other indefinites like a man, as shown in (2.49).)

The existential interpretation of the universal

As for the everyone, the A-not-A analysis produces the semantic representation in (2.60).

(2.60)

\[ \exists d \left( \text{tall}(o)(d) \land \neg (\forall x (x \neq o \rightarrow \text{tall}(x)(d))) \right) \]

A-not-A analysis: non-existent existential reading of the universal

By pushing negation in via De Morgan’s, the logical statement in (2.60) is equivalent to the one in (2.61).

(2.61)

\[ \exists d \left( \text{tall}(o)(d) \land \neg (\forall x (x \neq o \rightarrow \text{tall}(x)(d))) \right) \iff \exists d \left( \text{tall}(o)(d) \land \exists x (x \neq o \land \neg \text{tall}(x)(d)) \right) \]
2.6. THE QUANTIFIERS SOMEONE AND EVERYONE

The statement in (2.61) is true just in case there exists a degree of height that Orcutt possesses and (at least) one individual who is not Orcutt does not possess.

The A-not-A analysis incorrectly delivers a weakened, existential reading of the universal. And so does the Greater-than analysis, which is given in (2.62).

(2.62)

\[
\text{Orcutt is taller than everyone else } \leadsto \\
\max (\lambda d (\text{tall} \ (o) \ (d))) > \max (\lambda d' (\forall x \ (x \neq o \rightarrow \text{tall} \ (x) \ (d'))))
\]

Greater-than analysis: non-existent existential reading of the universal

The statement in (2.62) is equivalent to the set-theoretic statement in (2.63).

(2.63)

\[
\max (\lambda d (\text{tall} \ (o) \ (d))) > \max (\lambda d' (\forall x \ (x \neq o \rightarrow \text{tall} \ (x) \ (d')))) \iff \\
\max (\{d \in D \mid \text{tall} \ (o) \ (d)\}) > \max \left(\bigcap _{x \neq o} \{d' \in D \mid \text{tall} \ (x) \ (d')\}\right)
\]

The logical statement (2.63) is true just in case the maximal degree of height Orcutt possesses is greater than the maximal degree of height of the intersection of the sets of degrees of height all individuals but Orcutt possess. This intersection represents the degrees of height the shortest person other than Orcutt possesses. Both the representations in (2.60) and (2.62) are compatible with situations like the one shown in figure 2.20. So, just like the A-not-A analysis, the proponents of the Greater-than approach cannot account for universal interpretation of everyone. Again, it should be clear the locus of the relative failures of both the original A-not-A and Greater-than analyses: the fact that both negation and \text{max} out-scope the quantifiers under the scope of \text{than} result in a simple empirical failure, namely both under- and over-generation of meanings.
2.6. **THE QUANTIFIERS SOMEONE AND EVERYONE**

By letting negation out-scope the quantifiers, proponents of the A-not-A analysis make incorrect predictions. It is not surprising, then, that, given Larson’s (1988) semantics of adjectival comparatives involves forcing logical operators to take obligatory scope over negation, he cannot account for the strengthened, universal reading of *anyone* for precisely the same reason he was unable to capture the strengthened, conjunctive reading of *or*.

However, he can capture the weakened, existential interpretation of *someone*. The semantic representation he predicts for an adjectival comparative involving *anyone* is shown in (2.64).

\[
\text{Orcutt is taller than} \begin{cases} \text{anyone} \\ \text{someone} \end{cases} \quad \begin{cases} \text{else} \sim \\ \exists d (\text{tall}(o)(d) \land \exists x (x \neq o \land \text{tall}(x)(d))) \end{cases}
\]

**Larson’s (1988) analysis: existential reading of the existential**

The statement in (2.64) is true just in case there exists a degree of height that Orcutt possesses and (at least) one other person does not possess. This representation is compatible with the situations in figure 2.21.

To see clearly the effects of the relative scope of negation, compare (2.65) and (2.66).
2.6. THE QUANTIFIERS SOMEONE AND EVERYONE

(a) Situation 1

(b) Situation 2

(c) Situation 3

Figure 2.21: Possible situations of Orcutt’s, Smith’s, and Jones’ heights consistent with (2.64)

\[
(2.65) \quad \exists d \left( \text{tall} (o) (d) \land \neg \left( \exists x (x \neq o \land \text{tall} (x) (d)) \right) \right)
\]

Original A-not-A analysis: scope of negation

\[
(2.66) \quad \exists d \left( \text{tall} (o) (d) \land \exists x \neg \left( x \neq o \land \text{tall} (x) (d) \right) \right)
\]

Larson’s (1988) analysis: scope of negation

Because Larson’s (1988) revised A-not-A analysis forces the existential to out-scope negation, the latter operator cannot be pushed through via De Morgan’s laws, preventing a universal interpretation of the existential. Importantly, Larson (1988) is also able to capture the universal reading of everyone, as shown in (2.67).

(2.67)

Orcutt is taller than everyone else \( \sim \)

\[
\exists d (\text{tall} (o) d \land \forall x (x \neq o \rightarrow \neg \text{tall} (x) (d)))
\]

Larson’s (1988) analysis: universal reading of the existential
2.6. THE QUANTIFIERS SOMEONE AND EVERYONE

Figure 2.22: Possible situations of Orcutt’s, Smith’s, and Jones’ heights consistent with (2.67)

The statement in (2.67) is true just in case there exists a degree of height that Orcutt possesses that no one else but him does. Because the universal quantifier out-scopes negation, Larson (1988) prevents the universal from receiving an existential interpretation. The possible situation consistent with this statement is shown in figure 2.22.

Larson (1988) both under- and over-generates meanings with respect to the quantifier anyone; but his analysis perfectly fits the data when considering the quantifiers someone and everyone. Notice, however, that if we identify the semantics of anyone with the universal quantifier ($\forall$), then he is able to naturally account for its strengthened, universal interpretation in the obvious way.

2.6.2 Heim’s revised Greater-than analysis

Heim (2006) can account for the strengthened universal reading of anyone. It is shown in (2.68). (The particulars of the analysis itself, including a syntactico-semantic derivation are given in Beck (2010) and Aloni and Roelofsen (2011), so I will not go through the details here.)\(^\text{10}\)

\(^{10}\)Observe that the semantic representation Heim (2006) delivers in (2.68) is equivalent to the original Greater-than analysis shown in (2.59).
2.6. THE QUANTIFIERS SOMEONE AND EVERYONE

Figure 2.23: Possible situations of Orcutt’s, Smith’s, and Jones’ heights consistent with (2.68)

(2.68)

\[ \text{Orcutt is taller than } \{ \text{anyone, someone} \} \text{ else} \sim \]
\[
\max(\lambda d(\text{tall}(o))) >_H \max(\lambda d'(\exists x(x \neq o \land \text{tall}(x))(d'))))
\]

*Heim’s (2006) analysis: universal interpretation of the existential*

The representation in (2.68) is compatible with situations like those shown in figure 2.23.

*Heim’s (2006) analysis, however, simply over-generates meanings. For example, she predicts that, under the scope of *than*, anyone has a weakened, existential interpretation. It is shown in (2.69).

(2.69)

\[ \text{Orcutt is taller than } \{ \text{anyone, someone} \} \text{ else} \sim \]
\[
\exists x(\max(\lambda d(\text{tall}(o)(d))) >_H \max(\lambda d'(x \neq o \rightarrow \text{tall}(x)(d'))))
\]

*Heim’s (2006) analysis: existential interpretation of the existential*

The representation in (2.69) is compatible with situations like those shown in figure 2.24.

Compare the representation in (2.68) to the one in (2.69). In the case of the latter, the existential quantifier scopes out of the operator *max*. This statement is true just in case there exists an individual other than Orcutt such that the maximal degree of height Orcutt possesses is greater than the one that individual possesses.
Notice that for the same reasons Heim (2006) is able to derive two readings of comparatives involving anyone, she delivers two meanings for comparatives involving someone. The semantic representation of these meanings are themselves identical to the ones in (2.68) and (2.69). So, Heim (2006) predicts someone to have both a universal and existential interpretation—an ambiguity where there is none.

Similarly, she generates two meanings for comparatives involving everyone, which are shown in (2.70) and (2.71). Importantly, the former is compatible with situations depicted in figure 2.24, whereas the latter is compatible with the ones shown in figure 2.23.

\[(2.70)\]

Orcutt is taller than everyone else \(\leadsto\)
\[
\max (\lambda d (\text{tall} (o) (d))) >_H \max (\lambda d' (\forall x (x \neq o \land \text{tall} (x) (d'))))
\]

*Heim’s (2006) analysis: non-existent existential interpretation of the universal*

\[(2.71)\]

Orcutt is taller than everyone else \(\leadsto\)
\[
\forall x (\max (\lambda d (\text{tall} (o) (d))) >_H \max (\lambda d' (x \neq o \land \text{tall} (x) (d'))) )
\]

*Heim’s (2006) analysis: universal interpretation of the universal*
2.6. **THE QUANTIFIERS SOMEONE AND EVERYONE**

2.6.3 In sum

The successes and failures of the four semantic analyses are shown in table 2.6. In sum,

- both the original A-not-A and Greater-than analyses correctly predict the correct meaning of *anyone*; and both under- and over-generate with respect to *someone* and *everyone*.

- Larson’s (1988) revised A-not-A analysis incorrectly predicts the meaning of *anyone*;
but perfectly fits the data when it comes to someone and everyone.

- Heim’s (2006) revised Greater-than analysis over-generates meanings when it comes to anyone, someone, and everyone.

The empirical picture that emerges, here, is quite similar to the one outlined in §2.4.4 of this chapter. Again, both Larson’s (1988) and Heim’s (2006) revised analyses mark improvements over the original A-not-A and Greater-than analyses respectively. However, no analysis perfectly fits the attested entailment patterns.

If this overview teaches us anything, it is this: in adjectival comparatives, disjunctive meanings, including or and anyone, play badly under the scope of than: both expressions have a strengthened interpretation. Of course, I am not the first person to observe this. (See, for example, von Stechow (1984), Larson (1988), Beck (2010), van Rooij (2010a) and Alrenga and Kennedy (2013) among many more for discussions of some, if not all of the issues addressed here.) I want to review, now, possible explanations for as to why this is the case.

2.7 Downward entailment

It is possible that or and anyone receive strengthened interpretations under the scope of than in virtue of the fact that than creates a downward entailing context. This is hypothesis 1. Definition 3, taken from Condoravdi (2010; p. 878), makes clear what it means to be a downward entailing expression. (But see also Kadmon and Landman (1993), Krifka (1995) and von Fintel (1999) among many others for similar definitions.)

**Definition 3** (Downward entailing/monotonic expression). Let $\sqsubseteq$ a relation of semantic strength between two expressions of the same semantic type $\alpha$. An expression $A$ of type $(\alpha, \beta)$ is downward entailing just in case for all expressions $B$ and $C$ of type $\alpha$ such that $B \sqsubseteq C$, $A(C) \sqsubseteq A(B)$.

I think it is difficult to characterize the concept of semantic strength in an atheoretical way, and as of now, I do not have a generalized formal apparatus to provide such a definition. I think it is better, then, to define the concept implicitly. A clear example of a downward-entailing context is created by negation, as in (2.72).
2.7. DOWNWARD ENTAILMENT

(2.72)  
\begin{align*}
\text{a. Orcutt isn’t a spy} \\
\text{b. ⇒ Orcutt isn’t a spy that smokes}
\end{align*}

The idea is that the expression *a man that smokes* is semantically stronger, or rather (generally) entails the expression *a man*; and when the downward-entailing expression *isn’t* is applied to these two expressions, the entailment relation is reversed: (2.72a) entails (2.72b). If we understand the meaning of the relativizer *that* to be that of logical conjunction (\(\land\)), (2.72) would be understood as a **logical inference**.

I have schematized the inference in (2.72) in quasi-formal notation below. In regard to (2.72), \(A\) would be identified with *a spy*, \(B\) with *smoke*, ‘\(\neg\)’ with *is not*, and ‘\(\sqsubseteq\)’ would be the semantic strength relation. Below, \(DE\) is short for *downward-entailing*.

\[
\frac{(A \land B) \sqsubseteq A \ \neg (A)}{\neg (A \land B)} \quad DE
\]

Similarly, negation licenses **non-logical** inferences like the one below.

(2.74)  
\begin{align*}
\text{a. Orcutt isn’t a human} \\
\text{b. ⇒ Orcutt isn’t a man}
\end{align*}

Returning to the schema in (2.73), in this case, \(A\) would be identified with *a human*, \(B\) with *a man*, and again, ‘\(\neg\)’ with *is not*. Importantly, the expression *a man* entails (\(\sqsubseteq\)) *a human*. This is apparently a material fact about our world. However, there is nothing *a priori* about these two expressions that forces the extension of *a man* to be contained in the extension of *a human*. (See, for example, Carnap (1952) for a discussion on this point.) As such, (2.74) is a non-logical inference.

Another example of a downward-entailing expression is the quantifier *every* with respect to its first argument. This is made clear by the following valid strengthening inference.

(2.75)  
\begin{align*}
\text{a. Every man runs} \\
\text{b. ⇒ Every man that smokes runs}
\end{align*}

The idea, here, is that the expression *man that smokes* entails *man*. Because *every* is
2.7. DOWNWARD ENTAILMENT

downward-entailing in its first-argument, the inference is valid. Again, for the sake of clarity, I have schematized the inference below in (2.76).

\[(A \land B) \sqsubseteq A \quad \text{every } (A) \quad \text{DE} \quad \text{every } (A \land B)\]

2.7.1 Negative polarity items

Having made sense of the concept of downward entailment, we can now make sense of hypothesis 1. I want to begin, first, by explaining why authors think than creates a downward entailing environment. In adjectival comparatives than seemingly grammatically licenses so-called negative polarity items (NPIs). Some examples of this are shown in (2.77).

\[(2.77)\]

\n
a. Orcutt is taller than anyone else is
b. Orcutt is busier than he ever was before
c. Orcutt is richer than you care to know
d. Orcutt stole more than Orcutt has confessed yet

Heim (2006; (59)–(62))

NPIs like any, ever, care, and yet are said to be a lexical items that are grammatically licensed in downward entailing environments. This licensing condition is due to Fauconnier (1978) and Ladusaw (1979) and is stated clearly in definition 4.

**Definition 4** (Fauconnier-Ladusaw NPI licensing condition). An NPI is grammatically licensed only if it it is in the scope of a downward entailing expression.

As I showed, negation induces a downward-entailing context. Under the Fauconnier-Ladusaw licensing condition, it allows an NPI like ever to fall within its scope. This is made clear by the contrast between the acceptability of (2.78a) versus (2.78b).

\[(2.78)\]

\n
a. #Orcutt ever eats
b. Orcutt doesn’t ever eat

With this in mind, observe that, under hypothesis 1, one could explain the distribution of the NPIs in (2.77). Moreover, hypothesis 1 would go a long way in explaining the fact
2.7. **DOWNWARD ENTAILMENT**

that in an adjectival comparative, under the scope of *than*, *or* and *anyone* are interpreted conjunctively.

(2.79)  a. Orcutt is taller than Smith or Jones
       b. ⇔ Orcutt is taller than Smith is and Orcutt is taller than Jones is

Under hypothesis 1, (2.79) is now understood as an instance of the schema in (2.80).

(2.80)  

\[
\frac{(A \land B) \sqsubseteq A \lor B}{\text{than}(A) \land \text{than}(B)} \quad \frac{\text{than}(A) \land \text{than}(B)}{DE}
\]

Similar reasoning would explain the strengthening inference *anyone* licenses shown, again, in (2.81).

(2.81)  a. Orcutt is taller than anyone else
       b. ⇔ Orcutt is taller than everyone else
       c. ⇔ For everyone except Orcutt, Orcutt is taller than them

Again, interpreting the semantics of the existential quantifier as disjunction, under hypothesis 1, (2.81) is now understood as an instance of the schema in (2.82).

(2.82)  

\[
\frac{(A_1 \land \ldots \land A_n) \sqsubseteq (A_1 \lor \ldots \lor A_n)}{\text{than}(A_1) \land \ldots \land \text{than}(A_n)} \quad \frac{\text{than}(A_1) \land \ldots \land \text{than}(A_n)}{DE}
\]

for \(n < \omega\)

2.7.2 Adjectival comparatives are upward entailing

I think it is bad business to claim that *than* creates a downward entailing context. However, some authors do so. Take, for example, Morzycki (2013). He claims that the inference in (2.83) is evidence that *than* does in fact create a downward entailing context.

(2.83)  a. Orcutt is taller than any linguist
       b. ⇒ Orcutt is taller than any phonologist
To quote Morzycki (2013; p. 22)

[The example in (2.83)] invites one to examine all linguists, note the height of each, and pick the highest value. Because phonologists are a subset of linguists, the examination of all linguists included all phonologists. This in turn means that the maximum value initially arrived at could not be exceeded by looking only at phonologists.

However, it is important to point out that it is possible, and likely, that the reason the inference in (2.83) is downward entailing is because of the monotoncity behavior of the quantifier any. Just like every, any seems to be downward entailing in its first argument. It is reasonable to assume, then, that in (2.83), any monotonicity behavior takes precedence over than’s, which to my mind, is clearly upward monotonic. Of course, this suggests that a detailed investigation into the interaction between various lexical item’s monotonicity behavior is needed, which goes beyond the scope of this dissertation. Instead, I simply cite work on natural logic and monotonicity (van Benthem 1986; Valencia 1991; Dowty 1994; Gilad and Francez 2005; van Benthem 2008; Moss 2008; Pratt-Hartman and Moss 2009; MacCartney and Manning 2009; MacCartney 2009; Moss 2009, 2012, 2011; Icard and Holliday 2013).

As observed by authors like Schwarzschild and Wilkinson (2002), than actually seems to create an upward entailing context, where upward entailment is defined dually to definition 3 as below in definition 5.

**Definition 5** (Upward entailing/monotonic expression). Let \( \sqsubseteq \) a relation of semantic strength between two expressions of the same semantic type \( \alpha \). An expression \( A \) of type \( (\alpha, \beta) \) is **upward entailing** just in case for all expressions \( B \) and \( C \) of type \( \alpha \) such that \( B \sqsubseteq C \), \( A(B) \sqsubseteq A(C) \).

The idea is that upward entailing contexts do not license strengthening inferences but rather weakening ones. Consider, now, such an inference as it is shown in (2.84a).

(2.84) a. Orcutt is taller than a spy that smokes  
b. \( \Rightarrow \) Orcutt is taller than a spy

In this example, the expression *a spy that smokes* entails the expression *a spy*. If than were truly a downward entailing expression, then we would expect (2.84b) to entail (2.84a).
does not. Rather, the opposite is true, namely that (2.84a) entails (2.84b). To be clear, the inference in (2.84) is a instance of the schema in (2.85).

(2.85)\[
\frac{(A \land B) \sqsubseteq A \quad \text{than} \quad (A \land B)}{\text{than} \quad (A)} \quad DE
\]

However, just as in the case of (2.83), it is possible to attribute the inference in (2.84) to the monotonicity behavior of the quantifier $a$. Specifically, this quantifier is upward entailing in its first argument, so the data in (2.84) is not definitive evidence that than creates an upward entailing context.

Perhaps better evidence that the schema in (2.85) is valid is given by the following valid inferences in (2.86).

(2.86) a. Orcutt is taller than Smith and Jones
   b. $\implies$ Orcutt is taller than Smith
   c. $\implies$ Orcutt is taller than Jones

In fact, if than truly created a downward entailing context, as von Stechow (1984) alludes to, it would allow us to reason from (2.87a) to (2.87b), which is clearly not a valid inference.

(2.87) a. Orcutt is taller than Smith or Jones
   b. $\not\implies$ Orcutt is taller than everyone else

Why is this the case? Recall, first, that, under the scope of than, and is interpreted conjunctively. As I argued, the following inference is valid in general.

(2.88) a. Orcutt is taller than Smith and Jones
   b. $\implies$ Orcutt is taller than Smith and Orcutt is taller than Jones

Combining the insights of this section, the following proof demonstrates how it would be possible to reason from the premise in (2.88a) to the conclusion in (2.88b).
2.7. DOWNWARD ENTAILMENT

\[(2.89)\]
\[
\begin{align*}
(A_1 \land A_2) \subseteq (A_1 \lor A_2) & \quad \text{than} (A_1 \lor A_2) \\
\text{than} (A_1) \land \text{than} (A_2) & \quad \text{DE} \\
\text{than} (A_1) \land \text{than} (A_2) & \quad (2.88) \\
\text{than} (A_1) \land \text{than} (A_2) & \quad (A_1 \land A_2 \land A_3) \subseteq (A_1 \land A_2) \\
\text{than} (A_1) \land \text{than} (A_2) & \quad \text{DE} \\
\vdots & \\
\text{than} (A_1) \land \ldots \land \text{than} (A_n) & \\
\end{align*}
\]

where \(n < \omega\)

So, the idea would be that, through repeated applications of the proof rule \(DE\) and the equivalence in (2.83) as indicated by the vertical dots ‘\(\vdots\)’, we would invalidly be able to reason from a disjunction to a universal.

Stepping back, I believe the source of the strengthening inferences \(or\) and \(anyone\) license should be located elsewhere. Of course, rejecting hypothesis 1 means that there is no obvious explanation as to why the NPIs in (2.77), which appear under the scope of \(than\), are in fact felicitously licensed. Consequently, it allows for an independent explanation. See, for example, Giannakidou and Yoon (2010) for one such explanation.

2.7.3 The universal force of \(anyone\)

In this section, I will focus on providing an independent explanation for the fact that \(anyone\) can have a universal interpretation. First, I want to point out that, it is argued in the literature at least, that one must distinguish between two homophonous forms of \(any\): the so-called free choice item and the NPI. Examples of both are shown in (2.90) and (2.91) respectively.

(2.90) Orcutt can catch any raven

\textit{Free choice ‘any’}

(2.91) Orcutt didn’t see any pigs

\textit{Negative polarity item ‘any’}

Horn (2000; (1) & (2))
It is generally agreed that, under its free choice reading, *any* should be interpreted as a universal quantifier, while under its NPI reading, it is to be interpreted as an existential quantifier. However, this may seem odd, as in both (2.90) and (2.91), *any* seems to carry with it universal force.

Let me start with free-choice *any*: detecting it is tricky. However, tests have been identified for its identification in the literature. To begin with, it has been pointed out that universal quantifiers like *every* and *no* can be modified by *almost* but existential quantifiers like *some* cannot.

(2.92)  
\begin{align*}
&a. \text{ Almost every lawyer could answer that question} \\
&b. \text{ Almost no lawyer could answer that question} \\
&c. \#\text{Almost some lawyer could answer that question}
\end{align*}

Kadmon and Landman (1993; (13)–(15))

Similar to the above examples, the *any* that appears in (2.90) can be modified by *almost*, but the *any* that appears in (2.91) cannot.

(2.93)  
\begin{align*}
&a. \text{ Orcutt can catch almost any raven} \\
&b. \#\text{Orcutt didn’t see almost any pigs}
\end{align*}

So, there seems to be a distributional difference between free-choice and NPI *any*. However, is not there a clear meaning difference: again, both expressions seem to carry with them universal force. So, why not treat their semantic value as just being a universal quantifier?

This is the approach of Quine (1982). Specifically, he analyzes *anyone else*, as it occurs in (2.94), as being a universal quantifier.\footnote{See Boolos (1998) for a second-order analysis of (2.94) that involves existential quantification over properties.}
Some of Orcutt’s men entered the building unaccompanied by anyone else $\sim$

$$\exists x (\text{men}(x) \land \text{entered}(x) \land \forall y (\neg \text{accompanied}(x, y) \rightarrow \text{men}(y)))$$

Quine (1982; 197)

As Horn (2000) points out, a neo-Quinean would analyze both the NPI any and free choice any as the universal quantifier. According to a Quinean, the logical form of examples (2.90) and (2.91) are shown respectively below.

(2.95) Orcutt didn’t see any pigs $\sim \forall y (\neg \text{pig}(y) \rightarrow \text{see}(o, y)))$

Negative polarity item ‘any’

(2.96) Orcutt can catch any raven $\sim \forall y (\text{raven}(y) \rightarrow \text{catch}(o, y))$

Free choice ‘any’

Importantly, in both examples, the universal quantifier takes wide-scope, even over negation. However, as Horn (2000), citing Fauconnier (1978), points out, this analysis fails to account for the fact that, in certain contexts, any has a purely existential reading. This is made clear by the following example.

(2.97) Orcutt wonders if Smith married anybody

Fauconnier (1978)

Under a Quinean-style analysis of any, there is no obvious way to account for the existential force of examples like (2.95).\(^{12}\) Authors like Horn (2000) have argued that NPI any should be analyzed as the existential quantifier, and the apparent universal force of examples like (2.91) can be accounted for in the standard logical way, namely by pushing wide-scope negation through the quantifier via De Morgan’s laws.

\(^{12}\) Of course, the apparent existential interpretation of any in (2.97) could be due to the fact that the expression is embedded under wonders if.
2.7. **DOWNWARD ENTAILMENT**

\[(2.98) \quad \neg \exists y (\text{pig}(y) \land \text{see}(x)(y)) \Leftrightarrow \forall y \neg (\text{pig}(y) \land \text{see}(x)(y))\]

Horn’s (2000) critique of Quine’s (1982) original position is generally seen to have prevailed, and NPI *any* is analyzed semantically as the existential quantifier.

2.7.4 The revised theoretical landscape

It is important to point out there is no negation in (2.93a) involving free-choice *any* to obviously give that quantifier the universal force it carries. Consequently, many authors have assumed the existence of two homophonous forms of *any*, as shown in (2.99).

\[(2.99) \quad \begin{align*}
& \text{a. NPI *any* is to be analyzed as a existential quantifier;} \\
& \text{b. Free-choice *any* is to be analyzed as a universal quantifier.}
\end{align*}\]

Following Alrenga and Kennedy (2013) (but see also Heim (2006)), I want to pick up hypothesis 2 and argue that the *any* that appears under the scope of *than* is actually an instance of free-choice *any*, and thus, should be analyzed as in (2.99b). This is not an implausible hypothesis. Observe that the data in (2.100) patterns like the data in (2.92), suggesting that we truly are witnessing an instance of free-choice *anyone*.

\[(2.100) \quad \text{Orcutt is taller than almost } \begin{align*}
& \text{anyone} \\
& \text{everyone} \\
& \text{else} \\
& \#\text{someone}
\end{align*}\]

If authors like Alrenga and Kennedy (2013) are correct, and the *anyone* that appears under the scope of *than* is in fact free-choice *anyone*, new light is shed on the theoretical landscape for the various analyses of the semantics of adjectival comparatives considered in this chapter. In fact, Larson (1988) is the only analyst that gets the data exactly right. His revised analysis is shown in (2.101).
### 2.8. **FREE CHOICE**

<table>
<thead>
<tr>
<th>Attested entailments</th>
<th>Universal</th>
<th>Existential</th>
</tr>
</thead>
<tbody>
<tr>
<td>anyone</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>someone</td>
<td>x</td>
<td>✓</td>
</tr>
<tr>
<td>everyone</td>
<td>✓</td>
<td>x</td>
</tr>
<tr>
<td>A-not-A analysis</td>
<td>x</td>
<td>✓</td>
</tr>
<tr>
<td>Larson’s (1988) revised</td>
<td>✓</td>
<td>x</td>
</tr>
<tr>
<td>Greater-than analysis</td>
<td>x</td>
<td>✓</td>
</tr>
<tr>
<td>Heim’s (2006) revised</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

Table 2.7: This revised table represents the various entailment patterns the various analyses considered are able to capture.

\((2.101)\)

Orcutt is taller than anyone else \(\sim\)

\[
\exists d (\text{tally}(o)(d) \land \forall x (x \neq o \rightarrow \neg \text{tally}(x)(d)))
\]

Larson’s (1988) analysis: universal interpretation of anyone

Under this revised empirical picture, the relative successes and failures of the four semantic analyses are shown in table 2.7. In sum,

- both the original A-not-A and Greater-than analyses incorrectly predict the meaning of anyone; and both under- and over-generate with respect to someone and everyone.
- Larson’s (1988) revised A-not-A analysis correctly predicts the meaning of anyone and perfectly fits the data when it comes to someone and everyone.
- Heim’s (2006) revised Greater-than analysis over-generates meanings when it comes to anyone, someone, and everyone.

### 2.8 Free choice

I want to turn, now, to the strengthened interpretation of or as it appears under the scope of than, arguing that, just like anyone, it too should be understood as free-choice or. However,
2.8. FREE CHOICE

unlike free-choice anyone, I want to argue that the strengthened, conjunctive interpretation of or is a pragmatic phenomenon.

Dating back to at least Kamp (1973), authors have puzzled over inference patterns that stem from the interaction between deontic modals like may and or.

(2.102)a. You may go to the beach or the cinema
   
   b. \( \iff \) You may go to the beach and you may go to the cinema

Kamp (1973; (1))

In (2.102), we see that a conjunctive interpretation of disjunction is licensed. Consequently, authors like Kamp (1973) have argued that in (certain) modal contexts, a natural language like English allows for a sort of deficient De Morgan’s inference, often referred to as a

free-choice inference. The schema for such inferences is shown in (2.103).

(2.103)

\[ \text{Free-choice inference} \]

As the following examples would suggest, this type of inference extends beyond the domain of deontic modals to other modals, including epistemic ones.

(2.104)a. Orcutt might be in Victoria or in Brixton
   
   b. \( \iff \) Orcutt might be in Victoria and Orcutt might be in Brixton

Zimmermann (2001; (6))

(2.105)a. I would dance with Smith or Jones
   
   b. \( \iff \) I would dance with Smith and I would dance with Jones

(2.106)a. Orcutt or Smith could tell you that
   
   b. \( \iff \) Orcutt could tell you that and Smith could tell you that

Kadmon and Landman (1993; (18) & (19))

As pointed out by authors like Larson (1988), the data in (2.107) demonstrates that
before induces a context that licenses a strengthened, conjunctive interpretation of or.

(2.107)a. Orcutt left before Smith or Jones (did)
   b. ⇔ Orcutt left before Smith left and Orcutt left before Jones left

Similarly, certain temporal and modal auxiliaries induce similar contexts in which or is interpreted conjunctively.

(2.108)a. Orcutt \{ will\ \ can\ \ should\ \} help Smith or Jones
   b. ⇔ Orcutt \{ will\ \ can\ \ should\ \} help Smith and Orcutt \{ will\ \ can\ \ should\ \} help Jones

And finally, observe that other comparative constructions, including nominal and verbal ones, license the strengthened, conjunctive interpretation of or as well.

(2.109)a. Orcutt ate more apples than Smith or Jones (did)
   b. ⇔ Orcutt ate more apples than Smith ate apples and Orcutt ate more apples than Jones ate apples

(2.110)a. Orcutt ran more than Smith or Jones (did)
   b. ⇔ Orcutt ran more than Smith ran and Orcutt ran more than Jones ran

What the above examples show us is that the conjunctive interpretation of disjunction is ubiquitous and occurs across a myriad of clause types.

2.8.1 Free-choice as a pragmatic phenomenon

Under the standard Gricean assumption that the meaning of English or is equivalent to logical disjunction (\(\lor\)), as shown in table 2.8, the above data shows that or is misbehaved: it seems to have a strictly conjunctive interpretation. Given the systematicity of free-choice inferences, it is tempting to understand the conjunctive interpretation of or as being a semantic phenomenon, thus identifying the meaning of or (at least in these contexts) with
2.8. FREE CHOICE

logical conjunction (∧), as shown in the same table. I think this would be a mistake. That or plays badly from a semantic point-of-view is not a new observation and is arguably one of the motivations for Grice (1989) drawing the line between semantic and pragmatic phenomena generally.

To begin with, notice that there is a difference between free choice anyone and free-choice or. As I showed in (2.11) and (2.39), the strengthening inferences involving anyone cannot be cancelled, whereas the ones involving or can.

(2.111)a. #Orcutt is taller than anyone, but I don’t know who
    b. Orcutt is taller than Smith or Jones, but I don’t know who

If we identify the meaning of English or with logical conjunction (∧), it is not obvious how to account for the inference in (2.111b). Moreover, in many contexts, or seems to have an exclusive reading, not a conjunctive one, as in the case of the following example.

(2.112)a. There is a spade either in the attic or in the basement
    b. ⇒ The spade is not in both places and [the speaker] doesn’t know which

Levinson (2000; (7))

Drawing from real-world knowledge, (2.112b) follows from (2.112a), suggesting that, in this instance, the semantics of or should be analyzed as exclusive disjunction (∨), again, shown in table 2.8. Notice, however, that this inference is also cancelable.

(2.113) There is a spade either in the attic or in the basement, in fact, there could be one in both

Table 2.8: The truth-tables for classical disjunction (∨), conjunction (∧), and exclusive disjunction (∨) respectively
2.8. FREE CHOICE

<table>
<thead>
<tr>
<th>Conjunctive</th>
<th>Disjunctive</th>
</tr>
</thead>
<tbody>
<tr>
<td>or and</td>
<td>or and</td>
</tr>
<tr>
<td>Attested entailments revised</td>
<td>x</td>
</tr>
<tr>
<td>Larson’s (1988) revised</td>
<td>x</td>
</tr>
<tr>
<td>Heim’s (2006) revised</td>
<td>✓</td>
</tr>
</tbody>
</table>

Table 2.9: The various entailment patterns the various analyses considered are able to capture.

Again, if we do in fact decide to identify the meaning of *or* with exclusive disjunction (\(\lor\)) it’s not clear how to account for the data in (2.113).

Stepping back, cancellability is one of the hallmark properties of what (neo-) Griceans take to be pragmatic inferences, not semantic entailments. Specifically, the (neo-) Gricean begins from the premise that it is possible to maintain simple, unified, logical analyses of natural language expressions that capture all and only the entailment patterns such expressions are involved in. Any subsequent pragmatic meanings of these expressions are to be derived via general principles of context, conversation, or human rationality. In the case of English *or*, instead of positing (at least) three homophonous lexical entries corresponding to its disjunctive, conjunctive, and exclusive readings, the (neo-) Gricean looks to maintain the classical assumption that this expression is identified with logical disjunction (\(\lor\)) and that any subsequent meanings are derived extra-semantically.

2.8.2 The revised theoretical landscape

Under hypothesis 2, the strengthened, conjunctive interpretation of *or* as it occurs in comparative constructions is reduced to an instance of free-choice, which itself is understood to be an inherently post-semantic phenomenon. From a (neo-)Gricean perspective, it is not the responsibility of a semantic theory of comparatives to account for the strengthened interpretation of disjunction, as such an interpretation is understood to be an *implicature*, not an entailment. Hypothesis 2 has the added benefit of allowing us to maintain that *than* is an upward entailing expression, while still accounting for the inferential behavior of *or*.

Such a perspective sheds new light on the relative successes and failures of the semantic
analyses of comparatives considered. The revised picture that emerges is shown in table 2.9. In sum,

- both the original A-not-A and Greater-than analyses under-generate and over-generate meanings with respect to Boolean *or*;
- Larson’s (1988) revised A-not-A analysis perfectly fits when it comes to both *or* and *and*.
- Heim’s (2006) revised Greater-than analysis over-generates meanings when it comes to both *or* and *and*.

### 2.9 Negation

The previous discussion has demonstrated that there is good reason for adopting Larson’s (1988) revised A-not-A analysis: he just gets the facts right, particularly under the assumptions that under the scope of *than*

- *anyone* is to be understood as free-choice *anyone*, and consequently, assigned universal semantics; and
- *or* is to be understood as free-choice *or*, and consequently, its conjunctive interpretation is due to pragmatics, not semantics.

However, if we adopt Larson’s (1988) analysis, problems immediately arise. His analysis faces a real problem when considering the difference in meaning between sentential and differential negation, as shown in (2.114) and (2.115) respectively.

(2.114) Orcutt is not taller than Smith

\[ \text{Sentential negation} \]

(2.115) Orcutt is no taller than Smith

\[ \text{Differential negation} \]
On the face of it, there is no discernible truth-conditional difference between these two sentences: they mean the same thing. However, I claim that (2.114) is ambiguous between a narrow- and wide-scope interpretation; whereas (2.115) has no such scopal ambiguity. However, in what follows, I will show that, because Larson (1988) analysis is an A-not-A one, and posits negation at the level of logical form, he has no obvious way of accounting for the two readings of sentential negation, let alone the single interpretation differential negation licenses.

2.9.1 Negation and the Booleans

The meaning difference between the two types of negation is not apparent in the above examples. However, that there exists a difference is brought out most clearly by the two expression’s interaction with the Boolean connectives and various quantifiers under the scope of than. To begin with, consider, first, what I claim to be a difference in meaning between (2.116) and (2.117).

(2.116) Orcutt is not taller than Smith \{ or \}
\{ and \} Jones

\textit{Sentential negation and the Booleans}

(2.117) Orcutt is no taller than Smith \{ or \}
\{ and \} Jones

\textit{Differential negation and the Booleans}

I claim that the meaning difference between the two types of negation is due to the availability of negation to vary in scope in the case of sentential negation but not differential negation: sentential negation can take either narrow- or wide-scope (under or over) the Booleans.

Under its narrow-scope reading, the statements in (2.118a) are equivalent to their counterparts in (2.118b).
2.9. **NEGATION**

(2.118)a. Orcutt is not taller than Smith \{ or \}

\hspace{2cm} \{ and \} Jones

b. \iff Orcutt is not taller than Smith \{ or \}

\hspace{2cm} \{ and \} Orcutt is not taller than Jones

*Sentential negation and the Booleans: narrow-scope*

Notice that negation does not interact with the Booleans in any interesting way whatsoever. Contrast this with the data in (2.119). Here, negation takes wide-scope over the Booleans, thus causing the De Morgan’s effects observed in (2.119b).

(2.119)a. Orcutt is not taller than Smith \{ or \}

\hspace{2cm} \{ and \} Jones

b. \iff Orcutt is not taller than Smith \{ and \}

\hspace{2cm} \{ or \} Orcutt is not taller than Jones

*Sentential negation: wide-scope*

In (2.119b), see that *or* can flip so-to-speak, receiving a conjunctive interpretation (\(\land\)), and *and* can flip to a disjunctive interpretation (\(\lor\)). Differential negation, on the other hand, has no such ambiguity. Negation takes obligatory narrow-scope.

(2.120)a. Orcutt is no taller than Smith \{ or \}

\hspace{2cm} \{ and \} Jones

b. \iff Orcutt is no taller than Smith \{ or \}

\hspace{2cm} \{ and \} Orcutt is no taller than Jones

*Differential negation: narrow-scope*

These judgements are sensitive, no doubt muddied by the strengthened, conjunctive reading of disjunction. Let me focus, then, on the interaction between negation and *and*, as intuitions seem to be clearer here. To show that sentential negation allows for a wide-scope interpretation and differential negation does not, observe that there exists a difference in felicity between (2.121a) and (2.121b).
2.9. NEGATION

There exists a reading of (2.121a) in which the statement is interpreted disjunctively. This could only be the case if negation and *and* interacted in such a way to result in a disjunctive interpretation. Otherwise, the speaker would be calling into question a conjunctive assertion, thus violating his or her commitment to the truth of both propositions. Notice, however, that (2.121b) is strictly infelicitous; this statement does not allow for a disjunctive reading, suggesting that differential negation takes obligatory narrow-scope.

2.9.2 Negation and some quantifiers

As observed by Alrenga and Kennedy (2013), a similar effect can be seen in regard to the interaction between these two two types of negation and quantifiers. Notice, first, that in (2.122), sentential negation has a definite narrow-scope interpretation. It does not interact with the quantifiers *everyone* and *someone* as they appear under the scope of *than*.

Contrast these examples with the ones in (2.123). Here, it is quite clear that sentential negation has a wide-scope interpretation.
2.9. NEGATION

In these examples, we see that negation interacts with the quantifiers in a De Morgan’s like way. Specifically, the examples in (2.123a) and (2.123b) demonstrate that the interaction between sentential negation and the universal quantifier *everyone* yields an existential interpretation (∃), whereas the interaction between this type of negation and the existential quantifier *someone* yields a universal interpretation (∀). Observe, now, that no such interaction is available in the case of differential negation.

\[(2.123)\]
\[\begin{align*}
(2.123)a. \quad & \text{Orcutt is not taller than } \{ \text{everyone, someone} \} \text{ else} \\
 & \iff \text{For } \{ \text{someone, everyone} \} \text{ other than Orcutt, Orcutt is not taller than them} \\
\end{align*}\]

*Sentential negation and quantifiers: wide-scope interpretation*

Unlike sentential negation, differential negation does not induce a De Morgan’s effect, suggesting again that it only allows for a narrow-scope interpretation.

\[(2.124)\]
\[\begin{align*}
(2.124)a. \quad & \text{Orcutt is no taller than } \{ \text{everyone, someone} \} \text{ else} \\
 & \iff \text{For } \{ \text{everyone, someone} \} \text{ other than Orcutt, Orcutt is not taller than them} \\
\end{align*}\]

*Differential negation and quantifiers: narrow-scope interpretation*

2.9.3 The failure of the A-not-A analysis

The question now becomes, how could Larson (1988) potentially account

- the narrow- and wide-scope interpretation of sentential negation;

- and the narrow-scope interpretation of differential negation.

In this section, I want to show that he cannot obviously account for the scope ambiguity of sentential negation, and consequently, cannot account for the meaning of differential negation. In order to do this, I will consider all the scopal possibilities sentential negation could take at the level of logical form under the A-not-A analysis under the assumption that
the English expression *not* should in fact be analyzed as logical negation (¬). To be clearer, I consider all the possible logical forms the statement *Orcutt is taller than Smith or Jones* could take, disregarding the ways in which such logical forms could potentially be derived compositionally at the syntax/semantics interface.

The goal in this thought experiment is to show that Larson (1988) cannot obviously, in the case of an example like (2.125a), capture the fact that it has (at least) the readings in (2.125b) and (2.125c). In fact, he can only account for the reading in (2.125c).

(2.125)a. Orcutt is not taller than Smith or Jones
   
   b. ⇔ Orcutt is not taller than Smith or Orcutt is not taller than Jones
   
   c. ⇔ Orcutt is not taller than Smith and Orcutt is not taller than Jones

First, I begin by showing that Larson (1988) can account for the reading in (2.125c). Consider, now, the representation in (2.126). The idea is that sentential negation could take scope as indicated below.

(2.126)

\[
\neg \exists d (\text{tall} (o, d) \wedge (\neg \text{tall} (s, d) \vee \neg \text{tall} (j, d)))
\]

scope of sentential negation

*Pseudo-analysis 1*

By De Morgan’s laws, and the meaning of the material conditional (→), (2.126) is equivalent to (2.127).

(2.127)

\[
\neg \exists d (\text{tall} (o, d) \wedge (\neg \text{tall} (s, d) \vee \neg \text{tall} (j, d))) \iff \forall d (\neg \text{tall} (o, d) \vee \text{tall} (s, d) \wedge (\text{tall} (j, d))) \iff \\
(\forall d (\text{tall} (o, d) \rightarrow (\text{tall} (s, d) \wedge \text{tall} (j, d))))
\]
2.9. **NEGATION**

The logical statement in (2.127) works as an analysis of the interpretation of (2.125a) as it is given by (2.125c). This is because (2.127) is true just in case, if Orcutt possesses a degree of height, then both Jones and Smith also possesses that same degree. This is just to say that the statement in (2.127) is true just in case Orcutt is not taller than Smith and Orcutt is not taller than Jones.

Now, the question becomes whether Larson (1988) can give an analysis of (2.125a) as it is interpreted in (2.125b). I claim not. Here’s why. Obviously pseudo-analysis 1 does not correspond to the interpretation in (2.125b). So, it will not do as an analysis. Consider pseudo-analysis 2, as it is shown in (2.128).

(2.128)

Orcutt is not taller than Smith or Jones (is) \(\rightarrow\)

\[
\exists d \neg (\text{tall}(o,d) \land (\neg \text{tall}(s,d) \lor \neg \text{tall}(j,d)))
\]

*Pseudo-analysis 2*

Again, by De Morgan’s, (2.128) is equivalent to (2.129).

(2.129)

\[
\exists d \neg (\text{tall}(o,d) \land (\neg \text{tall}(s,d) \lor \neg \text{tall}(j,d))) \iff \\
\exists d (\neg \text{tall}(o,d) \lor (\text{tall}(s,d) \land \text{tall}(j,d))) \iff \\
\exists d \neg (\text{tall}(o,d) \rightarrow (\text{tall}(s,d) \land \text{tall}(j,d)))
\]

Observe that (2.129) will not suffice as an analysis of (2.125b), as it is consistent with the model \(\mathbb{M}\) such that \(I = \{o,s,j\}, D = \{d >_H d'\} \) and \([\text{tall}] = \{(o,d),(o,d'),(s,d'),(j,d')\}\). Specifically, (2.129) is true in \(\mathbb{M}\), but Orcutt is taller than both Smith and Jones.

Turning now to pseudo-analysis 3, as shown in (2.130), observe that it too will not do as an analysis of the interpretation in (2.125b).
2.9. NEGATION

Orcutt is not taller than Smith or Jones (is) \( \rightarrow \)
\[
\exists d (\text{tall} \ (o, d) \land (\neg (\text{tall} \ (s, d) \lor \text{tall} \ (j, d))))
\]
scope of sentential negation

Pseudo-analysis 3

The logical statement in (2.130) is consistent with the model \( \mathbb{M} \) such that \( I = \{o, s, j\} \), \( \mathbb{D} = \{d >_H d' >_H d''\} \) and \( [\text{tall}] = \{(o, d'), (o, d''), (s, d''), (j, d'')\} \). Again, (2.130) is true in \( \mathbb{M} \), but Orcutt is taller than both Smith and Jones. Pseudo-analysis 4 faces similar problems.

(2.131)

Orcutt is not taller than Smith or Jones (is) \( \rightarrow \)
\[
\exists d \left( \text{tall} \ (o, d) \land (\neg (\neg (\text{tall} \ (s, d) \lor \neg (\text{tall} \ (j, d)))) \right)
\]
scope of sentential negation

Pseudo-analysis 4

By De Morgan’s, (2.131) is equivalent to (2.132).

(2.132)

\[
\exists d (\text{tall} \ (o, d) \land \neg (\text{tall} \ (s, d) \lor \text{tall} \ (j, d))) \leftrightarrow \exists d (\text{tall} \ (o, d) \land (\text{tall} \ (s, d) \land \text{tall} \ (j, d)))
\]

Again, let \( \mathbb{M} \) be a model such that \( I = \{o, s, j\} \), \( \mathbb{D} = \{d >_H d'\} \) and
\( [\text{tall}] = \{(o, d), (o, d'), (s, d'), (j, d')\} \). (2.129) is true in \( \mathbb{M} \), but yet again, Orcutt is taller than both Smith and Jones. Pseudo-analyses 5–7, shown in (2.133)–(2.135) respectively, fail for analogous reasons.

(2.133)
Orcutt is not taller than Smith or Jones (is) $\sim$

$$
\exists d (\text{tall}(o,d) \land (\neg \text{tall}(s,d) \lor \neg \text{tall}(j,d))) \iff \\
\exists d (\text{tall}(o,d) \land (\neg \text{tall}(s,d) \lor \text{tall}(j,d)))
$$

$\text{Pseudo-analysis 5}$

(2.134)

Orcutt is not taller than Smith or Jones (is) $\sim$

$$
\exists d \left( \text{tall}(o,d) \land (\neg \neg \text{tall}(s,d) \lor \neg \text{tall}(j,d)) \right) \iff \\
\exists d (\text{tall}(o,d) \land (\text{tall}(s,d) \lor \neg \text{tall}(j,d)))
$$

$\text{Pseudo-analysis 6}$

(2.135)

Orcutt is not taller than Smith or Jones (is) $\sim$

$$
\exists d (\text{tall}(o,d) \land (\neg \neg \text{tall}(s,d) \lor \neg \text{tall}(j,d))) \iff \\
\exists d (\text{tall}(o,d) \land (\neg \text{tall}(s,d) \lor \text{tall}(j,d)))
$$

$\text{Pseudo-analysis 7}$

No matter where negation is posited at the level of logical form, Larson (1988) cannot account for the ambiguity of sentential negation seems to induce. More to the point, he cannot account for the narrow-scope reading of (2.125a), as it is shown in (2.125b). The source of the problem is clear: covert negation posited at the level of logical form plays badly with overt negation realized at the level of natural language. Moreover, given that differential no has only a narrow-scope interpretation, it is unclear how Larson’s (1988) analysis could be modified to account for its interpretation.
Similar thought experiments can be performed by considering the interaction between both sentential and differential negation and the ways in which they interact with the expressions and, everyone, and someone as they appear under the scope of than. So, there are problems for Larson’s (1988) analysis, leaving open the possibility for its revision or a completely new alternative. (See also van Rooij (2008) for other issues Larson’s (1988) account faces in light of quantifiers like exactly.)

2.10 Chapter summary and next chapter preview

The purpose of this chapter was to review previous analyses of the semantics of adjectival comparatives. Specifically, I looked at two classes of analyses and their revisions:

- the A-not-A analysis and (what can be seen as) Larson’s (1988) revision of it; and
- the Greater-than analysis and (what can be seen as) Heim’s (2006) revision of it.

As I demonstrated, under certain assumptions about what constitutes as semantic and/or pragmatic phenomena, namely the source of the strengthened, conjunctive interpretation of or and anyone, Larson’s (1988) analysis is to be preferred over its competitors, as it just gets the facts—the ones considered in this dissertation at least—correct. van Rooij (2008), in his review of various competing semantic analyses of adjectival comparatives comes to a similar conclusion; although he goes a step farther than I have done in this chapter, and proposes ways in which to augment the A-not-A analysis – specifically Klein’s (1980). However, as both Alrenga and Kennedy (2013) and myself point out, Larson’s (1988) analysis plays badly with different kinds of negation, something van Rooij (2008) does not consider. I take this to be a flaw of not only Larson’s (1988) analysis, but the class of A-not-A analyses more generally.

As I have said repeatedly, given the empirical inadequacies of both the A-not-A and Greater-than analyses, I take it as an opportunity to propose an alternative semantics for adjectival comparatives. The next chapter of this dissertation outlines the formal framework in which that analysis will be set.
3 | The basic system

3.1 Introduction

My syntactico-semantic analysis of adjectival comparatives will be cast in a Montagovian-style grammar (Montague 1974c). To quote Muskens (1995; p. 3), while authors like Heim and Kratzer (1998) “are revolutionary and seek to replace Montague Grammar by their new theory, my approach is evolutionary. I do not want to abandon Montague Grammar . . . I think we simply have not exploited Montague’s paradigm to the full as yet." Throughout the remainder of this dissertation, I assume basic familiarity with Montagovian semantics and everything that it entails, including categorial grammar, type-theory, and higher-order modal logic. (See, for example, Lewis (1970), Montague (1974a), Gallin (1975), Dowty et al. (1981), Oehrle (1988), Hendriks (1990), Gamut (1991), Morrill (1994), Carpenter (1997), Jäger (2005), Muskens (2007) and Morrill (2011) for introductions and explanations of these topics in various degrees of depth.)

The semantic representation language will take the form of the higher-order, typed logic $TT_2$ (Muskens 1995). $TT_2$’s type-theory will be built inductively out of two basic types: entities and Montagovian indices, which themselves will be understood as world/time pairs. Notice that I will be working in a degree-less semantics in the vein of Kamp (1975), Klein (1980), Larson (1988), van Rooij (2008), van Rooij (2010b) and van Rooij (2011). $TT_2$ terms will be interpreted model-theoretically over higher-order relational frames, differing from standard Montagovian assumptions that values of terms are themselves (higher-order) functions. Relational frames are non-standard; however, they will allow me to straightforwardly model the Cresswellian idea of a trans-world scale, which will be essential in my analysis of the semantics of adjectival comparatives.

My syntactic theory will take the form of a Gentzen-style sequent calculus (Gentzen
3.2. THE LOGIC $\text{TT}_2$

1934–35), namely the associative Lambek calculus. Following Barker and Shan (2013) and Barker (2013), I augment this syntax by adding continuations. The basic idea is to inductively build up a set of syntactic categories from a finite set of basic categories that includes all and only the categories $N$ (noun) and $S$ (sentence). Then, I will utilize a finite set of proof rules that govern the ways in which expressions of those categories can combine. Specifically, my system allows for two modes of combination: the merge mode and the continuation mode. Continuations are a relatively new idea in linguistics, but will prove absolutely essential in chapter 5, when I present a syntax for comparative ellipsis and various scopal facts involving adjectival comparatives.

3.2 The logic $\text{TT}_2$

Because $\text{TT}_2$ is a non-standard framework, I introduce it in its entirety now, but refer the reader to Muskens (1995) where all of the following definitions have been taken from.

To begin with, I assume the existence of a finite set of basic semantic types—entities $e$ and world-time indices $s$—out of which all other complex types are constructed per definition 6.

Definition 6 (Types). The set of $\text{TT}_2$ types is the smallest set of strings such that

- $e$ and $s$ are $\text{TT}_2$ types; and

- If $\alpha_1 \ldots \alpha_n$ are $\text{TT}_2$ types for $n \geq 0$, then $(\alpha_1 \ldots \alpha_n)$ is a $\text{TT}_2$ type.

So, for example, $(e)$ and $(es)$, are $\text{TT}_2$ types. Notice that I do not assume the existence of a type $t$ corresponding to truth-values. Instead, following Muskens (1995), I identify the type $(\ )$ as the type of formulae. Nor do I assume the existence of degrees. I work in a delineation semantics in the sense that I do not quantify over degrees in the object language. (See Lassiter (2011) for a good overview of delineation semantics and its competitors.)

As mentioned above, models for $\text{TT}_2$ are themselves sets of higher-order relations, which form a hierarchy that can be indexed naturally per definition 6. Such models are constructed by beginning with the idea of a frame. The idea of a frame is given explicitly by definition 7.
3.2. THE LOGIC TT$_2$

Definition 7 (Frames). A TT$_2$ frame $\mathcal{F}$ is a set $\mathcal{F} = \{D\alpha \mid \alpha$ is a TT$_2$ type} such that $D_e \neq \emptyset, D_s \neq \emptyset$ and

$$D(\alpha_1, \ldots, \alpha_n) = \mathcal{P}(D\alpha_1 \times \cdots \times D\alpha_n)$$

for all types $\alpha_1, \ldots, \alpha_n$.

Domains $D_e$ and $D_s$ consist of entities and indices respectively. Domains $D(\alpha_1 \ldots \alpha_n)$ are identified with all $n$-ary relations having $D\alpha_i$ as their $i$-th domain. So, for example, an object of type $(e)$ is an element of $\mathcal{P}(D_e)$, or rather a set of entities. The limiting case, here, is where $D() = \mathcal{P}() = \{0, 1\}$, the set of truth-values. Specifically, $0 = \emptyset$ and $1 = \{\emptyset\}$, the von Neumann definitions of 0 and 1.

As mentioned above, natural language expressions will be associated with a semantic representation, which itself takes the form of some TT$_2$ term or another. The syntax of the logical language TT$_2$ is familiar; it just takes the form of a typed $\lambda$-calculus. For each type, as given by definition 6, I assume the existence of a denumerable number of constants and variables of that type. The set of complex expressions is built up inductively by definition 8.

Definition 8 (Terms). Define for each TT$_2$ type a set of terms of that type as follows.

- Every constant or variable of any type is a term of that type;
- If $\varphi$ and $\psi$ are terms of type (), then $\neg \varphi$ and $(\varphi \land \psi)$ are of type ()
- If $\varphi$ is of type () and $x$ is a variable of any type, then $\forall x \varphi$ is of type ()
- If $A$ is a term of type $(\beta \alpha_1 \ldots \alpha_n)$ and $B$ is a term of type $\beta$, then $(AB)$ is a term of type $(\alpha_1 \ldots \alpha_n)$
- If $A$ is a term of type $(\alpha_1 \ldots \alpha_n)$ and $x$ is a variable of type $\beta$, then $\lambda x \beta (A)$ is a term of type $(\beta \alpha_1 \ldots \alpha_n)$
- If $A$ and $B$ are terms of the same type, then $(A = B)$ is of type ()

Logical operators not defined above have their usual definitions. For the sake of perspicuity, I reserve the right to omit and add parentheses as needed. I introduce some notational
3.2. THE LOGIC TT₂

conventions. Because I am working with relational frames, I need one auxiliary notion to interpret the expressions given by definition 8. Following Muskens (1995), I introduce the concept of a slice function, as given by definition 9.

**Definition 9 (Slice Functions).** Let $R$ be an $n$-ary relation for $n > 0$ and let $n \geq k > 0$. Define the $k$-th slice function of $R$ as

$$F^k_R (d) = \{ \langle d_1, \ldots, d_{k-1}, d_{k+1}, \ldots, d_n \rangle \mid \langle d_1, \ldots, d_{k-1}, d, d_{k+1}, \ldots, d_n \rangle \in R \}$$

$F^k_R (d)$ is the $n-1$-ary relation obtained from $R$ by fixing its $k$-th position by $d$. To see definition 9 in action, consider the binary relation $R = \{ \langle a, b \rangle, \langle b, b \rangle, \langle c, b \rangle \}$. The 1st slice function $F^1_R$ applied to $a$ is $F^1_R (a) = \{ b \}$, whereas the 2nd slice function $F^2_R$ applied to $b$ is $F^2_R (b) = \{ a, b, c \}$.

With this extra machinery in hand, terms are interpreted in the normal Tarskian-way with the aid of the interpretation function $I$ over a frame $\mathcal{F}$, which itself sends non-logical constants of type $\alpha$ to objects in $D_\alpha$. Moreover, I assume the existence of an assignment function $g$, which sends variables of type $\alpha$ to objects in $D_\alpha$. If $g$ is an assignment, I write `$g[d/x]$’ for the assignment $g'$ just like $g$ except possibly for the value assigned to $x$. The complete Tarskian truth-definition for TT₂ is given in definition 10.

**Definition 10 (Tarski truth-definition).** The value $[A]^{\mathcal{M}, g}_\mathcal{F}$ of a term $A$ on a model $\mathcal{M} = (\mathcal{F}, I)$ under an assignment $g$ is defined in the following way.

- $[c] = I(c)$, if $c$ is a constant;
- $[x] = g(x)$, if $x$ is a variable;
- $[\lnot \varphi] = 1 - [\varphi]$;
- $[\varphi \land \psi] = [\varphi] \cap [\psi]$;
- $[\forall x \alpha \varphi]^{\mathcal{M}, g} = \bigcap_{d \in D_\alpha} [\varphi]^{\mathcal{M}, g[d/x]}$;
- $[A(B)] = F^1_{[A]} ([B])$;
3.2. THE LOGIC TT

• \( [\lambda x \beta (A)]^{\mathbb{M}, g} = \) the relation \( R \) such that \( F^1_R (d) = [A]^{\mathbb{M}, g[d/x]} \) for all \( d \in D_\beta \);

• \([A = B] = 1\), if \([A] = [B]\) and 0 otherwise

Importantly, \([A (B)]\) is the result of applying the 1st slice function of \([A]\) to \([B]\), whereas \([\lambda x (A)]\) is gotten by the inverse procedure. To better understand these definitions, observe that the following equalities in (3.1) hold.

\[
\begin{align*}
\text{(3.1)} & \quad \text{a. } [A (B)] = F^1_{[A]} ([B]) = \{ \langle d_1, \ldots, d_n \rangle \mid \langle [B], d_1, \ldots, d_n \rangle \in [A] \} \\
& \quad \text{b. } [\lambda x \beta (A)] = \\
& \quad \text{the relation } R \text{ such that } F^1_R (d) = [A]^{\mathbb{M}, g[d/x]} \text{ for all } d \in D_\beta = \\
& \quad \{ \langle d, d_1, \ldots, d_n \rangle \mid d \in D_\beta \text{ and } \langle d_1, \ldots, d_n \rangle \in [A]^{\mathbb{M}, g[d/x]} \} \\
\end{align*}
\]

Muskens (1995; p. 16)

The move away from interpreting a \( \lambda \)-representational language over functional domains to relational ones may seem unnatural at first. However, it reflects an simplification of the semantic theory that is brought out most clearly by the way in which entailment is defined in this setting.

**Definition 11** (Entailment). Let \( \Gamma \) and \( \Delta \) be sets of terms of some type \( \alpha = (\alpha_1 \ldots \alpha_n) \). \( \Gamma \) is said to **entail** \( \Delta \), written \( \Gamma \models \Delta \), if

\[
\bigcap_{A \in \Gamma} [A]^{\mathbb{M}, g} \subseteq \bigcup_{B \in \Delta} [B]^{\mathbb{M}, g}
\]

for all models \( \mathbb{M} \) and assignments \( g \).

So, generalized entailment amounts simply to set-theoretic inclusion. Moreover, it is easy to define, for each type-domain, the generalized Boolean operators of conjunction (\( \lor \)), disjunction (\( \land \)), and negation (\( \neg \)) as their set-theoretic counterparts union (\( \cup \)), intersection (\( \cap \)) and complementation (\( \neg \)) respectively instead of relying on complex point-wise definitions that functional frames force upon us. (See, for example, Landman (1991) for such definitions.)
3.3. THE ASSOCIATIVE LAMBEK CALCULUS WITH CONTINUATIONS

<table>
<thead>
<tr>
<th>Category A</th>
<th>Semantic type ( \tau(A) )</th>
<th>Abbreviation</th>
<th>Traditional name</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S )</td>
<td>((s))</td>
<td></td>
<td>Sentence</td>
</tr>
<tr>
<td>( N/S )</td>
<td>((es))</td>
<td>CN</td>
<td>Common Noun</td>
</tr>
<tr>
<td>( N\backslash S )</td>
<td>((es))</td>
<td>IV</td>
<td>Intransitive Verb</td>
</tr>
<tr>
<td>( S/IV )</td>
<td>((es)(s))</td>
<td>NP</td>
<td>Noun phrase/Proper name</td>
</tr>
<tr>
<td>( NP/CN )</td>
<td>((es)(es)(s))</td>
<td>DET</td>
<td>Determiner</td>
</tr>
</tbody>
</table>

Table 3.1: Abbreviations for syntactic categories

Having laid out the semantic representation language \( \text{TT}_2 \) and its corresponding model-theoretic interpretation, I turn now to the theory of natural language syntax I will be working with.

### 3.3 The associative Lambek calculus with continuations

I begin, first, by defining an infinite set of syntactic categories of natural language expressions under the assumption of a finite set of basic categories noun \( N \), index (or world) \( I \), and sentence \( S \).

**Definition 12** (Categories). Define the set of syntactic categories inductively as follows.

- \( N \) is a category; \( I \) is a category; and \( S \) is a category;
- If \( \mathcal{A} \) and \( \mathcal{B} \) are categories, then \( \mathcal{B}/\mathcal{A}, \mathcal{A}\backslash \mathcal{B}, \mathcal{A}/\mathcal{B}, \) and \( \mathcal{A}\backslash \mathcal{B} \) are categories.

Not only do natural language expressions have a semantic representation, but also a a corresponding syntactic category built up by definition 12. So, for example, adjectives like *tall* will be of the category \( N/S \), whereas proper names like *Orcutt* will be of the category \( S/(N\backslash S) \). In table 3.1, I have listed several abbreviations for complex categories that will be relevant in this chapter. Importantly, I will move fluidly between abbreviations and their full-blown representations in order to aid in, what I take to be at least, conceptual clarity.

Notice that there is a tight-knit relationship between the categories created by definition 12 and the semantic types gotten by definition 6: category \( N \) can naturally be associated with type \( e \), and category \( S \) can itself be associated with type \( (s) \), which, interpreted in
its normal way, is the type of objects identified with propositions, namely sets of possible Montagovian indices. Building on this insight, following Muskens (1995), I define a translation function $\tau$ sending syntactic categories to their semantic types by recursion.

**Definition 13** (Category-to-type Rule). Let $\tau$ be a translation function defined by recursion sending syntactic categories to their semantic types as follows

- $\tau(N) = e$; $\tau(1) = s$; $\tau(S) = (s)$; and
- $\tau(B/A) = \tau(B/A) = \tau(A/B) = \tau(A\setminus B) = \tau(A) \ast \tau(B)$

where $\beta \ast (\alpha_1 \ldots \alpha_n) = (\beta \alpha_1 \ldots \alpha_n)$ for all types $\beta$ and $(\alpha_1 \ldots \alpha_n)$.

So, for example,

$$
\tau(N\setminus S) = \tau(N) \ast \tau(S) = (es)
$$

and

$$
\tau(N\setminus (N\setminus S)) = \tau(N) \ast \tau(N\setminus S) = \tau(N) \ast \tau(N) \ast (s) = (ees)
$$

It should be clear, from the above examples, that English expressions of a particular syntactic type will always be translated as semantic terms that denote intensional objects, not extensional ones. Of course the extension of a term can derived in the normal way, namely by applying an appropriate world to that term’s intension.

Notice that the translation function $\tau$ is not injective: multiple syntactic categories can receive the same semantic type. However, the category-to-term translation puts restrictions on the types of the potential semantic representations an English expression can receive. (See, for example, Jäger (2005; p. 12) for an excellent discussion on this point.)

In a normal categorial grammar, expressions of category $B/A$ and $B/B/A$ will combine with expressions of category $A$ on their right and return an expression of category $B$. 
3.3. THE ASSOCIATIVE LAMBEK CALCULUS WITH CONTINUATIONS

Similarly, expressions of category \( \text{A} \setminus \text{B} \) and \( \text{A} \setminus \text{B} \) combine with expressions of category \( \text{A} \) on their left and return an expression of category \( \text{B} \). Modes of combination, here, are slightly more complicated and will rely on the concept of a structure as given by definition 14, itself taken from Barker (2013).

**Definition 14 (Structures).** Let the set of structures be defined inductively as follows.

- If \( \Gamma \) is a category given by definition 12, then \( \Gamma \) is a structure;
- If \( \Gamma \) and \( \Delta \) are structures, then \( \Gamma \cdot \Delta \) is a merged structure;
- If \( \Gamma \) and \( \Delta \) are structures, then \( \Gamma \circ \Delta \) is a scoped structure; and
- If \( \Gamma(\Delta) \) is a structure, then \( \lambda \alpha \Gamma[\alpha] \) is a gapped structure.

The set of structures is the smallest such set containing the above structures.

I will use \( \Delta, \Gamma, \) and \( \Sigma \) as meta-variables ranging over structures. Specifically, \( \Delta(\Gamma) \) signifies a configuration \( \Delta \) with a distinguished sub-configuration \( \Gamma \). \( \alpha \) too is a meta-variable ranging over variables of the form \( X, X', Y, \) and \( Y' \). As is clear from the definition of a gapped structure, along with the semantic representation language, the syntactic component also takes the form of a \( \lambda \)-calculus, which is discussed extensively in Barker and Shan (2013). (But see also Muskens (2007) for more on \( \lambda \)-grammars.)

The symbols ‘/’, ‘\’, ‘\( /, \cdot \)’, ‘\( \setminus, \circ \)’, and ‘\( \cdot \)’ are referred to as modes of combination. These symbols form two natural classes: \{‘/’, ‘\’, ‘\( /, \cdot \)’, ‘\( \setminus, \circ \)’, and ‘\( \cdot \)’\}; which following Barker (2013), I will refer to as the merge mode of combination and \{‘\( /, \cdot \)’, ‘\( \setminus, \circ \)’, and ‘\( \cdot \)’\}; which, again, following Barker (2013), I will refer to the as continuation mode of combination. Structures of the form \( \Gamma \cdot \Delta \) will be understood as ‘\( \Gamma \) syntactically merged with \( \Delta \)’. From a natural language perspective, such structures correspond to the grammatical expressions of English. Structures of the form ‘\( \Gamma \circ \Delta \)’ will be read as ‘\( \Gamma \) taking scope over \( \Delta \)’. Such structures need not correspond to grammatical expressions of English and will provide the formal basis for treating scope and ellipsis in this setting. I hold off on explaining the intuitions behind gapped structures.

In this context, a lexical entry for a particular expression of a language like English is a triple \((\text{PRS}, \text{SYN}, \text{SEM})\), encoding for a prosodic element, a syntactic category, and a semantic representation. To establish some notational conventions, I will use variables \( P_i \) and \( P \) to range over elements of (an undefined) prosodic catalogue, \( A_i \) and \( B \) as a
meta-variables ranging over TT expressions, and $\mathcal{A}_i$ and $\mathcal{B}$ as meta-variables ranging over syntactic categories for $n \geq i \geq 1$, where $n$ is arbitrary. So, the idea, here, is that a natural language expression will be associated with the sequence $P_i : A_i : \mathcal{A}_i$ for $n \geq i \geq 1$, which itself represents each of these aspects of a particular natural language expression. Take, for example, the expression Orcutt: its full representation is shown in (3.2).\footnote{I will remain agnostic about the nature of each lexical entry’s prosodic element, referring the reader to their favorite phonological theory. Instead, I focus on the syntactic and semantic modes of combination and the sorts of models in which the semantic representations are interpreted on.}

(3.2)

| Lexical entry – | Orcutt : NP : $\lambda P (P (o))$ |

In traditional categorial grammars, authors generally work with combinators\footnote{In accordance with the type-logical tradition, I trade in parenthetical notation ‘$P_i, A_i, \mathcal{A}_i$’ for ‘$P_i : A_i : \mathcal{A}_i$’ to represent a triple.} or type-shifters. In the Lambek calculus, the work that combinators and type-shifters do is shifted to a set of proof rules, which themselves comprise a proof theory. I begin, then, by defining the objects of this theory, which themselves are normally referred to as sequents.

**Definition 15** (Sequent). Let $P_1 : A_1 : \mathcal{A}_1, \ldots, P_n : A_n : \mathcal{A}_n$ where $n \geq 1$ be a list of triples referred to as configurations.\footnote{In accordance with the type-logical tradition, I trade in parenthetical notation ‘$P_i, A_i, \mathcal{A}_i$’ for ‘$P_i : A_i : \mathcal{A}_i$’ to represent a triple.} Then a sequent is defined as

$$P_1 : A_1 : \mathcal{A}_1, \ldots, P_n : A_n : \mathcal{A}_n \vdash P : B : \mathcal{B}$$

where $P_1 : A_1 : \mathcal{A}_1 \ldots P_n : A_n : \mathcal{A}_n$ is the antecedent configuration of the sequent, $P : B : \mathcal{B}$ is its succedent configuration, and $\vdash$ is the derivability relation.

Having defined the objects of our proof theory, I lay out the rules in which those are composed, which are given by definition 16 and are a mixture of rules from Morrill (1994) and Barker (2013). For the sake of perspicuity, I drop reference to the irrelevant components of a configuration (for example, its prosodic element) when they are not relevant to the rule.
3.3. THE ASSOCIATIVE LAMBEK CALCULUS WITH CONTINUATIONS

Definition 16 (Proof rules).

\[
\begin{align*}
A : \mathcal{A} &\vdash A : \mathcal{A} & \text{Refl} \\
\Gamma \vdash \lambda A (B) : \mathcal{A} \downarrow B &\vdash \Gamma \vdash \lambda A (B) : \mathcal{A} \downarrow B & \text{I} \\
\Gamma \vdash A : \mathcal{A} \downarrow B : \mathcal{B} &\vdash \Gamma \vdash \lambda A (B) : \mathcal{A} \downarrow B / \mathcal{B} & \text{/I} \\
\end{align*}
\]

\[
\begin{align*}
\Gamma \vdash \lambda A (B) : \mathcal{A} \downarrow B &\vdash \Gamma \cdot \Delta \vdash (CA) : \mathcal{B} & \text{\textbackslash E} \\
\Gamma \vdash C : \mathcal{B} \downarrow \mathcal{A} &\vdash \Gamma \cdot \Delta \vdash \lambda A : \mathcal{A} & \text{/E} \\
\Gamma \vdash A : \mathcal{A} &\vdash \Gamma \cdot \Delta \vdash \lambda A : \mathcal{A} & \text{\textbackslash I} \\
\end{align*}
\]

To better understand what is going on, here, let me focus, now, on the following proof rule.

\[
\Gamma \vdash A : \mathcal{A} \quad \Delta \vdash C : \mathcal{A} \downarrow \mathcal{B} \quad \text{\textbackslash E}
\]

In words, this rule says that, ‘if the sequents \( \Gamma \vdash A : \mathcal{A} \) and \( \Delta \vdash C : \mathcal{A} \downarrow \mathcal{B} \) are provable from the proof theory, then the sequent \( \Gamma \cdot \Delta \vdash (CA) : \mathcal{B} \) is as well’. Focusing on the antecedent of the conclusion, \( \Gamma \) and \( \Delta \) have been merged per the operator \( \cdot \) forming the merged structure \( \Gamma \cdot \Delta \). As far as the succedent of the conclusion is concerned, we see that its syntactic component \( \mathcal{B} \) is a result of applying the antecedent, or rather premise category \( \mathcal{A} \downarrow \mathcal{B} \) to the other premise category \( \mathcal{A} \). Observe, here, that linear order matters: \( \mathcal{A} \downarrow \mathcal{B} \) applies to an argument on its left. (Compare the proof rule \( \text{\textbackslash E} \) to its counterpart \( \text{/E} \), which itself applies to an argument on its right.)

As for the succedent’s semantic component \( (CA) \), it is a result of applying the premise representation \( C \) to the other premise representation \( A \). Here, linear order does not matter, as functional application is commutative. However, this does raise an interesting point about the built in theory of semantic composition. By the Curry-Howard isomorphism there is a direct correspondence between syntactic and semantic composition. Notice, for example, that from a semantic perspective, the elimination rules \( \text{/E} \), \( \text{\textbackslash E} \), \( \text{\textbackslash I} \), \( \text{\textbackslash I} \), \( \text{\textbackslash I} \) and \( \text{\textbackslash J} \) correspond to functional application, whereas the introduction rules \( \text{\textbackslash I} \), \( \text{\textbackslash I} \), \( \text{\textbackslash I} \) and \( \text{\textbackslash I} \) correspond to functional abstraction. (See Morrill (1994), Jäger (2005) and Morrill (2011) for more on this point.)
3.3. THE ASSOCIATIVE LAMBEK CALCULUS WITH CONTINUATIONS

<table>
<thead>
<tr>
<th>Lexeme</th>
<th>Category/Type</th>
<th>Representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orcutt</td>
<td>NP</td>
<td>( \lambda P(P(o)) )</td>
</tr>
<tr>
<td>walk</td>
<td>IV</td>
<td>( \text{walk} )</td>
</tr>
<tr>
<td>Smith</td>
<td>NP</td>
<td>( \lambda P(P(s)) )</td>
</tr>
<tr>
<td>or</td>
<td>S ( /S )</td>
<td>( \lambda P ) ( \lambda q(p \lor q) )</td>
</tr>
<tr>
<td>every</td>
<td>DET</td>
<td>( \lambda P P \forall x(P(x) \rightarrow P'(x)) )</td>
</tr>
<tr>
<td>man</td>
<td>CN</td>
<td>( \text{man} )</td>
</tr>
<tr>
<td>love</td>
<td>( (N/S)/N )</td>
<td>( \lambda y \lambda x (\text{love}(x)(y)) )</td>
</tr>
<tr>
<td>some</td>
<td>DET</td>
<td>( \lambda P \lambda P' \exists x(P(x) \land P'(x)) )</td>
</tr>
<tr>
<td>woman</td>
<td>CN</td>
<td>( \text{woman} )</td>
</tr>
</tbody>
</table>

Table 3.2: A sample lexicon

3.3.1 Merge mode

Let’s see how the proof rules work in practice. I begin, first, by defining a sample lexicon as shown in table 3.2. Now consider the statement and its semantic representation shown in 3.3. Using the proof rules laid out in definition 16, it is now possible to derive this representation. This derivation is shown in (3.3b). (Again, I freely omit irrelevant components of various configurations to improve readability.)

\[(3.3) \quad \text{a. Orcutt walks } \sim \text{ walk}(o) \]
\[\]
\[
\text{b.}
\]
\[
\frac{
\text{Orcutt : } S/IV \vdash \lambda P(P(o)) : S/IV}{\text{walk : } IV \vdash \text{walk} : IV}
\]
\[\]
\[
\frac{
\text{Lex}}{
\text{Orcutt : } S/IV \cdot \text{walk : } IV \vdash \lambda P((o)) (\text{walk}) : S}
\]
\[\]
\[\]

A few words on (3.3b). First, the proof rule \textit{Lex}, short for \textit{lexical entry}, is a particular instance of \textit{Refl}. Nothing special there. As the last sequent in the above derivation would indicate, the two configurations represented by the triples ‘Orcutt : \( S/IV : \lambda P(P(o)) \)’ and ‘walk : \( IV : \text{walk} \)’ form a merged structure via \( \cdot \) per the application of the proof rule \( /E \). Therefore, the string \textit{Orcutt walks} is predicted (correctly) to be a grammatical string of English. Finally, it is worth commenting on the built in theory of semantic composition at work. By \( \beta \)-reduction, the semantic component of the last succedent in the above proof reduces to the representation shown below. The value of the representation is the set of all indices in which Orcutt walks.

\[(3.4) \quad \lambda P(P(o)) (\text{walk}) \Leftrightarrow \text{walk}(o) \]
Because Gentzen-style sequent calculi are often cumbersome and difficult to read, for the remainder of this dissertation, I will be a constant abuser of notation. I will often write proofs in a Gentzen/Prawitz-style natural deduction. (See Morrill (1994), Jäger (2005) and Morrill (2011) for the relation between sequent calculi and natural deduction.)

\[
\frac{\text{Orcutt}}{\lambda P(P(o))} \quad \frac{\text{walk}}{\text{Lex}} \quad \frac{\text{walk}}{\text{IV}} \quad /E
\]

And oftentimes, to emphasize the syntactic component of the proof theory, I will invert proof trees like the one in (3.4) as in (3.5).

\[
S \\
S/\text{IV} \\
\text{IV} \\
\text{Orcutt} \quad \text{walks}
\]

The above tree is illuminating in the following sense: in the Gentzen-style sequent calculus, derivations, or rather proofs like the one in (3.4) double as the syntactico-semantic composition trees linguists are familiar with.

### 3.3.2 Continuation mode

I now turn to exploring the continuation mode. I begin first by adding the following proof in definition 17 rule taken from Barker (2013) to the set laid out in definition 16.

**Definition 17** (Continuations (Barker 2013)).

\[
\Gamma(\Delta) \equiv \Delta \circ \lambda \alpha \Gamma[\alpha]
\]
Notice that this proof rule is the first to make reference to a gapped structure as defined in 14. Generally, this proof rule allows us to move fluidly between (non-) gapped structures and the continuation mode. Specifically, to quote Barker (2013; p. 197), the proof rule given by definition 17 “if a structure \([\Gamma]\) containing within it a structure \(\Delta\), then \(\Delta\) can take scope over the rest of \([\Gamma]\), where ‘the rest of \([\Gamma]\)’ is represented as a gapped structure of the form \(\lambda \alpha \Gamma[\alpha]\).” Here, gapped structures will play the role of quantifier raising or Montague’s so-called quantifying in technique.

**Quantifier scope as continuations**

To see continuations in action, let us consider the timeless example in (3.7).

(3.7) Every man loves some woman

It is generally agreed that this statement is ambiguous between two readings: a narrow-scope one and a wide-scope one, shown in (3.8) and (3.9) respectively.

(3.8)

Every man loves some woman \(\rightarrow \forall x (\text{man} (x) \rightarrow \exists y (\text{woman} (y) \land \text{love} (x) (y)))\)

*Narrow-scope reading*

(3.9)

Every man loves some woman \(\rightarrow \exists y (\text{woman} (y) \land \forall x (\text{man} (x) \rightarrow \text{love} (x) (y)))\)

*Wide-scope reading*

The difference in meaning between the representations in (3.8) and (3.9) is simple. In the case of the former, every man loves some, not necessarily the same, woman. In the case of the latter, there is a single woman such that every man loves her. The question now
becomes: how do we derive both representations compositionally using continuations? In figure 3.1, I have presented the derivations of both of these representations side-by-side, tacitly performing \( \beta \)-reduction in each succedent’s semantic component. (But see also Barker and Shan (2013; pp. 145-46) for similar derivations.)

Representing continuations syntactically

Again, Gentzen sequent calculi are extremely difficult to parse. In (3.10), I present what I take to be a clearer method of representing the syntactic intuitions behind the derivation of the narrow-scope reading in (3.8). A few notes on my notational decisions are given below.

(3.10)

1. First, the transitive verb *love* combines with the phonetically null nominal expression ‘∅’ on its right, yielding the quasi-intransitive verb *loves ∅*. 
3.3. THE ASSOCIATIVE LAMBEK CALCULUS WITH CONTINUATIONS

Figure 3.1: Narrow- and wide-scope reading of sentences involving quantifiers using continuations
2. Next, this quasi-intransitive verb combines with the phonetically null nominal expression $\emptyset$ on its left, yielding the quasi-sentence $\emptyset$ loves $\emptyset$.

3. The phonetically null expression introduced in step (1) is continuized. In the syntactic tree, this is reflected by adopting the notational convention ‘$N \hookrightarrow X$’. This is read ‘the configuration $\emptyset : N : x$ is replaced with the variable $X$’. The expression $\emptyset$ loves $\emptyset$ is subsequently lifted via the introduction rule $\land I$, resulting in the gapped structure $\lambda X (\emptyset$ loves $X)$ of category $N \land S$.

4. The noun phrase some woman combines with this gapped structure on its right, yielding a sentence. In order to capture word order facts, this expression may be reduced by the proof rule $\equiv$ as follows

$$\textit{some woman}(\lambda X (\emptyset$ loves $X)) \iff \emptyset$ loves some woman$$

This reduction is indicated by the dashed line in the tree.

5. The phonetically null expression introduced in step (2) is continuized. In the syntactic tree, this is reflected by adopting the notational convention ‘$N \hookrightarrow Y$’. The expression $\emptyset$ loves some woman is subsequently lifted via the introduction rule $\land I$, resulting in the gapped structure $\lambda Y (Y$ loves some woman) of category $N \land S$.

6. The noun phrase every man combines with this gapped structure on its right, yielding a sentence. Again, in order to capture word order facts, this expression may be reduced by the proof rule $\equiv$ as follows

$$\textit{every man}(\lambda Y (Y$ loves some woman)) $\iff$ every man loves some woman$$

Again, this reduction is indicated by the dashed line in the tree.
Representing continuations semantically

The linguist’s tree illuminates what is going on syntactically in regard to continuations at the expense of semantic clarity. In order to remedy that, I have also provided both a Gentzen/Prawitz-style deduction of this narrow-scope reading in (3.11).

(3.11)

A few words on my notation. No doubt it is difficult to fully represent continuations using deductions. To aid in readability, I have combined the application of both the proof rules $\equiv$ and $\otimes I$ into the meta-rule $\text{Cont}^n$ for $n < \omega$. The application of this rule is super-scripted with a natural number $n$ to indicate the other configurations that depend on it. Such dependent configurations are also super-scripted with $n$ to make this point formally obvious.

3.4 Chapter summary and next chapter preview

In this chapter, I laid out the formal machinery, which my analysis of the syntax and semantics of adjectival comparatives will be framed in. Specifically, I laid out

- my semantic representation language, which takes the form of Muskens’s (1995) logic $\text{TT}_2$; and
- my syntactic representation language, which takes the form of a Gentzen-style sequent calculus, augmented with Barker’s (2013) continuations.
Although both representation languages are fairly non-standard, they will provide me with concise and elegant means for capturing the syntactico-semantic behavior of adjectival comparatives.

In the next chapter, I will provide such an analysis of what are often referred to as phrasal comparatives. There, I will showcase both TT$_2$ and the sequent calculus defined, here, sans continuations.
4 | First steps

4.1 Introduction

In this section, I will present my syntactico-semantic analysis of what is often referred to as a phrasal comparative like the one in (4.1).

(4.1) Orcutt is taller than Smith

Phrasal comparative

At a high level, my analysis can be thought of as a hybrid approach in the following sense.

• Following Kamp (1975), Klein (1980), Larson (1988), van Rooij (2008) and van Rooij (2011) among many more authors, I work in a delineation, i.e., degree-less, semantics.

• Following Cresswell (1976), von Stechow (1984), Kennedy (1997) and Heim (2006) among many more authors, the logical shape of my analysis is a Greater-than one.

• Following Larson (1988), my analysis begins from the premise that the simplest possible analysis of the verb phrase comprised of be + the comparative form of an adjective is treating that phrase as a quasi-transitive verb phrase. (But see Heim (2006) for potential reasons for rejecting a similar hypothesis.)

From a semantic perspective and simplifying things a bit, I treat the denotation of a comparative adjective as denoting an order, i.e., a binary relation (with certain mathematical properties) over the domain of individuals relative to a Montagovian index. In Sassoon’s (2007) terminology, my analysis is an ordinal one in that, to know the the meaning
4.2. **PHRASAL COMPARATIVES**

Table 4.1 shows the abbreviations for syntactic categories. Adopting such notational conventions will no doubt make things conceptually easier to understand at the expense of formal perspicuity. On the semantic side of things, I will use the following stock of variables.

1. $x, y, z, x', y', z', \ldots$ are variables of type $e$;

2. $u, v, w, u', v', w', \ldots$ are variables of type $s$;

3. $P, P', \ldots$ are variables of type $(es)$;

---

1Effectively, all delineation based approaches to the semantics of adjectival comparatives are in fact ordinal.
4.2. PHRASAL COMPARATIVES

Table 4.2: A sample lexicon for phrasal comparatives

<table>
<thead>
<tr>
<th>Lexeme</th>
<th>Category/Type</th>
<th>Representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orcutt</td>
<td>NP</td>
<td>( \lambda P(P(o)) )</td>
</tr>
<tr>
<td>is</td>
<td>IV/A</td>
<td>( \lambda P(P) )</td>
</tr>
<tr>
<td>tall</td>
<td>A</td>
<td>tall</td>
</tr>
<tr>
<td>er</td>
<td>A ((A/PP))</td>
<td>( \lambda P\lambda \lambda x (Q\lambda y(\delta (P)(x)(y))) )</td>
</tr>
<tr>
<td>than</td>
<td>P</td>
<td>( \lambda Q(Q) )</td>
</tr>
<tr>
<td>Smith</td>
<td>NP</td>
<td>( \lambda P(P(s)) )</td>
</tr>
</tbody>
</table>

• \(Q, Q', \ldots\) are variables of type \(((es)s)\); and

• \(\varepsilon\) is a variable of type \(((((es)s)(es))\)

Having established some notational conventions, I claim that the appropriate lexical entry for the comparative morpheme -er is given in (4.2).

(4.2)

\[
\text{Lexical entry —} \quad \text{er} : A \backslash A/PP : \lambda P\lambda \lambda x (Q\lambda y(\delta (P)(x)(y)))
\]

I refrain from explication of the semantic representation in (4.2) for the time being, particularly the meaning of the \(\delta\) operator. I will return to it in §4.3 of this chapter. I begin, first, by showing how this lexical entry behaves in the type-logical setting.

To get the analysis off the ground, I assume, further, the lexical entries specified in table 4.2. (Note that this table replaces the sample one shown in table 3.2.) With these in hand, I give the Gentzen-Prawitz style natural deduction of (4.3) in figure 4.1. (See, again, §4.3.1 for a discussion on deductions.)

(4.3) Orcutt is taller than Smith

The proof tree in figure 4.1 can be inverted, and what we are left with is a more familiar linguist’s tree shown in (4.4). (See, again, §3.9 on the inversion of a Gentzen-Prawitz proof to derive a linguist’s tree.)
4.2. PHRASAL COMPARATIVES

(4.4)

From a syntactic perspective, in words, the derivation in (4.4) can be understood as follows.

1. First, the comparative morpheme -er combines with the adjective tall on its left, yielding the comparative adjective taller.

2. In parallel, the preposition than combines with the noun phrase Smith on its right, yielding the prepositional phrase than Smith.

3. The comparative adjective taller then combines with the prepositional phrase than Smith, yielding the adjective taller than Smith.

4. The verb is combines with the adjective taller than Smith, yielding the intransitive verb phrase is taller than Smith.

5. Finally, the noun phrase Orcutt combines with the intransitive verb phrase is taller than Smith, yielding the sentence Orcutt is taller than Smith.

I will admit right now that my syntactic analysis of the adjectival comparatives is non-standard and problematic in certain ways. However, due to the formal transparency of the system I am working in, it is more-or-less easy to identify where my analysis misfires; and I do my best throughout the remainder of this dissertation to identify the places in which it does, offering sketches of possible solutions or references to possible solutions in the literature. However, I will investigate now the morpho-syntactic dependency between the comparative morpheme -er and than.
Figure 4.1: A complete derivation of a phrasal comparative
4.2. PHRASAL COMPARATIVES

4.2.1 The morpho-syntactic dependency between -er and than

In the introduction of this section, I claimed that it is natural to think of a comparative adjective like taller as a transitive verb, at least from a semantic perspective. This is only an analogy but a productive one at that. To see what I mean, let’s compare my analysis of the expression taller to a run-of-the-mill transitive verb. Assume, now, that the lexical entry for the verb find is as in (4.5).

(4.5)

\[
\text{Lexical entry —}
\]
\[
\text{find : IV/NP : } \lambda Q \chi (Q \lambda y (\text{find}(x)(y)))
\]

The syntactic derivation of the statement in (4.6a) is given in (4.6b).

(4.6) a. Orcutt found Smith
    b. 

\[
\begin{array}{c}
S \\
\text{NP} & \text{IV} \\
\text{IV/NP} & \text{NP} \\
\text{Orcutt} & \text{found} & \text{Smith}
\end{array}
\]

There are two things to observe here.

- While find and taller differ in their syntactic categories, they are of the same semantic type.
4.2. PHRASAL COMPARATIVES

\[ \tau(IV/\text{NP}) = ((es)s)es \]
\[ = \tau(A/\text{PP}) \]

- While *Smith* and *than Smith* differ in their syntactic categories, they too are of the same semantic type.

\[ \tau(\text{NP}) = ((es)s) \]
\[ = \tau(\text{PP}) \]

From a semantic perspective, it makes sense to think of the expression *taller* as transitive-like, or stated another way a quasi-transitive verb phrase. Its semantic value is a binary relation between two individuals indexed, of course, to a Montagovian index. Although the effects of *than* are purely syntactic, transforming a noun phrase into a prepositional one the expression *than Smith* can be thought of as quasi-direct object. This is because it denotes a generalized quantifier meaning. I take it that these syntactico-semantic intuitions are at the heart of Larson’s (1988) proposal for the syntax and semantics of adjectival comparatives.

Related to this point, notice that I have exploited the fact that the translation function \( \tau \) is not injective so that the expression *taller* must obligatorily combine with the prepositional phrase *than Smith*; it cannot combine with *Smith* directly. Consequently, I make a prediction, namely, that the morpheme *-er* and *than* co-occur in general, which is evidenced by examples like (4.7).

(4.7) #Orcutt is taller Smith

However, as it stands, the system I have presented is too coarse-grained to block the ungrammaticality of statements like (4.8).

(4.8) #Orcutt is taller to Smith
4.2. PHRASAL COMPARATIVES

This is because I have no obvious way of distinguishing, either at the level of syntactic types or at the level of the way in which those types are combined, i.e., the level of syntactic modes of combination, the fact that the comparative morpheme -er/more must combine with only the preposition than and not to. It simply could be a lexical fact about English that -er/more co-occur with than, or the explanation could lie at a much deeper morpho-syntactic level. I refer the reader to Hendriks (1995) for an alternative but similar explanation on why the comparative morpheme and than co-occur. Her analysis involves three modes of syntactic combination in the type-logical setting whereas mine only involves two. However, for my present purposes, introducing an additional mode of syntactic combination à la Hendriks (1995) will only complicate syntactic matters and detract from the primary purpose of this dissertation, which is to give a semantic analysis of adjectival comparatives.

4.2.2 The synthetic and analytic form of the comparative morpheme

While many adjectives have a synthetic form, i.e., that adjective A plus (+) the comparative morpheme -er, many do not. Adjectives like intelligent only have an analytic form, i.e., more plus (+) that adjective A. An example is shown (4.9).

(4.9) Orcutt is more intelligent/#er than Smith

The analytic form of the comparative is easy to account for in this system. I simply posit the lexical entry for more as in (4.10).

(4.10)

\[
\text{Lexical entry – more} : (A/PP) / A : \lambda P \lambda Q x (Q \lambda y (\delta (P) (x)) (y))
\]

The only difference between the synthetic and analytic form of the comparative is that the former applies to an adjective on its left, whereas the latter applies to one on its right, thus accounting for word order facts.

\[^2\text{See, again, \S 3.3.1 and \S 3.3.2 for a discussion of the two modes of combination I deploy in this dissertation.}\]
4.3 Scalar dimensions

I turn, now, to an exposition of the semantic representation of a phrasal comparative like (4.11).

(4.11) Orcutt is taller than Smith \( \sim \delta \text{[tall]} (o) (s) \)

In the above example, the value of \( \delta \) is a function—a function that takes as an argument the semantic representation of an adjective like tall, which in this case would be the logical constant tall of semantic type (es), and returns a value of type (ees). This is shown pictorially in figure 4.2. This value is an ordering over the domain of entities \( D_e \) fixed at an index \( w \), which itself is identified, at least in part, with the scalar dimension that adjective encodes for. (I will make clear my formal views on scalar dimensions in what follows.) In (4.11), the value of \( \delta \) applied to the value of tall would be the scale associated with HEIGHT.

There is much talk about the relationship between gradable predicates like tall and the gradable properties they make reference to like HEIGHT. This relation makes reference to is a subtle one but completely non-trivial. To see what I mean, consider the comparative constructions in (4.12).

(4.12) Orcutt is \( \left\{ \begin{array}{ll} \text{taller} \\
\text{shorter} \end{array} \right. \) than Smith (is)
4.3. SCALAR DIMENSIONS

There is no mention of height in (4.12). However, there is little doubt that what is being compared is not the two men’s tallness, but rather their height. Evidently, in comparative constructions, gradable adjectives like tall and short tacitly make reference to, or rather lexically encode for a more abstract gradable property like height.

Specifically, gradable properties are often said to characterize a scale. Consequently, they are often referred to as scalar dimensions. To quote Kennedy (2007b; p. 34)

> the [scalar] dimension indicates the kind of measurement that the scale represents, and is the most obvious parameter of scalar variation, since it both distinguishes different adjectives from each other (e.g. expensive measures an object along a dimension of COST while fast measures an object along a dimension of SPEED) . . .

Gradable properties tend to be associated with at least one antonym pair (but oftentimes more) (Leher and Lehrer 1982; Cruse and Togia 1995; Kennedy 2001; Kennedy and McNally 2005; Kennedy 2007b). The adjectives tall and short are associated with the property of height; heavy and light with the property of weight; and cold and hot with the property of temperature. Sometimes, more than one antonym pair is associated with the same gradable property. This is the case with hot and cold and warm and cool, which are both associated with the gradable property of temperature.

In figure (4.3), I have depicted a useful but rudimentary picture of the gradable property of temperature, showing clearly (some of) the values of its associated predicates: cold, cool, warm, and hot. The individual notches can be interpreted as degrees Celsius but need
Note that, oftentimes, a language like English does not explicitly lexicalize a gradable property like it does with *height* and *HEIGHT* or *temperature* and *TEMPERATURE*. Take, as an example, the expression *handsome*.

(4.13) Orcutt is more handsome than Smith (is)

It is unclear what the name of the gradable property associated with handsome actually is—possibly (physical) *ATTRACTIVENESS*. But if that is the case, what then is the gradable property made reference to by *physically attractive* as it appears in (4.14)?

(4.14) Orcutt is more (physically) attractive than Smith (is)

As far as I can tell, the property being compared in both (4.13) and (4.14) is the same, and its not *ATTRACTIVENESS* per se. Rather, the property being compared in these two examples is something more abstract than attractiveness—perhaps *PHYSICAL CHARACTER*. Whatever it is, it does not appear to be lexicalized by English, and we as theoreticians are reduced to giving a sort of roundabout description of a gradable property like this one.

Some adjectives like *clever* are referred to as multi-dimensional adjectives (Kamp 1975; Klein 1980; Sassoon 2012), as they encode for different dimensions of evaluation. Take, for example, *clever*: one may be clever in regard to his/her mathematical problem solving abilities or one may be clever in regard to his/her quick-wittedness. The adjective *healthy* is another good example of a multi-dimensional adjective. As Sassoon (2012; p. 2) points out

An adjective like *healthy*, for example, may be associated with many different dimensions simultaneously, such as blood pressure, cholesterol, cancer, lung functions, pneumonia, chickenpox, and so forth. One can be healthy with respect to blood pressure, but not with respect to cholesterol.

Adjectives like *clever* and *healthy* are ubiquitous: others include, but are not limited to *similar, identical, typical, normal, good, talented, happy, human, and healthy* (Sassoon 3

Importantly, I do mean to suggest that this diagram is the way to understand the scalar dimension of temperature, as it seems to suggest that denotations of predicates like *cold, cool, warm, and hot* are non-vague and exhaustified. This is, of course, does not seem to be the case, and a complete picture of this particular scalar dimension should take these facts into consideration.
4.3. SCALAR DIMENSIONS

Figure 4.4: This diagram represents the relation between gradable adjectives & scalar dimensions. The mapping from sick to PSYCHOLOGICAL HEALTH and PHYSICAL HEALTH is a one-to-many map. The mapping from tall and short is a many-to-one map

2012). Generally, the relation between adjectives and scalar dimensions is understood as a mapping from the set of adjectives of a particular natural language like English to the set of scalar dimensions and vice versa. This mapping may be many-to-one as in the case of tall and short, or one-to-many as in the case of sick. This is captured diagramatically in figure (4.4).

4.3.1 Impartiality

From a natural language perspective, the obvious question becomes, is there any evidence to support the idea that, in comparative constructions, gradable adjectives lexically encode for scalar dimensions? Yes, but in a sort of circuitous way. Cruse and Togia (1995) claim that gradable adjectives are (often) impartial in comparative constructions like (4.15), in that they do not require the individuals mentioned by that comparative to possess the property denoted by that adjective.

(4.15) The box is quite light, but it is heavier than that one

In this example, it need not be the case that the boxes be heavy. In fact, the statement claims explicitly they are light: what is at stake is whether one box possesses more weight than the other.

Authors in the formal semantic tradition discuss the concept of impartiality in different terms. It has been pointed out that adjectival comparatives do not entail their positive
4.3. **SCALAR DIMENSIONS**

form (Bartsch and Vennemann (1972), Kennedy (1997), Bale (2008) and Morzycki (2011) among many more).

\[ (4.16) \]
\[
\begin{align*}
\text{a. Orcutt is taller than Smith} \\
\neg \text{Orcutt is tall}
\end{align*}
\]

The idea, here, is that while it very well may be the case that Orcutt is taller than Smith in that Orcutt is 5′10 and Smith is 5′9, it may very well be the case that Orcutt is not tall in some contextually determined sense but of only average height.

Building on this insight, observe that the statement in (4.17a) is compatible with the situations in (4.17b)–(4.17d), but not the one in (4.17e).

\[ (4.17) \]
\[
\begin{align*}
\text{a. Orcutt is taller than Smith and . . .} \\
\text{b. both Orcutt and Smith are tall} \\
\text{c. Orcutt is tall but Smith isn’t} \\
\text{d. neither Orcutt nor Smith are tall} \\
\# \text{Orcutt isn’t tall but Smith is}
\end{align*}
\]

That the statement in (4.17a) is compatible with the situation depicted by (4.17d) suggests that the property of tallness is not being compared in (4.17a). After all, how can Orcutt be taller than Smith if neither man is are in fact tall?\(^4\)

Importantly, being tall is not a property intrinsic to the material world in the same way that the property of having height is. Certainly, evolution did not conspire to give us the property of being tall in the way in which it can said to have extended us linearly in space. Being tall is a property of the material world only derivatively in the sense that it derives its meaning from the property of having height: to attribute to an individual the property of being tall is to attribute of that person the property of having a certain contextually defined standard of height—a point Kennedy (2007b) argues at length.

There is little question that standards, by their very nature, do not exist independently of human cognition; they are context dependent, conceptual categories created by humans for humans to impose upon the world to make useful distinctions. However, given that both Orcutt and Smith are both material objects bound in space-time, we can be sure that both

\(^4\)It is important to point out that, at some level, all current semantic theories of the semantics of adjectival comparatives agree that it is not the property of TALLNESS that is being compared in (4.17a).
4.3. SCALAR DIMENSIONS

men possess a property related to being tall, namely height.

The same can be said about most if not all gradable adjectives, including but not limited to short, fat, skinny, obese, thin, hot, cold. These properties are contributions of the mind: nothing more, nothing less. However, many, but not all gradable properties that gradable adjectives make reference to, like HEIGHT, MASS, TEMPERATURE are, as far as I can tell, fundamental to the makeup of our world.

4.3.2 A formal model of scalar dimensions

Having established some linguistic and philosophical intuitions regarding the nature of scalar dimension, I turn now to formalizing this concept. (See, however, Kennedy (2007b) for an alternate formalization.) In this section, I will provide various formalizations of scalar dimensions, revising them in light of new linguistic data.

Importantly, it should be understood from the outset that it is my intention to provide a framework on the formalization for scalar dimensions, providing only minimum mathematical requirements on the structure of scalar dimensions generally. As will become clear through the course of this section, although I will attempt to provide a full-blown characterization of the scalar dimension of HEIGHT, I do not mean to suggest this is the way to characterize all scalar dimensions. Such characterizations will necessarily draw from the hard sciences, including the cognitive sciences, as well as psychology.

I choose the scalar dimension of HEIGHT, or equivalently LENGTH, not only because it has played an important role in the study of comparatives, but it illustrates a nice philosophical point that underlies my formalization of this particular dimension. Following Carnap (1966), let us consider a world consisting of ‘length bearing objects’—rods—as depicted in figure (4.5(a)). Cresswell (1976), Bale (2008) and van Rooij (2010b) all take it as a near conceptual primitive that humans are able to rank-order rods with respect to the extent to which those rods possess length. Bale (2008; p. 9; replacing rod for individual and beauty for length), citing Cresswell (1976), claims

If one has the conceptual abilities to determine who has more of a certain quality than another, then one can develop a scale based on this distinction. For example, most people are able to determine whether one [rod] has as much [length] as another.
What the cognitive mechanisms underlying this ordering process are exactly, I do not know. I do, however, agree with Bale (2008): there is little doubt that the reader will be able to survey the ‘rods’ in figure 4.5(a) and rank-order them from shortest-to-longest as in figure 4.5(b) without the aid of a full-blown mathematical measurement system. And, there’s an intuition that, interpreting ‘conceptual abilities’ in its ordinary sense, even if the antecedent of Bale’s (2008) conditional is false, that is to say, even if the length of the rods were indistinguishable to the human eye, we can appeal to some tool—say a microscope—that can determine whether there exists a difference in length between the ‘rods’ on this page. Stated more clearly, there’s an intuition that humans can always order a set of objects, for example, a set of rods, from shortest-to-longest (or longest-to-shortest), without using degrees or numbers or measures or the like.

The above thought experiment has important consequences for our formal semantic theory of adjectival comparatives: in this and chapter 5, I will show that we get a more general treatment of comparatives if one adopts a view that the orderings, not degrees or numbers or measures, are relevant to capture the logic of comparatives. That we do not necessarily
appeal to degrees will be reflected in my initial and subsequent formalizations of scalar dimensions generally, and the scalar dimension of \textit{HEIGHT} in particular. I begin, then, by defining the concept of a scalar dimension generally, which itself is given preliminarily in definition 18.

**Definition 18** (Scalar dimension (Preliminary version)). Let a scalar dimension $\mathcal{S}$ be the pair $\mathcal{S} = (\mathcal{A}, \mathcal{S})$. Here, $\mathcal{A} \subseteq D_{(es)}$ is a system of sets referred to as $\mathcal{S}$’s \textbf{adjectival component}, whose elements correspond to adjectival meanings associated with $\mathcal{S}$. $\mathcal{S} \subseteq D_{(ees)}$ is also a system of sets referred to as $\mathcal{S}$’s \textbf{scalar component} and whose elements include at least the ternary relations $>_{\mathcal{S}}, <_{\mathcal{S}}$, and $=_{\mathcal{S}}$ such that for all $x, y, z \in D_e$ and $w \in D_s$

1. $\neg (x >_{\mathcal{S}} x @ w)$  \hspace{1cm} \textit{Irreflexivity}
2. $(x >_{\mathcal{S}} y @ w) \leftrightarrow (y <_{\mathcal{S}} x @ w)$  \hspace{1cm} \textit{Duality}
3. $x =_{\mathcal{S}} x @ w$  \hspace{1cm} \textit{Reflexivity}
4. $(x =_{\mathcal{S}} y @ w) \rightarrow (y =_{\mathcal{S}} x @ w)$  \hspace{1cm} \textit{Symmetry}
5. $(x =_{\mathcal{S}} y @ w) \land (y =_{\mathcal{S}} z @ w) \rightarrow (x =_{\mathcal{S}} z @ w)$  \hspace{1cm} \textit{Transitivity}
6. $(x >_{\mathcal{S}} y @ w) \land (x =_{\mathcal{S}} z @ w) \rightarrow (z >_{\mathcal{S}} y @ w)$

Importantly, in adopting definition 18, I am making a tacit claim, namely that all scalar dimensions have the structure imposed by these axioms. This is an important point, and one that I will return to often. Abusing notation, I will often write `$x >_{\mathcal{S}} y @ w$’, which is to be read $x$ \textit{has more $\mathcal{S}$ than $y$ at index $w$}. In order-theoretic terminology, $>_{\mathcal{S}}$ and $<_{\mathcal{S}}$ are \textbf{order duals}; and the idea, here, is that antonyms, or rather \textbf{polar opposites} like \textit{tall} and \textit{short}, which themselves lexically encode for the same scalar dimension, are associated by the value of $\delta$ with one ordering $>_{\mathcal{S}}$ or the other $<_{\mathcal{S}}$ (Rullmann (1995) but see, also, Kennedy (2001) for potential problems to this approach).

So, for example, the formalization of the scalar dimension of \textit{HEIGHT} would be $\mathcal{S}_H = (\{[\text{tall}], [\text{short}]\}, \{>_{\mathcal{S}_H}, <_{\mathcal{S}_H}, =_{\mathcal{S}_H}\})$, where the value of $\delta$ associates the value of \textit{tall} with $>_{\mathcal{S}_H}$ and the value of \textit{short} with $<_{\mathcal{S}_H}$. Finally, $=_{\mathcal{S}_H}$ will be referred to as the \textbf{indifference relation} of $\mathcal{S}_H$, which is to be read \textit{having the same height as}. Formally, it is an equivalence relation over the domain of individuals $D_e$ relative to some index $w$. In sum, all scalar dimensions have
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- an adjectival component encoding for the adjectival meanings that lexically encode for that scalar dimension; and

- a scalar component encoding for the orderings those adjectival meanings are associated with.

Because scalar dimensions like HEIGHT and WEIGHT are formalized independently of each other, their respective mathematical properties must be governed by the adoption of a series of axioms. In definition 19, I give a sample formalization of HEIGHT.

**Definition 19** (Scalar dimension of HEIGHT (Preliminary version)). Let a scalar dimension of HEIGHT be the tuple \( S_H = (\mathcal{A}, \{ >_H, <_H, =_H \}) \) as given by definition 18 such that for all \( x, y, z \in D_e \), and \( w \in D_s \) the relation \( >_H \) obeys the axioms in (1)–(3).

**Intra-world axioms:**

1. \( (x >_H y @ w) \land (y >_H z @ w) \rightarrow (x >_H z @ w) \)\) \hspace{1cm} **Transitivity**

2. \( (x >_H y @ w) \lor (y >_H x @ w) \lor (x =_H y @ w) \) \hspace{1cm} **Totality**

Again, by formalizing a scalar dimension like HEIGHT independently from all other scalar dimensions, I am making a tacit claim, namely that the study of specific scalar dimensions is a lexical semantic enterprise. Of course, the program of modeling independent scalar dimensions must draw on the resources of other sciences, including but not limited to the hard sciences. However, this enterprise can be informed and guided productively by natural language phenomena. More to the point, I will show how (in)valid natural language inferences involving particular gradable adjectives like taller or shorter, and consequently, the scalar dimensions those adjectives encode for like HEIGHT, can inform us, as lexically semantic-minded formal semanticists, about the structure of these scalar dimensions, at least in the way in which competent speakers of a natural language like English envisage them. With this in mind, the remainder of this section will be spent investigating the formal properties of the scalar dimension HEIGHT, as it is the one dealt with primarily in the literature. I make no strong claims about the structure of other scalar dimensions unless otherwise indicated.

Returning to definition 19, in order theory, there is a powerful concept referred to as duality (Davey and Priestley 2002). Duality says that, given any statement \( \varphi \) about elements
4.3. SCALAR DIMENSIONS

$x$ and $y$ of a set ordered under a relation $>$, we obtain the dual statement $\varphi^D$ by replacing each occurrence of $>$ with its dual $<$. By duality, lemma 1 holds in general.

**Lemma 1** (Duality). The following statements hold for all $TT_2$ models $\mathbb{M}$ satisfying axioms (1)–(5) of definition 18 and axioms (1)–(3) of definition 19, $x, y, z \in D_e$, and $w \in D_s$

**Intra-world duals:**

1. $\neg (x <_S x \mathbin{@} w)$  \hspace{1cm} Irreflexivity
2. $(x <_S y \mathbin{@} w) \land (y <_S z \mathbin{@} w) \rightarrow (x <_S z \mathbin{@} w)$  \hspace{1cm} Transitivity
3. $(x <_S y \mathbin{@} w) \lor (y <_S x \mathbin{@} w) \lor (x =_S y \mathbin{@} w)$  \hspace{1cm} Totality
4. $(x <_S y \mathbin{@} w) \land (x =_S z \mathbin{@} w) \rightarrow (z <_S y \mathbin{@} w)$

In conjunction with the axioms laid out by 18, the axioms (1)–(3) given by definition 19 force $>_S$ and $<_S$ to behave as a **intra-index scales**. Specifically, the relation $>_S$, and consequently $<_S$ form **strict weak orders**, which themselves are a particular type of order generally used to model scales (Krantz et al. 1971; Sassoon 2010; van Rooij 2010b; Lassiter 2011; van Rooij 2011).

**Definition 20** (Strict weak order). A relation $R$ over a set $X$ is said to be a **strict weak order** just in case for all $x, y, z \in X$

1. $\neg (xRx)$  \hspace{1cm} Irreflexivity
2. $(xRy) \land (yRz) \Rightarrow (xRz)$  \hspace{1cm} Transitivity
3. $(xRy) \Rightarrow (xRz) \lor (zRy)$  \hspace{1cm} Almost connected

Irreflexivity and transitivity jointly imply that $R$ is asymmetric. Specifically,

3. $xRy \Rightarrow \neg (yRx)$  \hspace{1cm} Asymmetry

**Lemma 2.** For all indices $w \in D_s$, the relation $>_S$, as given by definition 19, is a strict weak order over $D_e$ with respect to $w$. 
Proof. Irreflexivity and transitivity are immediate per definition 19. For the almost connected property, fix an arbitrary index $w$. Suppose that $x >_{J_2} y @ w$. Consider an arbitrary $z$. By the totality condition, there are three cases to consider.

- If $x >_{J_2} z @ w$, then the proof is completed.
- If $z >_{J_2} x @ w$, given that $x >_{J_2} y @ w$, by transitivity, $z >_{J_2} y @ w$, and the proof is again complete.
- If $x =_{J_2} z @ w$, given that $x >_{J_2} y @ w$, by axiom (5) of definition 18, it follows that $z >_{J_2} y @ w$. So, the proof is completed.

To better understand what I mean by the relations $>_{J_2}$ and $<_{J_2}$ forming (intra-index) scales, consider the model in example 1.

**Example 1.** Let $\mathcal{M} = (\mathcal{F}, I)$ be, at least in part, such that

1. $D_e = \{o, s, j\}$;

2. $D_s = \{w\}$; and

3. $J_2 = \{[\text{[tall]}], [\text{[short]}], \{\langle o, s, w \rangle, \langle s, j, w \rangle, \langle o, j, w \rangle \},$

   \[
   \{\langle s, o, w \rangle, \langle j, s, w \rangle, \langle j, o, w \rangle \}, \emptyset \}
   \]

Here,

- $[\delta]$ will associate $[\text{[tall]}]$ with the set $\{\langle o, s, w \rangle, \langle s, j, w \rangle, \langle o, j, w \rangle \}$; and

- $[\delta]$ will associate $[\text{[short]}]$ with the set $\{\langle s, o, w \rangle, \langle j, s, w \rangle, \langle j, o, w \rangle \}$.

So, in this model,

- Orcutt has more height than Smith, who in turn has more height than Jones.
• Similarly, Jones has less height than Smith, who in turn has less height than Orcutt.

• And no one is as tall as anyone else.

The scalar dimension of \textsc{height} $\mathfrak{h}$, as shown in (3) can be represented graphically as in figure 4.6. Fixing index $w$, we have two scales, whose edges represent the relations \textsc{having more or less height than} and whose elements are entities, not degrees.\footnote{Following Cresswell (1976), it is possible to form equivalence classes of individuals under an indifference relation like $\equiv_{\mathfrak{h}}$, and identify those equivalences classes with the concept of, or as just being a degree. As such, many authors have claimed Cresswell’s (1976) analysis is a degree-based one. However, there is an important difference between my analysis and current degree-based analyses of adjectival comparatives like von Stechow’s (1984), Kennedy’s (1997) or Heim’s (2006): I do not quantify over these equivalence classes in the object language; nor do I have any operators at the level of logical form that manipulate these equivalence classes. That is to say, the meta-linguistic identification of these equivalence classes with degrees on the part of the theoretician has no formal ramifications in the analysis presented here. To make this point even clearer, observe that the relations $\mathfrak{h}$ and $\mathfrak{h}$ are defined over entities (and worlds), not equivalence classes of entities. This is a philosophically and formally subtle point, but one that I take to be important.}

\begin{figure}[h]
\centering
\begin{tikzpicture}
    \node (o) at (0,0) {$o$};
    \node (s) at (1,0) {$s$};
    \node (j) at (2,0) {$j$};
    \node (j') at (3,0) {$j'$};
    \node (s') at (4,0) {$s'$};
    \node (o') at (5,0) {$o'$};
    \draw[->] (o) -- (s) node[midway, above] {$>_{\mathfrak{h}}$};
    \draw[->] (s) -- (j) node[midway, above] {$<_{\mathfrak{h}}$};
    \end{tikzpicture}
\caption{The scalar dimension of \textsc{height} depicted at index $w$}
\end{figure}

Now it should be easier to understand the semantic representation of the comparative adjectives \textit{taller} and \textit{shorter} as they are given in (4.18a) and (4.18b) respectively.

\begin{equation}
\begin{gathered}
\text{a.}
\text{Orcutt is taller than Smith} \sim\!
\delta (\text{tall}) (o) (s) = \{ w \in D_s \mid o >_{\mathfrak{h}} s @ w \}
\end{gathered}
\end{equation}

\begin{equation}
\begin{gathered}
\text{b.}
\text{Orcutt is shorter than Smith} \sim\!
\delta (\text{short}) (o) (s) = \{ w \in D_s \mid o <_{\mathfrak{h}} s @ w \}
\end{gathered}
\end{equation}
The set denoted by the semantic representation in (4.18a) is the set of indices in which Orcutt has more height than Smith. Similarly, the denotation of the representation in (4.18b) is the set of indices in which Orcutt has less height than Smith. With respect to the model laid out in example 1, the set denoted in (4.18a) would be \( \{w\} \), whereas the one denoted in (4.18b) would be \( \emptyset \).

Given the way I have axiomatized \( \text{HEIGHT} \), I predict a series of trivial facts, including the equivalence in (4.19) to hold in general.

(4.19)  
\[
\begin{align*}
\text{a. Orcutt is taller than Smith} & \quad \text{(is)} \\
\text{b. } & \quad \iff \text{Smith is shorter than Orcutt} \quad \text{(is)}
\end{align*}
\]

Moreover, I predict the transitive inference in (4.20) to hold by Axiom (3) of definition 19.

(4.20)  
\[
\begin{align*}
\text{a. Orcutt is taller than Smith} \\
\text{b. Smith is taller than Jones} \\
\text{c. } & \quad \Rightarrow \text{Orcutt is taller than Jones}
\end{align*}
\]

That I have chosen to model the scalar dimension of \( \text{HEIGHT} \) independently from other scalar dimensions has important ramifications at the level of natural language inference. Consider, now, an invalid inference brought to my attention by Christopher Potts (p.c.), involving the comparative adjective \emph{more powerful}.

(4.21)  
\[
\begin{align*}
\text{a. Rock is more powerful than scissors} \\
\text{b. Scissors is more powerful than paper} \\
\text{c. } & \quad \not\Rightarrow \text{Rock is more powerful than paper}
\end{align*}
\]

Notice that, given the rules of the game Rock, Paper, Scissors, the transitive inference depicted in (4.21) does not hold. It would follow, then, that whatever scalar dimension \emph{more powerful} encodes for in this context is not transitive in general, which suggests that not all scalar dimensions are even scalar! This is an important empirical point that is particularly evident in the context of multi-dimensional adjectives.
Transitivity and multi-dimensional adjectives

In the previous section, I remarked that, at a bare minimum, scales are to be understood as strict orders. Cresswell (1976; p. 266) is indifferent on the nature of the type of ordering a scalar dimension is modeled as: “whether it should be strict or not or total or not seems unimportant”. However, according to Kennedy (2007b; p. 34)

> There are a number of different ways that adjectival scales can be formalized, but minimally they must be triples \((D, \prec, \delta)\) where \(D\) is a set of points, \(\prec\) is a total ordering on \(D\), and \(\delta\) is a dimension.

No doubt no one can be \(A\)-er than themselves for any gradable adjective \(A\), irrespective of whether this schema is instantiated by tall, short, or clever. That the scalar dimensions associated with a particular comparative adjective be irreflexive seems very much true of all scalar dimension. The question is whether all scalar dimensions are transitive. To quote Wheeler (1972; p. 320)

> “I should give some account of why, if someone invents the word “glof” and says the truths “[Orcutt] is glofer than [Smith]” and “[Smith] is glofer than [Jones]”, we can know that [Orcutt] is glofer than [Jones] even though we don’t know what “glof” means.”

I take Wheeler’s (1972) point (and Kennedy’s (2007b) for that matter) to be this: that the scalar dimension associated with a gradable adjective is transitive is too part of the meaning of the comparative form in general. However, I think Wheeler (1972) is wrong: multi-dimensional adjectives like cleverer are not associated with scalar dimensions that denote transitive orders.

To see this, notice that the dimension of a gradable adjective can be fixed both linguistically and extra-linguistically (Kamp 1975; Sassoon 2012). In the below examples, the phrases in the sense that and in regard to can be used to explicitly fix or rather, disambiguate the dimension the adjective is being evaluated on.

\[(4.22)\] a. Orcutt is cleverer than Smith in the sense that he\(_1\) is more mathematically savvy than Smith
b. Orcutt₁ is cleverer than Smith in regard to his₁ mathematical problem solving abilities

Tests like the above have been proposed by Sassoon (2012) to separate the classes of single- and multi-dimensional adjectives. Another reasonable diagnostic seems to be the use of consider, as the following examples would suggest.

\[
\begin{align*}
\text{All things considered} & \quad \{ \text{When considering } x₁, \ldots xₙ \} \\
\text{Orutt is} & \quad \{ \text{taller} \quad \text{cleverer} \} \quad \text{than Smith (is)}
\end{align*}
\]

where \( x₁ \ldots xₙ \) are some boolean combination of adjectival dimensions

The data in (4.23) shows that we can consider all or some dimensions when comparing one individual to another with respect to the gradable property made reference to by a particular multi-dimensional adjective. I stress boolean combination because Sassoon (2012) points out that some number of dimensions taken disjunctively (or conjunctively) affect our judgments of the truth of statements like those in (4.23).

Given that dimensions of evaluation can be fixed overtly or contextually, let us consider a thought experiment taken from Kamp (1975; p. 140) which shows that not all scalar dimensions are in fact transitive. “Suppose for example that [Orcutt], though less quick-witted than Jones, is much better at solving mathematical problems”.

(4.24)

\[
\begin{align*}
a. \quad \text{Wymann: Orcutt is cleverer than Smith (is); Smith is cleverer than Jones (is); therefore, Orcutt is cleverer than Jones (is)} \\
b. \quad \text{McX: No. Orcutt is not cleverer than Jones because . . .}
\end{align*}
\]

No doubt McX and Wymann are in the midst of a real debate here. Is McX right, and Orcutt wrong? Is Orcutt right, and McX wrong? Is there even a fact of the matter?

(4.25)

\[
\begin{align*}
a. \quad \text{Orcutt is cleverer than Smith (is)} \\
b. \quad \text{Smith is cleverer than Jones (is)} \\
c. \quad ? \Rightarrow \text{Orcutt is cleverer than Jones (is)}
\end{align*}
\]
4.3. SCALAR DIMENSIONS

Table 4.3: Orcutt’s, Smith’s and Jones’ relative cleverness

<table>
<thead>
<tr>
<th></th>
<th>Orcutt</th>
<th>Smith</th>
<th>Jones</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>QW</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

Here, ✓ is to be read *a is cleverer than b in regard to X*, and x is to be read *a is less clever than b in regard to X*, where X is either *mathematical problem solving abilities* (MP) or *quick-wittedness* (QW).

While it very well may be true that Orcutt is cleverer than Smith in regard to his quick-wittedness, and Smith is cleverer than Jones in the same regard, we cannot conclude anything about the relation between Orcutt and Jones stand in. Recall that clever encodes for multiple dimensions, so, at face value, it’s unclear what dimension we’re being asked to evaluate the inference in (4.25) on. Consequently, the validity of (4.25c) is unclear. Assume, as an example, that we are talking about cleverness in regard to the men’s mathematical problem solving abilities. Then (4.26) is apparently valid.

(4.26) a. Orcutt is cleverer than Smith (is) in regard to his mathematical . . .
   b. Smith is cleverer than Jones (is) in regard to his mathematical . . .
   c. ⇒ Orcutt is cleverer than Jones (is) in regard to his mathematical . . .

However, I think this example misses the heart of Kamp’s (1975) point. To round out the example, suppose Orcutt is cleverer than Smith in regard to his mathematical problem solving ability but less quick-witted. Finally, assume that Smith, though also less quick-witted than Jones, is more mathematically gifted than him. Such a situation is depicted in table (4.3).

As Kamp (1975; p. 140) points out, there are a variety of ways in which the above issue can be settled, depending, of course, on how we weight quick-wittedness versus mathematical problem solving abilities.
This is perhaps not clear, for we usually regard quick-wittedness and problem-solving facility as indications of cleverness, without a canon for weighing these criteria against each other when they suggest different answers. When faced with the need to decide the issue, various options may be open to us. We might decide that really only problem-solving counts, so that after all, [Orcutt] is cleverer than Jones; or we might decide on a particular method for weighing the two criteria - so that [Orcutt]’s vast superiority at solving problems will warrant that in spite of Jones’s slight edge in quick-wittedness [Orcutt] is cleverer than Jones; or we might decide that only quick-wittedness counts; and this time Jones will come out as the cleverer of the two.

I take Kamp (1975) to mean this: whether or not Orcutt is cleverer than Jones is determined by considering, in some way or another, all of the dimensions of evaluation of cleverness, not just one. This is made clearer by the following example in (4.27).

(4.27) All things considered, Orcutt is cleverer than Jones

I think it’s wrong to understand the phrase all things considered, as it is used in (4.27), as a sort of universal quantifier over adjectival dimensions. Rather, what I have in mind is something a little more nuanced and is made clearer by van Rooij (2011; p. 354).

It is then standardly assumed that each ‘dimension’ gives rise to a separate ordering and that whether somebody is more clever big than somebody else depends then, somehow, on these separate orderings.

The idea, here, is that, in some way or another, a strict order ‘$>^C$’, corresponding to, in this case, the meaning of cleverer can be determined from a set of independent strict orders $\mathcal{C} = \{>^C_1, \ldots, >^C_n\}$ for $n < \omega$ that correspond to the dimensions clever encodes for. Stated more informally in the context of this thought experiment, the meaning of cleverer, which understood as a strict order over individuals, is determined in some form or another, by a set of criteria, for example, being cleverer in regard to mathematical problem solving abilities or being cleverer in regard to quick-wittedness, themselves also understood as rank orders over individuals. That is to say the meaning of cleverer is a function of the meanings of the members $>^C_i \in \mathcal{C}$ for $i \leq n$.

---

6 Thanks to Justin D’Ambrosio for pointing out this example to me.
4.3. SCALAR DIMENSIONS

How, then, is function determined? What does it look like? With respect to the above example, when Kamp (1975) says, ‘we might decide that really only problem-solving counts’, what he has in mind is a sort of quasi-lexicographic order over dimensions of evaluation as defined in definition 21.

**Definition 21** (Lexicographic order). Let \( I \) be the set of *individuals* and \( \mathcal{C} = \{ >_1^A, \ldots, >_n^A \} \) for \( n < \omega \) be a set of rank orders over \( I \) intuitively understood as *criteria* or *dimensions* corresponding to some adjective \( A \). For \( x, y \in I \), define \( x >^A y \iff \exists m > 0 \forall i < m \left( (x >_m^A y) \land (x =_i y) \right) \)

Abstractly, the way to think about a lexicographic order is a sort of meta-order—an order over orders if you will. In Kamp’s (1975) example, being cleverer in regard to mathematical problem solving abilities is ranked higher than being cleverer in regard to quick-wittedness. So, while both Smith and Jones outrank Orcutt in regard to their quick-wittedness, because being cleverer in regard to mathematical problem solving abilities is ranked higher than the former, Orcutt is judged to be cleverer than both Smith and Jones (see van Rooij (2011) for more on this point).

However, it’s important to point out here, that a lexicographic order is only one way in which to derive a single strict order from a set of orders. There are a *multitude* of ways in which we can devise methods to determine who is cleverer—Orcutt or Jones (again, see van Rooij (2011) for several suggestions). From an order-theoretic perspective, we may as well take the disjoint union, linear sum, or even cartesian product of members of the set of criteria (see Davey and Priestley (2002) for more information on each). Some of these methods may, from the perspective of, say a philosopher or mathematician, seem principled and some may seem more-or-less arbitrary. That’s not the point. The point is simply that there exists a plethora of independent ways to derive a single strict order from an arbitrary finite number of them.

However, we have to be willing to accept the fact that, all things considered, we may not be able to make a determination in regard to who is cleverer—Orcutt or Smith, Smith or Jones, or Orcutt or Jones—because we decide that the properties being cleverer in regard to mathematical problem solving abilities or being cleverer in regard to quick-wittedness are simply incommensurable, or rather, incomparable. Kamp (1975; p. 141) considers this situation and concludes it just is true “neither that [Orcutt] is cleverer than Jones nor that Jones is cleverer than [Orcutt]”.
4.3. SCALAR DIMENSIONS

That, oftentimes, it just may very well be the case that two individuals cannot be ordered with respect to a particular gradable property or properties is referred to as incomensurability. Now, while this thought-experiment might feel manufactured, the debate surrounding the question, Who is cleverer – Orcutt or Jones? captures the spirit of the sorts of arguments real people have all of the time. To be more topical, suppose, now, that the United Nations (UN) has devised a metric for ordering the worlds countries based on wealth. Suppose, further, that the UN holds a press conference and a representative from the UN asserts (4.28).

(4.28) The United States is wealthier than China

Now suppose that a representative from an independent organization, call it XYZ, stands up at the conference and responds angrily by asserting (4.29).

(4.29) Based on our metric, we have determined that China is wealthier than the United States

The disagreement between our two representatives is not uncommon. How to appropriately quantify wealth is something economists regularly debate. Again, I ask you, who’s right: the UN or XYZ? Even more to the point, if you or I were watching such a press-conference from home, can we say that we’ve learned an extensional fact about the world, namely a fact about the relative wealth of the United States as compared to China?

To push this thought experiment a bit farther, both representatives from the UN and XYZ can lay out their criteria for all the world to see, explaining how each criteria themselves was determined, and why they settled on the set of criteria that they did. Moreover, each representative can explain how they utilized those criteria to determine whether the United States is wealthier than China or China is wealthier than China.

Of course, even in light of all of the facts, XYZ’s representative can take issue with the UN in (at least) the following ways.

• She can dispute the set of criteria the UN settled on, arguing that the UN failed to include, for example, a ranking of the world’s countries based on each country’s citizen’s average income; and

• She can dispute the way in which those criteria were utilized to determine the rank
order corresponding to wealthier. That is to say, she can dispute the UN’s δ function.

Similarly, the UN’s representative can take issue with XYZ for the same exact reasons. And there may never be a resolution to the matter; there’s no expectation that either UN or XYZ will revise their criteria or metric. So, from my perspective, what looked like a disagreement about the facts in (4.29) amounted to a meta-methodological disagreement about how such ‘facts’ were determined.

What the two above thought experiments demonstrate is that, oftentimes, it is not possible to determine whether one individual has more (or less) of a particular gradable property than another. Of course, an epistemicist like Williamson (1994), Kennedy (1997), Kennedy (2001) and Kennedy (2007b) would presumably argue that, in these instances, there is a fact of the matter that is hidden from us. That is to say, someone like Kennedy (1997), Kennedy (2001) and Kennedy (2007b) would argue that it must be the case, for example, that Orcutt is cleverer than Smith, or Smith is cleverer than Orcutt, or Orcutt and Smith are as clever as each other irrespective of whether we, as humans are in an epistemic position to determine which one is which. Maybe so. But if we as pragmatic-minded semanticists are interested in modeling the ways in which humans use words in practice, I think its bad business to model scalar dimensions as always being transitive.

In the formal analysis of scalar dimensions presented in this chapter, I do not claim as a general fact about scalar dimensions that they are transitive. (See, again, definition 18.) The previous discussion shows that teasing apart the mathematical properties of various scalar dimensions is to be preferred.

Moreover, I should point out that I allow for the value of δ to associate the values of more than one adjective with the same ordering of a particular scalar dimension. Take, for example, the scalar dimension of Temperature. It would be formalized as $\mathcal{T} = ([\text{warm}], [\text{hot}], [\text{cool}], [\text{cold}]), \{>, <, =\}$, where the value of δ associates both the values of warm and hot with $>$, and both the values of cool and cold with $<$.

This modeling decision captures another important fact about gradable adjectives. Consider the adjectives warm and hot. I doubt anyone would claim that these two adjectives are in fact synonymous, as the following (in)valid inferences would suggest.

(4.30) a. It’s warm out in San Francisco
   b. $\not\Rightarrow$ It’s hot out in San Francisco
4.3. SCALAR DIMENSIONS

Take, for example, a 70° degree day in San Francisco: I think native San Franciscans would generally agree that such a day is warm but not hot. (Although I do know some San Franciscans who would call this a hot day.) Notice, however, that in their comparative form, the two adjectives do in fact appear to be equivalent.

\begin{align}
(4.31) & \quad \text{a. San Francisco is warmer than Los Angeles (is)} \\
& \quad \text{b. \Leftrightarrow San Francisco is hotter than Los Angeles (is)}
\end{align}

The meanings of the comparative forms of two adjectives like warm and hot are more coarse-grained than their corresponding positive forms in that in their comparative form, the two meanings are indistinguishable. By relying on the value $\delta$ to associate the values of multiple gradable adjectives with their appropriate orders over a particular scalar dimension, I am able to remain agnostic about the meanings of those adjective’s positive forms, while still accounting for the apparent validity of inferences like those in (4.31).

I should point out, however, that the reason I choose to axiomatize each scalar dimension independently is because, as I will argue in the next chapter, not all gradable adjectives like clever should be thought of as being associated with a scale, i.e., a strict weak order. What this section shows is that, in some sense, there is very little we, as semanticists, can say about the properties scalar dimensions all have in common. However, the formal framework which I have chosen to adopt is robust and flexible enough to account for the many interesting facts individual scalar dimensions exhibit from the perspective of natural language inference.

4.3.3 Extending a scalar dimension

I conclude this section by investigating some nice formal properties and consequences of the theory of scalar dimensions I have laid out. I begin, first, by pointing out that as as it appears in as . . . as can now easily be defined as in (4.32).

\begin{align}
(4.32) & \quad \text{Lexical entry —} \\
& \quad \text{as : (A/PP) / A : } \lambda P \lambda Q \lambda x (Q \lambda y (\delta'(P)(x)(y)))
\end{align}
The second occurrence of *as* as it appears in the expression *as ... as* can be defined as in (4.33).

(4.33)

\[
\text{Lexical entry — } \\
\text{as : } p : \lambda Q (Q)
\]

In (4.32), the value of $\delta'$ is understood as a function akin to the value of $\delta$. Specifically, the value of $\delta'$ takes as an argument the value of an adjective of semantic type ($es$) and returns the relation $\geq_{\delta}$ of type ($ees$). Focusing on the adjective *tall*, the value of $\delta'$ takes the value of *tall* as an argument and returns the relation $\geq_{\delta}$ as defined in (4.34).

(4.34)

\[
(x \geq_{\delta} y @ w) \leftrightarrow (x >_{\delta} y @ w) \lor (x =_{\delta} y @ w)
\]

for all $x, y \in D_e$ and $w \in D_s$

The relation $\geq_{\delta}$ is read *having more height or having the same height*. In this way, I capture the equivalence between the statements in (4.35).

(4.35)  

a. Orcutt is as tall as Smith  
b. $\iff$ Orcutt is taller than Smith or Orcutt is the same height as Smith

Similarly, $\delta'$ will associate *short* with the relation $\leq_{\delta}$ defined in (4.36).

(4.36)

\[
(x \leq_{\delta} y @ w) \leftrightarrow (x <_{\delta} y @ w) \lor (x =_{\delta} y @ w)
\]

for all $x, y \in D_e$ and $w \in D_s$

The relation $\leq_{\delta}$ is read *having more height or having the same height*. Similarly, I capture the equivalence between the statements in (4.37).

(4.37)  

a. Orcutt is as short as Smith  
b. $\iff$ Orcutt is shorter than Smith or Orcutt is the same height as Smith

Moreover, by duality, I predict correctly the following equivalence to hold.
4.3. SCALAR DIMENSIONS

(4.38) a. Orcutt is as tall as Smith
    b. Smith is as short as Orcutt

All of this leads to a revision of the preliminary version of the properties all scalar dimensions have as proposed in definition 18. The revised version of a scalar dimension generally is given in definition 22.

**Definition 22** (Scalar dimension (Revised version 1)). Let a scalar dimension \( \mathcal{S} \) be the pair \( \mathcal{S} = (\mathcal{A}, \mathcal{S}) \). Here, \( \mathcal{A} \subseteq D_{(es)} \) is a system of sets referred to as \( \mathcal{S} \)'s adjectival component, whose elements correspond to adjectival meanings associated with \( \mathcal{S} \). \( \mathcal{S} \subseteq D_{(ees)} \) is also a system of sets referred to as \( \mathcal{S} \)'s scalar component and whose elements include at least the ternary relations \( >_{\mathcal{S}}, <_{\mathcal{S}}, =_{\mathcal{S}}, \geq_{\mathcal{S}} \) and \( \leq_{\mathcal{S}} \) such that for all \( x, y, z \in D_e \) and \( w \in D_s \):

1. \( \neg (x >_{\mathcal{S}} x \at w) \) \hspace{1cm} *Irreflexivity*
2. \( (x >_{\mathcal{S}} y \at w) \leftrightarrow (y <_{\mathcal{S}} x \at w) \) \hspace{1cm} *Duality*
3. \( x =_{\mathcal{S}} x \at w \) \hspace{1cm} *Reflexivity*
4. \( (x =_{\mathcal{S}} y \at w) \rightarrow (y =_{\mathcal{S}} x \at w) \) \hspace{1cm} *Symmetry*
5. \( (x =_{\mathcal{S}} y \at w) \land (y =_{\mathcal{S}} z \at w) \rightarrow (x =_{\mathcal{S}} z \at w) \) \hspace{1cm} *Transitivity*
6. \( (x >_{\mathcal{S}} y \at w) \land (x =_{\mathcal{S}} z \at w) \rightarrow (z >_{\mathcal{S}} y \at w) \)
7. \( (x \geq_{\mathcal{S}} y \at w) \leftrightarrow (x >_{\mathcal{S}} y \at w) \lor (x =_{\mathcal{S}} y \at w) \)
8. \( (x \leq_{\mathcal{S}} y \at w) \leftrightarrow (x <_{\mathcal{S}} y \at w) \lor (x =_{\mathcal{S}} y \at w) \)

In modeling a scalar dimension in this way, I make a claim: all scalar dimensions obey the axioms in (1)–(6) as laid out in definition 22. However, for any scalar dimension \( \mathcal{S} \) the individual properties of \( >_{\mathcal{S}} \), and consequently \( <_{\mathcal{S}} \), must be governed by dimension-specific axioms like HEIGHT is in definition 19.
4.3. SCALAR DIMENSIONS

4.3.4 The relation between the positive and comparative form of an adjective

I choose to axiomatize the behavior of a scalar dimension like HEIGHT via the adoption of axioms. Consequently, I stipulate the scalar behavior of scalar dimensions. Degree semanticists like Kennedy (1997) are no different. They too must stipulate, either through the adoption of axioms in the object language, or through meta-linguistic stipulation, the inherent structure of the set of degrees.

However, authors like Klein (1980), van Benthem (1982), van Rooij (2010b) and van Rooij (2011) have pursued a slightly different route. The idea for them is to derive the structure of a scalar dimension an adjective like tall encodes for from the meaning of that adjective’s positive form. As far as I understand it, this program stems from an argument made by Klein (1980; p. 4) where he claims that, under standard Fregean assumptions about compositionality, the meaning of the comparative form of an adjective must be determined from the meaning of its positive form.

Yet we also require a semantic theory for English to analyze the interpretation of complex expressions in terms of the interpretations of their components. An expression of the form \([AP \text{ A-er than } X]\) is clearly complex. How do its components contribute to the meaning of the whole?

Klein’s (1980) answer to this question, which is taken up and extended by van Rooij (2010b) and van Rooij (2011), is both elegant and mathematically sophisticated. However, to review it in detail would take me too far afield. There are two points I want to raise here. In order for authors working in the Kleinian tradition to derive the structure of a scalar dimension like HEIGHT, they too must stipulate certain things through the adoption of axioms, namely the semantic behavior of the value of a gradable adjective’s positive form. From a modeling perspective, as far as I can tell, every author working on the semantics of adjecival comparatives will have to stipulate something at some level to govern the behavior of a scalar dimension.

The second point I want to make is that my analysis also derives the meaning of a comparative adjective from the meaning of that adjective’s positive form, albeit somewhat indirectly. I argued at length that a gradable adjective lexically encodes for (at least one) scalar dimension. My inclusion of \(\delta\) in the semantic representation of the comparative
4.4. BOOREANS

<table>
<thead>
<tr>
<th></th>
<th>Conjunctive</th>
<th>Disjunctive</th>
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<tbody>
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<td>or</td>
<td>and</td>
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<td>x ✓</td>
<td>✓ x</td>
</tr>
<tr>
<td>Greater-than analysis</td>
<td>✓ x</td>
<td>x ✓</td>
</tr>
<tr>
<td>Heim’s (2006) revised</td>
<td>✓ ✓ ✓ ✓</td>
<td>✓ ✓ ✓ ✓</td>
</tr>
</tbody>
</table>

Table 4.4: The various entailment patterns the various analyses, including my own, are able to capture

morpheme means that the scalar dimension in which the truth of an adjectival comparative is determined is a function—quite literally—of the value of that adjective’s positive form.

As is clear from table 4.2, I assume that the semantic representation of the positive form of a comparative adjective like *tall* is to be understood as TT₂ type *(es)*, whose value is a set of individuals relative to an index w. I should say, however, that, in this dissertation, I am actually quite indifferent as to how exactly the semantics of the positive form of a gradable adjective should in fact be analyzed. This field of inquiry is quite complicated and involves the issue of vagueness (Fine 1975; Kamp 1975; Williamson 1994; Kennedy 2007b), which also extends beyond the scope of this dissertation. However, nothing about my analysis rests on the fact that adjectives are to be understood as type *(es)*. If, for example, we assume that gradable adjectives, as they are instantiated in the lexicon, should in fact be understood as higher-order modifiers of type *(es) es*, all of the definitions I propose, here, could be augmented accordingly without changing anything fundamental to my proposal.
4.4 Booleans

In this section, I will extend my analysis to the cases of the Boolean connectives *or* and *and* as they appear under the scope of *than*. Specifically, I will show that my analysis captures

- the weakened, disjunctive reading of *or*; and
- the normal conjunctive interpretation of *and*.

This is summarized clearly in table 4.4. Moreover, I will investigate a variety of nice consequences of my system: I will show how I can capture general facts about coordination and comparative adjectives—facts I have not seen researchers discuss anywhere at length.

4.4.1 Boolean *or*

I turn, now, to the semantics of *or* under the scope of *than*. Given that a natural language like English allows for coordinated syntactic categories of a variety of types, we would like our type-logical grammar to reflect this. We have several options afforded us. Instead of stipulating a variety of lexical entries for a coordinator like *or*, differing in their syntactic category and semantic type, following Jäger (2005) we can simply introduce *or* syncategorematically by adopting the polymorphic inference schema given in definition 23.\(^7\)

---

\(^7\)I make no conceptual claims in regard to whether boolean connectives like *or* should be introduced syncategormatically or lexically. It may well be desirable to introduce an expression like this lexically and derive the fact that it can conjoin expressions of a variety of syntactic categories proof-theoretically. (This amounts to type-shifting in systems with combinators.) However, such derivations would ultimately be more involved and obfuscate my formal point. I rely on the fact that both can be done in practice, and stick with syntcategormatic definitions.
4.4. **BOOLEANS**

**Definition 23** (Disjunction).

\[
X : \varphi : A \quad Y : \psi : A \\
\frac{}{X \cdot \text{or} \cdot Y : \varphi \lor \psi : A}
\]

where

\[
\varphi \lor \psi = \begin{cases} 
\varphi \lor \psi & \text{if } \varphi, \psi \text{ are type } () \\
\lambda x_1 \ldots \lambda x_n \lambda w (\varphi (x_1) \ldots (x_n)(w) \lor \psi (x_1) \ldots (x_n)(w)) & \text{if } \varphi, \psi \text{ are type } (\alpha_1 \ldots \alpha_n)
\end{cases}
\]

for all types \((\alpha_1 \ldots \alpha_n)\) where \(n \geq 1\)

The schema says that two expressions can be coordinated with `or` just in case they have the same syntactic category \(A\). Given the boolean character of our \(T T_2\) frames \(\lor\) is interpreted as \(\cup\) in general.

It is now easy to account for the normal disjunctive reading of disjunction in comparative clauses shown originally in (2.11), repeated again in (4.39) below.\(^8\)

(4.39) \hspace{1cm} a. Orcutt is taller than Smith or Jones

b. \(\Leftrightarrow\) Orcutt is taller than Smith is or Orcutt is taller than Jones is

The semantic representation of (4.39a) is given in (4.40a); the relevant portion of its Gentzen/Prawitz derivation is given in (4.40b); and its linguist’s tree is shown in (4.40c).

---

\(^8\)But see, again, §2.8 for a pragmatic argument as to why a semantic theory should only capture this reading and not the strengthened, conjunctive interpretation in (2.5), repeated again below

(i) \hspace{1cm} a. Orcutt is taller than Smith or Jones

b. \(\Leftrightarrow\) Orcutt is taller than Smith is and John is taller than Jones is

Without further assumptions, I have formal mechanisms to derive the above interpretation. However, I assume that my semantic theory can be augmented with the pragmatic one laid out in van Rooij (2010a).
4.4. **BOOLEANS**

(4.40)  a. Orcutt is taller than Smith or Jones \(\sim\)

\[
[\lambda w (\delta (\text{tall})(o)(s)(w) \lor \delta (\text{tall})(o)(j)(w))] =
\{w \in D_s \mid o >_{S} s \land w \lor o >_{S} j \land w\}
\]

b. \(\begin{array}{c}
\frac{\text{Smith}}{\overline{\lambda P (P(s))}} \text{Lex} \\
\frac{\text{Jones}}{\overline{\lambda P (P(j))}} \text{Lex}
\end{array}\)

\begin{array}{c}
\frac{\text{than}}{\overline{\lambda Q (Q)}} \text{Lex} \\
\frac{\text{NP}}{\overline{\lambda P \lambda w (P(s)(w) \lor P(j)(w))}} \lor \\
\frac{\text{NP}}{\overline{\lambda P \lambda w (P(s)(w) \lor P(j)(w))}} / E
\end{array}

The set denoted by this representation is the set of indices in which Orcutt has more height than Smith or Orcutt has more height than Jones. Situations such indices would represent are reflected in figure 4.7.

Moreover, given the generality of the schema laid out in definition 23, I correctly predict the semantic representation in (4.41).
4.4. BOOLEANS

(4.41)

Orcutt or Smith is taller than Jones

\[
\lambda w (\delta(tall)(o)(j)(w) \lor \delta(tall)(s)(j)(w)) = \{w \in D_s \mid o >_j w \lor s >_j w\}
\]

The value of its semantic representation is the set of all indices in which Orcutt has more height than Jones or Smith has more height than Jones. The example is the mirror image of the one in (4.39). This is because, like Larson (1988), under my analysis, the than phrase of a comparative constructions is a generalized quantifier, and, in coordinate constructions, behaves analogously to subjects both syntactically and semantically.

4.4.2 Boolean and

My analysis of conjunction in comparative clauses is analogous to that of disjunction. It is accomplished by adding an additional proof rule just like definition 23 substituting \( \land \) for \( \lor \).
4.4. **BOOLEANS**

**Definition 24** (Conjunction).

\[
\begin{align*}
X : \varphi : A & \quad Y : \psi : A \\
\quad & \quad \frac{\quad}{X \cdot \text{and} \cdot Y : \varphi \land \psi : A \quad \land}
\end{align*}
\]

where

\[
\varphi \land \psi = \begin{cases} 
\varphi \land \psi & \text{if } \varphi, \psi \text{ are type } () \\
\lambda x_1 \ldots \lambda x_n \lambda w (\varphi(x_1) \ldots (x_n)(w) \land \psi(x_1) \ldots (x_n)(w)) & \text{if } \varphi, \psi \text{ are type } (\alpha_1 \ldots \alpha_n)
\end{cases}
\]

for all types \((\alpha_1 \ldots \alpha_n)\) where \(n \geq 1\)

Now I can account for the normal conjunctive interpretation of conjunction first shown in (2.24), repeated below in (4.42).

(4.42)  
\begin{align*}
a. & \quad \text{Orcutt is taller than Smith and Jones} \\
b. & \quad \text{⇔ Orcutt is taller than Smith is and John is taller than Jones is}
\end{align*}

My semantic representation for (4.42) is shown in (4.43a). For completion’s sake, but at the risk of redundancy, the relevant portion of its Gentzen/Prawitz derivation is given in (4.43b); and its linguist’s tree is given in (4.43c).
4.4. BOOLEANS

(4.43)  

a. Orcutt is taller than Smith and Jones $\rightarrow$

\[ \left[ \lambda w (\delta (\text{tall})(o)(s)(w) \land \delta (\text{tall})(o)(j)(w)) \right] = \{w \mid o >_S s @ w \land o >_S s @ w \} \]

b. $\lambda Q(Q) \over \lambda P(P(s)) \over \lambda P\lambda w (P(s)(w) \land P(j)(w)) \over \lambda P\lambda w (P(s)(w) \land P(j)(w)) / E$

c. The set denoted by this representation is the set of indices in which Orcutt has more height than Smith and Orcutt has more height than Jones. Such a situation is shown in figure 4.8.

4.4.3 Comparative adjectives and transitive verbs

I now investigate some nice formal consequences of my analysis that bring together the tight-knit relationship between the verb phrase be + a comparative adjective and transitive
verbs. To begin with, there is a syntactic phenomenon involving coordinated transitive verbs referred to as right and left node raising. A clear example of the former is given in (4.44).

(4.44)  

   a. Orcutt likes broccoli and Smith detests broccoli  
   b. \( \Leftrightarrow \) Orcutt likes broccoli and Smith detests broccoli

\textit{Right node raising (Jäger 2005; (14))}

In (4.44a), there appears to be deleted material in the first conjunct, namely the object of the second conjunct broccoli. That one would suspect there exists deleted material, here, is because (4.44a) and (4.44b) mutually entail each other.\(^9\) Despite the fact that broccoli does not appear overtly in (4.44a), the statement means the same thing as (4.44b). (I struck through broccoli in (4.44a) merely to emphasize this point and am not suggesting that this ‘deleted’ material exists in some quasi-realist sense.)

Similarly, (4.45) is an example of left node raising. In (4.45a), there appears to be deleted material in the second conjunct, namely the subject and verb of the first conjunct Orcutt introduced.

(4.45)  

   a. Orcutt introduced Smith to Jones and Orcutt introduced Wymann to McX  
   b. \( \Leftrightarrow \) Orcutt introduced Smith to Jones and Orcutt introduced Wymann to McX

\textit{Left node raising (Jäger 2005; (16))}

\(^9\)It is important to point out, however, that whether there actually exists deleted material as a syntactic fact is a theory internal question that depends on the sort of elliptical mechanisms assumed by that theory, if they are assumed at all. For the purposes of this section, I will speak as if the deleted material exists, understanding that, from a type-logical perspective it does not. See Jäger (2005) for more on this point.
In both (4.44) and (4.45), the deleted materially crucially depends on (non-deleted) material from the adjacent conjunct. With this in mind, observe that transitive verbs are interesting, because they allow for both right and left node raising simultaneously, as evidenced by (4.46).

(4.46)  
\begin{align*}
\text{a. Orcutt hates Smith or Orcutt loves Smith} \\
\text{b. } \iff \text{Orcutt hates Smith or Orcutt loves Smith}
\end{align*}

*Simultaneous right and left node raising*

What makes (4.46) tricky is not its syntax per se: it’s simply an instance of V-level coordination. Rather, it is compositionally accounting for the semantic reflexes of the fact that Smith is functioning as an object in the first conjunct and Orcutt is functioning as the subject of the second conjunct. Adjectival comparatives behave similarly, as the valid inference pattern in (4.46) would suggest.

(4.47)  
\begin{align*}
\text{a. Orcutt is taller than Smith or Orcutt is shorter than Smith} \\
\text{b. } \iff \text{Orcutt is taller than Smith or Orcutt is shorter than Smith}
\end{align*}

*Simultaneous right and left node raising*

Extending my fragment with the boolean connectives as I have done in definitions 23 and 24 gives me everything I need to account for simultaneous left and right node raising as it occurs in adjectival comparatives. Following Jäger (2005), such statements can naturally be derived in the type-logical setting without any reliance on, say, ellipsis. In (4.48a), I provide the semantic representation of (4.46) whose linguist’s tree can be seen in (4.48b), and a partial derivation of which is given in figure 4.9.
4.4. BOOLEANS

(4.48) a.

Orcutt is taller or shorter than Smith \( \rightarrow \)

\[ \{ w \mid o >_S s \land w \lor o <_S s \land w \} \]

b.

The semantic representation in (4.48a) denotes the set of indices in which Orcutt has more height than Smith or Orcutt has less height than Smith.

Building on these insights, let us consider a more complex construction. Imagine, now, that Orcutt, a clever thief, has recently become aware that he is being monitored by the (less clever) federal agent Smith. McX’s utterance in (4.49) is perfectly sensible.

(4.49) a. **Wymann:** Does Orcutt fear Smith\(_1\)?

b. **McX:** Orcutt fears Smith but Orcutt is nonetheless cleverer than him\(_1\)

In (4.49b), not only do we witness simultaneous left and right node raising, but also that the transitive verb *fears* and quasi-transitive verb phrase *is cleverer* are coordinated via *but*.

In order to capture this fact, I will assume, for the sake of simplicity, that the lexical entry for *fear* is as in (4.50).
Figure 4.9: A partial derivation of node raising in a phrasal comparative
Lexical entry —

\[ \text{fear} : \text{IV/\text{NP}} : \lambda Q \lambda x (Q \lambda y (\text{fear} (x) (y))) \]

It is not difficult to show that the derived expression *is cleverer* is represented as in (4.51).

\[ \text{clever} : \text{A/\text{PP}} : \lambda Q \lambda x (Q \lambda y (\text{clever} (x) (y))) \]

Similar to the syntactico-semantic relationship between *find* and *taller* discussed in §4.2 of this chapter, the syntactic categories of *fear* and *clever* also differ, even though both categories are mapped to the same semantic type via the translation function \( \tau \). Consequently, the two expressions cannot be coordinated by a rule like definition 23 or its conjunctive analog. From a semantic perspective, given that boolean coordination is defined over all type domains, in principle, there should be nothing preventing us from coordinating the semantic representation of both expressions via conjunction \( \land \).

As it turns out, I can account for the above coordination fact rather easily, with a little trick from the type-logical tradition. (In Montagovian terms, this trick is effectively quantifying in.) The semantic representation of (4.49b) is given in (4.52a). The linguist’s tree is in (4.52b), and the relevant portion of this representation’s derivation is shown in figure 4.10.
(4.52)  a.

Orcutt fears but is cleverer than Smith \( \sim \)

\[
\left[ \lambda w \,(\text{fear}(o)\,(s)\,(w)) \land \delta (\text{clever})(o)\,(s)\,(w)) \right] = \\
\{ w \in D_s \mid o \, \text{fear} \, s \, @ \, w \, \land \, o > \epsilon \, s \, @ \, w \}
\]

b.

Notice that its derivation in figure 4.10 relies on the immediate application of semantic representation of \textit{than} to the ‘dummy’ variable \( Q \), which, again is the semantic representation of the phonetically null expression \( \varnothing \), having syntactic category \( \text{NP} \). The variable \( Q \) is subsequently \( \lambda \)-abstracted over per inference rule \( /I \) of definition 16. From a proof-theoretic perspective, \( \varnothing : \text{NP} : Q \) is a hypothesis that is withdrawn during the course of the deduction via the application of the inference rule \( /I \). From a syntactic perspective, such a withdrawal means that the introduction of this null expression \( \varnothing \) has no long-term syntactic effects: it is not present in the final string derived by the calculus. To be clearer, my analysis generates the string in (4.53a), not the one in (4.53b).
Figure 4.10: A partial derivation of a transitive verb and a verb phrase containing a comparative adjective
4.4. **BOOLEANS**

(4.53)  

a. Orcutt fears and is cleverer than Smith  
b. Orcutt fears and is cleverer than $\emptyset$ Smith

The process described above has the syntactic consequence of causing the transitive verb and comparative adjective to be of the same syntactic category $\text{IV/}\text{NP}$. Consequently, the two expressions can be coordinated per definition 24. This is a common technique in the type-logical setting and one I will use readily and often.

Notice that I make a syntactic prediction. My analysis requires the comparative morpheme -$\text{er}$ to be applied to the dummy, or rather unsaturated PP (to borrow a phrase from Frege). This is to force the syntactic categories of the expressions $\text{fear}$ and $\text{is taller than}$ to be of the same syntactic category. Consequently, I predict the comparative morpheme -$\text{er}$ and $\text{than}$ to co-occur in coordinated environments, which is borne out by fact.

(4.54)  

a. Orcutt fears but is nonetheless cleverer than Smith  
b. # Orcutt fears but is nonetheless cleverer Smith  
c. Orcutt is cleverer than but nonetheless fears Smith  
d. # Orcutt is cleverer but nonetheless fears Smith

Importantly, the derivation of (4.54c) can be gotten analogous to (4.54a), except reversing the order of the coordinates.

**Constraining the syntax**

The type-logical trick I deployed to account for the aforementioned coordination facts should give us pause. Unconstrained, it seemingly massively over-generates. What is to prevent my grammar from generating ungrammatical strings like the one below?

(4.55)  

#Orcutt fears but is nonetheless cleverer $\emptyset$

To be clearer, the pertinent question becomes: what forces our grammar to withdraw the hypothesis $\emptyset : $NP : $Q$ through the course of the derivation thus preventing the generation of strings like (4.55)? As it stands, nothing, which is very problematic. However, we can resolve this problem, at least conceptually, by analogizing our linguistic theory with the proof-theoretic tradition. Given a particular proof or derivation, the idea is to draw a distinction between original or starting hypotheses (or premises) and hypotheses assumed
4.5. NEGATION

through the course of the derivation, which themselves must be subsequently discharged in order to obtain a valid proof.

To better understand what I mean, it is important to point out that, from a type-logical perspective, the set of lexical entries taken in its entirety (and all the subsets thereof) constitutes the set of original or starting hypothesis in which all grammatical strings of English must subsequently be derived via only the proof rules laid out in §3.3, and of course the additional ones assumed through the course of this chapter. For present purposes, table 4.2 represents all and only the elements of this set of original hypothesis. So, for example, if we want to account for the fact that (4.49b), repeated below in (4.56), is a grammatical string of English, we must do it by assuming only the lexical entries in table 4.2 and the ways in which they can combine via the aforementioned proof rules.

(4.56) Orcutt fears but is nonetheless cleverer than Smith

Notice that the dummy lexical entry ‘∅ : NP : Q’ used in the derivation shown in (4.52b) and figure 4.10 is not an element of the original hypotheses set: this lexical item is assumed through the course of the derivation and subsequently discharged. If, following the spirit of the proof-theoretic tradition, we impose the meta-logical constraint on our theory of English syntax that the only hypotheses that do not have to be discharged through the course of the derivation are the originally assumed ones, then we constrain our grammar appropriately. That is to say, we block the generation of statements like (4.55) in virtue of the fact that the dummy lexical entry ‘∅ : NP : Q’ is not an original hypothesis nor has it been discharged.

4.5 Negation

In order to differentiate my analysis from Larson’s (1988), it is necessary to show that there exist a class of empirical phenomena that distinguishes the two analyses. In this section, I will extend my analysis to the cases of sentential and differential negation. Specifically, I will show that my analysis captures

• both the narrow- and wide-scope readings of sentential negation; and

• the narrow-scope reading of differential negation.
The fact that I can capture the different readings of sentential negation is important, as it distinguishes, from an empirical perspective, my analysis from Larson’s (1988). Recall that, in §2.9.3, I showed that, given the fact that negation (¬) is posited covertly at the level of logical form, Larson (1988) has no obvious way of capturing the narrow-scope interpretation of sentential negation, and consequently, no way of capturing the semantics of differential negation. Here, I will show how to do both. I will do this by

- handling sentential negation by positing a type-polymorphic schema, analyzing its narrow-scope reading by deploying the type-logical trick utilized in §4.4.3; and

- positing a lexical entry for differential negation, which forces it to take obligatory narrow-scope.

As I will show, my analysis (correctly) predicts that the meaning of sentential negation under its narrow-scope negation converges with that of differential negation in general.

Moreover, just as in the previous section, I will investigate a variety of consequences of my system: I will show how my theory of negation naturally captures the relationship between comparative constructions and those constructions’ polar opposites.

4.5.1 Sentential negation

I begin, first, by analyzing sentential negation, as it appears in (2.114), repeated below in (4.57).

(4.57) Orcutt is not taller than Smith

\textit{Sentential negation}

As in the case of §4.4.1 and §4.4.2, I introduce a single syncategormatic rule for negation, as given by definition 25.
Definition 25 (Negation).

\[
\frac{X : \varphi : A}{\text{not} \cdot X : \neg \varphi : A}
\]

where

\[
\neg \varphi = \begin{cases} 
\neg \varphi & \text{if } \varphi \text{ is type } () \\
\lambda x_1, \ldots, \lambda x_n \lambda w (\neg (\varphi (x_1) \ldots (x_n)) (w)) & \text{if } \varphi \text{ is type } (\alpha_1 \ldots \alpha_n)
\end{cases}
\]

for all types \((\alpha_1 \ldots \alpha_n)\) where \(n \geq 1\)

I introduce this rule as a type-polymorphic schema, because natural language negation, at least in English, can target a variety of syntactic expressions of a variety of different types. Moreover, given the Boolean nature of the semantic models I am working with, negation can be interpreted as set-theoretic complementation \((\neg)\) in general. Now, I claim that the addition of the proof rule above to my pre-existing proof-theory gives me everything I need to account for the variability of scope as exhibited by sentential negation. I begin, first, by considering the way in which I can capture the narrow-scope reading.

**Sentential negation: narrow-scope interpretation**

To see how I capture the narrow-scope interpretation of negation, a semantic representation of (4.57) is given in (4.58a); its linguist’s tree is in (4.58b); and a Gentzen/Prawtiz style natural deduction is in figure 4.11.
(4.58) a.

Orcutt is not taller than Smith \( \sim \)

\[ [\lambda w \neg (\delta \text{tall})(o)(s)(w))] = \]

\[ \{ w \in D_s \mid \neg (o > s_\delta s @ w) \} \]

b.

Sentential negation: narrow-scope derivation

The semantic representation in (4.58a) denotes the set of indices in which it is not the case that Orcutt has more height than Smith. Some situations representative of such indices are shown in figure 4.11.

A careful following of the proof in figure 4.12 will reveal that the application of the proof rule \( \neg \) is illicit. Technically, the proof should proceed as follows.
4.5. NEGATION

Figure 4.11: Possible situations consistent with (4.58a)

\[
(4.59)
\]

\[
\begin{align*}
\lambda x (Q \lambda y (\delta (\text{tall}) (x) (y))) \\
\lambda x \lambda w (\neg Q \lambda y (\delta (\text{tall}) (x) (y) (w))) \\
\lambda x \lambda y (\delta (\text{tall}) (x) (y)) \\
\lambda x \lambda y (\delta (\text{tall}) (x) (y)) \\
\end{align*}
\]

However, as will become clear in §4.66 and §4.71, in order to capture the semantic difference between the narrow- and wide-scope readings of sentential negation as they relate to Boolean or, I need logical negation (\(\neg\)) in figure 4.12 to target the atomic statement \(\delta (\text{tall}) (x) (y) (w)\) directly. In order to account for narrow-scope readings, I need negation to scope under the generalized quantifier denoted by the than phrase.

As it stands, I have no way of doing this. Definition 25 forces negation (\(\neg\)) to target the atomic statement \(Q \lambda y (\delta (\text{tall}) (x) (y))\), which means that negation will always scope over the generalized quantifier denoted by the than phrase. This formal problem can be surmounted, however, by assuming a more simple semantic representation for the comparative morpheme -er. Such a representation is shown in (4.60).

\[
(4.60) \quad \text{er} \sim \lambda P \lambda x \lambda y (\delta (P) (x) (y))
\]

Given such a representation, the licit application of the proof rule \(\neg\) would be as follows, delivering precisely what I need.
Figure 4.12: A derivation of the narrow-scope reading of sentential negation
4.5. NE\textsc{GATION}

(4.61)

\[ \begin{array}{c}
\lambda x \lambda y (\delta (\text{tall})(x)(y)) \\
\Downarrow \\
\lambda x \lambda y \lambda w (\neg \delta (\text{tall})(x)(y)(w)) \\
\end{array} \]

However, in adopting the representation in (4.60), the ramifications are quite severe in the sense that many of the lexical entries assumed, e.g., the lexical entries for the various proper names like Orcutt, must be altered, such that their semantic representations take the form of individual constants of type $e$. But then, I no longer treat the semantic representations of noun phrases uniformly; and so on and so forth. The point is, there are formal trade-offs no matter which approach I take.\footnote{Readers familiar with the Montagovian tradition will understand the trade-offs in attempting to give the semantics of all noun-phrases uniformly as type $((e,t), t)$, while still accounting for the semantics of transitive verbs.}

Observe, though, that the semantic representation in (4.2)—the one I actually assume—can be derived from the more simple one in (4.60) using the proof rules laid out in chapter 3. (This amounts to type-shifting in type-logical systems that adopt combinators.) This is shown explicitly in (4.62). Here, I omit syntactic categories for perspicuity’s sake.

(4.62)

\[ \begin{array}{c}
\emptyset \frac{P}{Lex^4} \\
\emptyset \frac{er}{Lex^3} \\
\emptyset \frac{\lambda P \lambda x \lambda y (\delta (P)(x)(y))}{Lex} \\
\emptyset \frac{\lambda x \lambda y (\delta (P)(x)(y))}{x/E} \\
\emptyset \frac{\lambda y (\delta (P)(x)(y))}{y/E} \\
\emptyset \frac{Q \lambda y (\delta (P)(x)(y))}{I^1} \\
\emptyset \frac{\lambda x (Q \lambda y (\delta (P)(x)(y)))}{I^2} \\
\emptyset \frac{\lambda Q \lambda x (Q \lambda y (\delta (P)(x)(y)))}{I^3} \\
\emptyset \frac{\lambda P \lambda Q \lambda x (Q \lambda y (\delta (P)(x)(y)))}{I^4} \\
\end{array} \]

Consequently, for the remainder of this section, I will perform illicit applications of the proof-rule $\neg$ with the knowledge that I may as well have assumed the representation in (4.60) and subsequently derived the more complex one in (4.2) proof-theoretically.
Sentential negation: wide-scope interpretation

To see how I capture the wide-scope interpretation of negation, the semantic representation of (4.57) under this reading is given in (4.63a); its linguist’s tree in (4.63b); and a Gentzen/Prawitz-style derivation is shown in figure 4.13.

(4.63)  a.

Orcutt is not taller than Smith \(\sim\)

\[\lambda w \neg (\delta (\text{tall}) (o, s) (w)) = \{w \in D_s | \neg (o >_{w} s \circ w)\}\]

b. 

![Linguist’s tree](image)

Sentential negation: wide-scope derivation

Observe that the semantic representations in (4.58a) and (4.63a) are logically equivalent. This is as it should be: without more articulate syntactic structure under the scope
Figure 4.13: A derivation of the wide-scope reading of sentential negation
4.5. NEGATION

of than, it is impossible at this point to tease apart the meaning difference between the narrow- and wide-scope readings of sentential negation. What is important, for the present purposes, is the way in which the derivations differ reflected clearly in (4.58b) and (4.63b). Observe that in the case of the latter, that is to say, the narrow-scope interpretation of sentential negation, I deploy the type-logical trick explained in detail in §4.4.3 of this chapter.

The difference in these derivations will prove essential in capturing the meaning difference between narrow- and wide-scope negation as they interact with the Booleans or and and, as well as the quantifiers anyone, everyone, and someone, as they appear under the scope of than.

Negation and polar opposites

Before turning to the interaction between negation various complex constructions under the scope of than, I investigate, now, some pleasant properties of the formal system I have laid out as they relate to negation and polar opposites generally. (See Bierwisch (1989) for a similar discussion.) Introducing negation into the object language has immediate albeit trivial consequences with respect to polar opposites like taller and shorter. Observe that, by the axioms laid out in definition 19 and lemma 1, the following chain of equivalences holds in general.

\[ (4.64) \]

\[ \left[ \lambda w \neg (\delta (\text{tall})(o)(s)(w)) \right] \Leftrightarrow \{ w \in D_s \mid \neg (o >_s s @ w) \} \]

\[ \Leftrightarrow \{ w \in D_s \mid s \leq_o s @ w \} \]

\[ \Leftrightarrow \left[ \lambda w (\delta'(\text{short})(o)(s)(w)) \right] \]

In words, I predict that the equivalence in (4.65) to hold in general.

\[ (4.65) \]

a. Orcutt is not taller than Smith

b. \( \Leftrightarrow \) Orcutt is as short as Smith

I also predict the following equivalence.
(4.66)  a. Orcutt is not shorter than Smith
       b. ⇔ Orcutt is as tall as Smith

This is again is welcome result, because, as far as I can tell at least, both (4.65) and (4.66)
are valid in general. Consequently, these results show us that we may substitute ‘x ≤_{y @ w}’
y for ‘¬(x >_{y @ w})’ and ‘x ≥_{y @ w}’ for ‘¬(x <_{y @ w})’ for all x, y ∈ De and
w ∈ Ds in our semantic representation language while still preserving validity on the natural
language side of things.

**Sentential negation and Boolean or: narrow-scope reading**

I now give the two interpretations of sentential negation involving Boolean or. I begin,
first, with the narrow-scope interpretation shown originally in (2.118) and repeated again
in (4.67).

(4.67)  a. Orcutt is not taller than Smith or Jones
       b. ⇔ Orcutt is not taller than Smith or Orcutt is not taller than Jones

**Sentential negation and Boolean ‘or’: narrow-scope interpretation**

The semantic representation and linguist’s tree of (4.67) shown in (4.68). Its complete
derivation is shown in figure 4.14.
Figure 4.14: A derivation of the narrow-scope interpretation of sentential negation and Boolean or
(4.68) a. Orcutt is not taller than Smith or Jones \(\sim\)

\[
\lambda w \neg \delta (\text{tall}) (o) (s) (w) \lor \neg \delta (\text{tall}) (o) (j) (w)
\]

\[
\{ w \in D_s \mid \neg o > s @ w \lor \neg o > j @ w \}
\]

b. Sentential negation and Boolean ‘or’: narrow-scope derivation

Notice that or out-scopes negation, and consequently, the two expressions’ semantic translations do not interact with each other in any meaningful way at the level of logical form. The semantic representation in (4.68a) denotes the set of indices in which it is not the case that Orcutt has more height than Smith or it is not the case that Orcutt has more height than Jones. (There are too many situations consistent with (4.68a) to picture graphically in a meaningful way.)

Under the narrow-scope reading of sentential negation, I predict the validity involving polar opposites shown below.
4.5. NEGATION

(4.69)  
a. Orcutt is not taller than Smith or Jones  
b. $\Leftrightarrow$ Orcutt is as short as Smith or Orcutt is as short as Jones

Sentential negation and polar opposites: narrow-scope interpretation

This is justified by the following equational reasoning.

\[
(4.70)
\]

\[
\lambda w \delta (\text{tall})(o)(s)(w) \lor \neg \delta (\text{tall})(o)(j)(w) = \{ w \in D | (o > s @ w) \lor (o > j @ w) \}
\]

\[
= \{ w \in D | o \leq s @ w \lor o \leq j @ w \}
\]

\[
= \lambda w (\delta'(\text{short})(o)(s)(w) \lor \delta'(\text{short})(o)(j)(w))
\]

And, so by duality (see, again, lemma 1), I also predict the next validity.

(4.71)  
a. Orcutt is not shorter than Smith or Jones  
b. $\Leftrightarrow$ Orcutt is as tall as Smith or Orcutt is as tall as Jones

Sentential negation and polar opposites: narrow-scope interpretation

This is a nice consequence of the system I have proposed here, as it accords with pre-theoretical intuitions regarding the meaning of these statements.

Sentential negation and Boolean or: wide-scope reading

Turning attention, now, to the wide-scope interpretation of sentential negation, it is now easy to derive the De Morgan’s effects first discussed in (2.119), repeated again in (4.72).

(4.72)  
a. Orcutt is not taller than Smith or Jones  
b. $\Leftrightarrow$ Orcutt is not taller than Smith and Orcutt is not taller than Jones

Sentential negation and Boolean ‘or’: wide-scope interpretation

A sample derivation of the reading in (4.72) is given in figure 4.15. Its semantic representation is in (4.73a), and its linguist’s tree is in (4.73b). It is clear in (4.73a) that, by De Morgan’s laws, negation has been pushed through, flipping disjunction to conjunction.
Figure 4.15: A complete derivation of the wide-scope interpretation of sentential negation and Boolean or
The semantic representation in (4.73a) denotes the set of indices in which it is not the case that Orcutt has more height than Smith and it is not the case that Orcutt has more height than Jones. Situations such indices characterize are shown in figure 4.16.

Moreover, under the normal, disjunctive reading of or, I predict the validity of (4.74).

(4.74)  a. Orcutt is not taller than Smith or Jones
b. $\Leftrightarrow$ Orcutt is as short as Smith and Orcutt is as short as Jones

Sentential negation and polar opposites: wide-scope interpretation
No doubt (4.74b) is a reading of (4.74a), so this is a welcome result. Again, my prediction follows from the interaction between negation and polar opposites shown below. This time, however, De Morgan’s laws play a role in the equational reasoning.

\[(4.75)\]

\[
\lambda w \neg (\delta (\text{tall}) (o) (s) (w) \lor \delta (\text{tall}) (o) (j) (w)) = \{ w \in D_o \mid \neg (o > H_s @ w) \land \neg (o > H_j @ w) \}
\]

\[
= \{ w \in D_o \mid o \leq H_s @ w \land o \leq H_j @ w \}
\]

\[
= \lambda w (\delta' (\text{short}) (o) (s) (w) \land \delta' (\text{short}) (o) (j) (w))
\]

I also predict the validity in (4.76).

\[(4.76)\]

a. Orcutt is not shorter than Smith or Jones

b. \(\iff\) Orcutt is as tall as Smith and Orcutt is as tall as Jones

*Sentential negation and polar opposites: wide-scope interpretation*
It should be clear, now, that the theory of sentential negation presented, here, and the principle of duality accord with each other in such a way as to deliver simple, albeit deep results about adjectives and their polar opposites.

**Sentential negation and Boolean and: narrow-scope reading**

For the sake of completeness, I consider the interaction between sentential negation and Boolean and, although from a formal perspective, this simply amounts to replacing or/∨ with and/∧ in the above derivations. All the same, to begin with, I consider the narrow-scope reading of sentential negation and how it interacts with and under the scope of than. Consider, again the relevant example in (2.118), repeated for convenience below in (4.77).

\[(4.77)\]

\[\begin{align*}
\text{a. Orcutt is not taller than Smith and Jones} \\
\text{b. } \Leftrightarrow \text{Orcutt is not taller than Smith and Orcutt is not taller than Jones}
\end{align*}\]

**Sentential negation and Boolean ‘and’: narrow-scope interpretation**

The semantic representation of (4.77) is given in (4.78a), and its linguist’s tree is given in (4.78b). (I refrain from giving a Gentzen/Prawtiz derivation and refer the reader to figure 4.14 for how such a derivation would proceed.)
4.5. NEGATION

(4.78)  a. Orcutt is not taller than Smith and Jones \( \sim \)

\[
\begin{align*}
[\lambda w \neg \delta (\text{tall})(o)(s)(w) \land \neg \delta (\text{tall})(o)(j)(w)] &= \\
\{w \in D_s | \neg(o > s \# w) \land \neg(o > j \# w)\}
\end{align*}
\]

b. 

\[ \text{Sentential negation and Boolean 'and': narrow-scope derivation} \]

The semantic representation in (4.78a) denotes the set of indices in which it is not the case that Orcutt has more height than Smith and it is not the case that Orcutt has more height than Jones. Situations such indices characterize are shown in figure 4.17.

Under its narrow-scope reading, the statement in (4.78) involving sentential negation and and is equivalent to the one in (4.73), which itself is the wide-scope reading of sentential negation and or. It makes sense, then, the following validity is predicated to hold.

(4.79)  a. Orcutt is not taller than Smith and Jones

\[ \iff \text{Orcutt is as short as Smith and Orcutt is as short as Jones} \]
4.5. NEGATION

Figure 4.17: Possible situations consistent with (4.78a)

Sentential negation and polar opposites: narrow-scope interpretation

Of course, from a formal perspective, this fact is derivable in a similar fashion to the above.

(4.80)

\[
[\lambda w \neg (\text{tall}) (o) (s) (w) \land \neg (\text{tall}) (o) (j) (w)] = \{ w \in D_s | \neg(o >_{D_s} s \land w) \land \neg(o >_{D_s} j \land w) \} \\
= \{ w \in D_s | o \leq_{D_s} s \land o \leq_{D_s} j \land w \} \\
= [\lambda w (\delta' (\text{short}) (o) (s) (w) \land \delta' (\text{short}) (o) (j) (w))] 
\]

I predict the following equivalence, which again seems to accord with pre-theoretical intuitions.

(4.81) a. Orcutt is not taller than Smith and Jones
    b. \(\Leftrightarrow\) Orcutt is as short as Smith and Orcutt is as short as Jones

Sentential negation and polar opposites: narrow-scope interpretation
4.5. NEGATION

**Sentential negation and Boolean and: wide-scope reading**

To conclude, consider the wide-scope reading of negation as it interacts with Boolean and as it appears under the scope of than. Originally shown in (2.119), I repeat the relevant example below in

\[(4.82)\]

<table>
<thead>
<tr>
<th>a. Orcutt is not taller than Smith and Jones</th>
</tr>
</thead>
<tbody>
<tr>
<td>b. $\Leftrightarrow$ Orcutt is not taller than Smith or Orcutt is not taller than Jones</td>
</tr>
</tbody>
</table>

*Sentential negation and Boolean ‘and’: wide-scope interpretation*

The semantic representation of (4.82) is given in (4.83a). Again, it is clear that, by De Morgan’s laws, negation has been pushed through flipping conjunction to disjunction. The representation’s linguist’s tree is given in (4.83b). (I have also not chosen to give the Gentzen/Prawtiz derivation for this example, referring the reader, again, to figure 4.15 for the way in which it would be done.)
4.5. NE\textsc{GATION}

(4.83)  

\begin{align*}
\text{Orcutt is not taller than Smith and Jones } & \sim \\
\left[ \lambda w \neg (\delta (\text{tall})(o)(s)(w) \wedge \delta (\text{tall})(o)(j)(w)) \right] = \\
\{ w \in D_s \mid \neg (o > s @ w) \lor \neg (o > j @ w) \}
\end{align*}

b.

\begin{tikzpicture}
    \node {S}
    child {node {NP}
        child {node {IV/A} edge from parent node[above]{IV}}
        child {node {A} edge from parent node[above]{A}}
        child {node {A/PP} edge from parent node[above]{A/PP}}
    }
    child {node {PP} edge from parent node[above]{PP}}
    child {node {NP} edge from parent node[above]{NP}}
    child {node {NP} edge from parent node[above]{NP}}
    edge from parent node[above]{\text{Sentential negation and Boolean ‘and’: wide-scope derivation}}
\end{tikzpicture}

The semantic representation in (4.83a) denotes the set of indices in which it is not the case that Orcutt has more height than Smith or it is not the case that Orcutt has more height than Jones. (Again, there are too many situations consistent with (4.83a) in which to picture graphically in a meaningful way.)

Notice, again, that (4.83a) is logically equivalent to the logical statement in (4.68a). Under its wide-scope reading, a statement involving sentential negation and \textit{and} is equivalent to the narrow-scope reading of a statement involving sentential negation and \textit{or}. Consequently, I predict the validity in (4.84).
4.5. NEGATION

(4.84) a. Orcutt is not taller than Smith and Jones
     b. ⇔ Orcutt is as short as Smith or Orcutt is as short as Jones

Sentential negation and polar opposites: wide-scope interpretation

This is justified by the following equational reasoning.

(4.85)

\[
[\lambda w \neg (\delta \text{(tall)})(o)(s)(w) \land \delta \text{(tall)})(o)(j)(w))] = \{w \in D_s | \neg(o >_s s @ w) \lor \neg(o >_j j @ w)\}
= \{w \in D_s | o \leq_s s @ w \lor o \leq_j j @ w\}
= [\lambda w (\delta'(\text{short})(o)(s)(w) \lor \delta'(\text{short})(o)(j)(w))]\]

I also predict the validity in (4.86).

(4.86) a. Orcutt is not taller than Smith and Jones
     b. ⇔ Orcutt is as short as Smith or Orcutt is as short as Jones

Sentential negation and polar opposites: wide-scope interpretation

4.5.2 Differential negation

I consider, now, the semantics of so-called differential negation, introduced in (2.120) and repeated below in (4.87).

(4.87) Orcutt is no taller than Smith

Differential negation

Schwarzschild (2008; p. 318) points out “possible differentials include many (many more eggs), no (no hotter), any (was not any happier), and noun phrases headed by a unit of measure (2 pounds heavier). Recall that, unlike sentential negation, differential negation is not ambiguous: it only has a narrow-scope interpretation.

In order to account for this fact, instead of introducing a type-polymorphic proof-rule, I account for this fact lexically by adopting the entry for no as in (4.88).
(4.88)

<table>
<thead>
<tr>
<th>Lexical entry —</th>
</tr>
</thead>
<tbody>
<tr>
<td>no : (A/PP) / (A/PP) :</td>
</tr>
<tr>
<td>( \lambda e \lambda Q \lambda x (Q \lambda y (e \lambda P \lambda w (\neg (P (y)) (w))) (x)) )</td>
</tr>
</tbody>
</table>

This representation looks daunting. From a syntactic perspective, differential negation is nothing more than a comparative adjective modifier. Semantically, differential no introduces the negation operator, which has the effect of forcing negation to take obligatory narrow-scope under the than phrase. This will ultimately block any sort of De Morgan’s effect(s) from being licensed.

To get used to how differential negation works in the type-logical setting, I analyze (4.87) by giving its semantic representation in (4.89a); the relevant portion of its Gentzen-Prawitz derivation in (4.89b); and its linguist’s tree in (4.89c).
(4.89) (4.89) a.

Orcutt is no taller than Smith

\[ \lambda w (\neg \delta (\texttt{tall}) (o) (s)) (w) \] =
\[ \{ w \in D_s \mid \neg (o > s \ where \ w) \} \]

b. [Diagram]

c. [Diagram]

The semantic representations in (4.89a), (4.63a), and (4.63a) are all logically equivalent. This is because, without more articulate structure under the scope of \textit{than}, it is impossible to tease apart the meaning difference between differential negation on the one hand and the two readings of sentential negation on the other. Just as in the case of the latter, we must consider more diverse data to capture the relevant semantic differences. Some situations (4.89a) are consistent with are shown in figure 4.18
The lexical entry for differential negation shown in (4.88) forces *no* to take obligatory narrow-scope under the *than* phrase. To see this, consider, first, Boolean *or* as it appears under the scope of *than*. Recall that in (2.120), repeated below in (4.90), it is clear that differential negation does not interact with *or* in any interesting way whatsoever.

(4.90)  

(a) Orcutt is no taller than Smith or Jones  

(b) ⇔ Orcutt is no taller than Smith or Orcutt is no taller than Jones

**Differential negation and Boolean *or***

A sample derivation (4.90) is given in figure 4.19. Notice, here, that because negation takes narrow-scope, De Morgan’s laws are not applicable at the sentential level. To be even clearer, in (4.91a), I have provided the semantic representation of (4.90). Its linguist’s tree is shown in (4.91b).
Figure 4.19: A complete derivation of differential negation and Boolean or
4.5. NEGATION

(4.91) a. Orcutt is no taller than Smith or Jones

\[ \lambda w (\neg \delta (\text{tall})(o)(s)(w) \lor \neg \delta (\text{tall})(o)(j)(w)) = \{ w \in D_s \mid \neg (o > s @ w) \lor \neg (o > j @ w) \} \]

b.

Differential negation and Boolean ‘or’: narrow-scope derivation

The statement in (4.91a) denotes the set of indices in which it is not the case that Orcutt has more height than Smith or it is not the case that Orcutt has more height than Jones. (Again, there are too many situations consistent with (4.91a) in which to picture graphically in a meaningful way.)

Observe that the semantic representations in (4.91a) and (4.68a) are logically equivalent. I predict that adjectival comparatives involving differential negation plus Boolean or under the scope of than are synonymous with adjectival comparatives involving sentential negation plus Boolean or under the scope of than.

(4.92) a. Orcutt is no taller than Smith or Jones

b. \( \Leftrightarrow \) Orcutt is as short as Smith or Orcutt is as short as Jones

Differential negation and polar opposites: narrow-scope interpretation
4.5. NEGATION

And again by duality I predict the validity in (4.93).

(4.93)  a. Orcutt is no shorter than Smith or Jones
   b. \(\iff\) Orcutt is as tall as Smith or Orcutt is as tall as Jones

\textit{Differential negation and polar opposites: narrow-scope interpretation}

In light of the equivalency between (4.95a) and (4.68a), for formal justification of the validities in (4.92) and (4.93), we need only turn to the chain of equivalences shown in (4.70) plus an application of the duality principle.

\textbf{Differential negation and Boolean \textit{and}}

Again, for the sake of completeness, I consider the interaction between differential negation and Boolean \textit{and} as it appears under the scope of \textit{than} even though, at this point, it should be obvious that the semantic analysis of this section will follow that of the narrow-scope reading of sentential negation and \textit{and}. (See, again, §4.76 for such an analysis.) As was shown in (2.120), but repeated again in (4.94), differential \textit{no} does not interact with \textit{and} in any interesting way à la De Morgan’s laws.

(4.94)  a. Orcutt is no taller than Smith and Jones
   b. \(\iff\) Orcutt is no taller than Smith and Orcutt is no taller than Jones

\textit{Differential negation and Boolean \textit{‘and’: narrow-scope interpretation}

To capture the syntax and semantics of (4.94), I provide the semantic representation of (4.94), and has become customary, in (4.95b), I give its linguist’s tree.
4.5. NEGATION

(4.95) a.

Orcutt is no taller than Smith and Jones \( \sim \)

\[
\left[ \lambda w (\neg \delta (\text{tall}) (o) (s) (w) \land \neg \delta (\text{tall}) (o) (j) (w)) \right] = \\
\{ w \in D_s \mid \neg (o \triangleright s @ w) \land \neg (o \triangleright j @ w) \}
\]

b.

Differential negation and Boolean ‘and’: narrow-scope derivation

The semantic representation in (4.95b) denotes the set of indices in which it is not the case that Orcutt has more height than Smith and it is not the case that Orcutt has more height than Jones.

Observe that the semantic representations in (4.95a) and (4.68a) are logically equivalent. I predict that adjectival comparatives involving differential negation plus Boolean and under the scope of than are synonymous with adjectival comparatives involving sentential negation plus Boolean and under the scope of than with respect to sentential negation’s narrow-scope reading.

(4.96) a. Orcutt is no taller than Smith and Jones

b. \( \Leftrightarrow \) Orcutt is as short as Smith and Orcutt is as short as Jones

Differential negation and polar opposites: narrow-scope interpretation
4.6 QUANTIFIERS

<table>
<thead>
<tr>
<th>Attested entailments</th>
<th>Universal</th>
<th></th>
<th>Existential</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>anyone</td>
<td>✓</td>
<td>x</td>
<td>✓</td>
<td>x</td>
</tr>
<tr>
<td>someone</td>
<td>x</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>everyone</td>
<td>✓</td>
<td></td>
<td>✓</td>
<td>x</td>
</tr>
</tbody>
</table>

| My analysis          | ✓         | x | ✓           | x |

| A-not-A analysis     | x         | ✓ | x           | ✓ |
| Larson’s (1988) revised | ✓   | x | ✓           | ✓ |

| Greater-than analysis | x         | ✓ | x           | ✓ |
| Heim’s (2006) revised | ✓         | ✓ | ✓           | ✓ |

Table 4.5: The various entailment patterns the various analyses, including my own, are able to capture, under the assumption that anyone is given universal semantics ($\forall$)

And again, by duality, I predict the following equivalence holds in general.

(4.97) a. Orcutt is no shorter than Smith and Jones

b. $\Leftrightarrow$ Orcutt is as tall as Smith and Orcutt is as tall as Jones

Differential negation and polar opposites: narrow-scope interpretation

4.6 Quantifiers

In this section, I will extend my analysis to the cases of the Boolean quantifiers anyone, someone, and everyone as they appear under the scope of than. Specifically, I will show that my analysis captures

- the universal interpretation of anyone under the assumption that it is to be given universal semantics ($\forall$);

- the normal existential interpretation of someone; and

- the normal universal interpretation of everyone.
4.6. QUANTIFIERS

This is summarized clearly in table 4.5. Moreover, I will investigate a variety of nice consequences of my system, showing that I can capture the semantics of adjectival comparatives involving non-monotonic quantifiers and downward monotonic quantifiers, which themselves have proven problematic for a variety of semantic analyses.

4.6.1 The quantifier anyone

I turn now to exploring the syntax and semantics of the quantifier anyone as it appears under the scope of than. Recall that in (2.37), repeated below in (4.98), it was shown that, in this syntactico-semantic context, anyone has universal force.

(4.98) a. Orcutt is taller than anyone else
   b. $\iff$ Orcutt is taller than everyone else
   c. $\iff$ For everyone else other than Orcutt, Orcutt is taller than them

Following Alrenga and Kennedy (2013) (but see, also, Aloni and Roelofsen (2011)), I analyze anyone under the scope of than as free choice anyone. (See, again, §2.7 and §2.8 for reasons to do so.) Its lexical entry is as in (4.99).\textsuperscript{11}

(4.99)

\begin{center}
\textit{Lexical entry —}
\end{center}
\begin{center}
anyone else : DET : $\lambda P\lambda w \forall x \ (x \neq o \rightarrow P(x)(w))$
\end{center}

In (4.100a), I provide the semantic representation of an adjectival comparative involving free choice anyone as it appears in (4.98). Its linguist’s tree is in (4.100b).

\footnote{A full analysis of the meaning of the expression anyone else and its anaphoric properties extend beyond the scope of this dissertation.}
4.6. QUANTIFIERS

Figure 4.20: Possible situation consistent with (4.100a)

(4.100)a.

Orcutt is taller than anyone else $\sim$

$$\left[\lambda w \forall x (x \neq o \rightarrow \delta (\text{tall}) (o) (x) (w))\right] =$$

$$\{w \in D_s \mid \forall x (x \neq o \rightarrow o > H x @ w)\}$$

b.

The semantic representation in (4.100a) denotes the set of indices in which, for everyone other than Orcutt, Orcutt has more height than them. A possible situation characterized by these indices is shown in figure 4.20.
4.6.2 The quantifier *someone*

Similarly, I am able to derive the meaning of an adjectival comparative involving the quantifier *someone* under the scope of *than*. Recall that, in (2.46), repeated below in (4.101), it was shown that, in this particular syntactico-semantic context, *someone* has its normal, existential interpretation, not a strengthened universal interpretation.

(4.101)a. Orcutt is taller than someone else  
b. ⊖ Orcutt is taller than everyone else  
c. ⊖ For everyone₁ other than Orcutt, Orcutt is taller than them₁

In order to account for the invalidities in (4.101), I assume the lexical entry for *someone* in (4.102).

(4.102)

\[
\text{Lexical entry –} \quad \text{someone else} : \text{DET} : \lambda P \lambda w \exists x (x \neq o \land P(x)(w))
\]

In (4.103a), I provide the semantic representation (4.101), and the linguist’s tree is in (4.103b).
4.6. QUANTIFIERS

Figure 4.21: Possible situations consistent with (4.103a)

(4.103)a.

Orcutt is taller than someone else \(\sim\)

\[
[\lambda w \exists x (x \neq o \land \delta(tall)(o)(x)(w))] = \\
\{w \in D_s \mid \exists x (x \neq o \land o >_x x @ w)\}
\]

b.

The semantic representation in (4.103a) denotes the set of indices in which there exists someone other than Orcutt such that Orcutt has more height than them. Some possible situations characterized by these indices are shown in figure 4.21.
4.6. QUANTIFIERS

Sentential negation and the quantifier *someone*

Recall that in §2.9.2, I discussed the interaction between sentential negation and quantifiers like *someone*, which appear under the scope of *than*. Recall that sentential negation has two interpretations in general: a narrow- and a wide-scope reading. Originally shown in (2.122) and (2.123), the relevant examples are repeated below in (4.104) and (4.105), respectively.

\[(4.104)\]
\[
a. \text{ Orcutt is not taller than someone} \\
 b. \Leftrightarrow \text{ For someone}_1, \text{ Orcutt is not taller than them}_1
\]

*Sentential negation and ‘someone’: narrow-scope interpretation*

\[(4.105)\]
\[
a. \text{ Orcutt is not taller than someone} \\
 b. \Leftrightarrow \text{ For everyone}_1, \text{ Orcutt is not taller than them}_1
\]

*Sentential negation and ‘someone’: wide-scope interpretation*

It is clear that, under its wide-scope interpretation, sentential negation induces a sort of De Morgan’s effect, flipping so-to-speak the existential to a universal. No such effect can be seen in the case of the narrow-scope reading of sentential negation.

As I showed in §4.5.1 of this chapter, the difference in interpretations of sentential negation can naturally be accounted for in the type-logical system I have laid out here. It should come as no surprise, then, that I am naturally able to account for the difference in meaning between (4.104) and (4.105). I begin, first, by deriving the narrow-scope reading of sentential negation. The semantic representation of (4.104) is shown clearly in (4.106a), and its linguist’s tree is given in 4.106.
(4.106)a.

Orcutt is not taller than someone else $\sim$

$$[\lambda w \exists x \neg (x \neq o \land \delta (\text{tall}) (o) (x) (w))] =$$

$$\{w \in D_s \mid \exists x (x = o \lor \neg (o >_s x @ w))\}$$

b.

The semantic representation in (4.106a) denotes the set of indices in which, for someone other than Orcutt, it is not the case that Orcutt has more height than them.

Turning, now, to the wide-scope reading of sentential negation, the semantic representation of (4.105) is given in (4.107a), and its linguist’s tree is in (4.107b). Notice, here, that because negation takes wide-scope over the existential, it can subsequently be pushed through by De Morgan’s laws, thus flipping the existential to a universal.
(4.107)a.

Orcutt is not taller than someone else \( \sim \)

\[ [\lambda w \exists x (x \neq o \land \delta (\text{tall})(x)(w))] = \{ w \in D_s \mid \forall x (x \neq o \rightarrow \neg (o > x \cdot w)) \} \]

b.

Sentential negation and Boolean ‘someone’: wide-scope derivation

The semantic representation in (4.107a) denotes the set of indices in which, for everyone other than Orcutt, it is not the case that Orcutt has more height than them. Some representative situations characterized by the set of indices above are shown in figure 4.22.

Differential negation and the quantifier someone

Recall that, because differential negation takes obligatory narrow-scope, we do not witness a sort of De Morgan’s effect between no and someone as that quantifier appears under the scope of than. This was originally shown in (2.124) and is repeated below in (4.108).
Figure 4.22: Possible situations consistent with (4.107a)

(4.108)a. Orcutt is no taller than someone else

b. \(\Leftrightarrow\) For someone\(_1\) other than Orcutt, Orcutt is not taller than them\(_1\)

**Differential negation and ‘someone’: narrow-scope**

The semantic representation of (4.108) I deliver is provided in (4.109a), and its linguist’s tree is given in (4.109b). Notice, here, that, because negation takes narrow-scope, is not available to be pushed through by De Morgan’s laws.
(4.109)a.

Orcutt is no taller than someone else \( \sim \)

\[
[\lambda w \exists x (o \neq x \land \neg (\text{tall}) (o) (x)) = \\
\{w \in D_s | \exists x (o \neq x \land \neg (o >_\exists x @ w))\}
\]

b.

The semantic representation in (4.109a) denotes the set of indices in which there exists someone other than Orcutt such that it is not the case that Orcutt has more height than them. (Again, there are too many situations consistent with (4.109a) to picture them all graphically in a meaningful way.)

Notice that (4.109a) and (4.106) are equivalent. This is as to be expected. As was discussed extensively in §4.5.1 and §4.5.2, the narrow-scope interpretation of sentential negation and differential negation are equivalent in general.

4.6.3 The quantifier *everyone*

At this point, there should be nothing surprising about my analysis of the quantifier *everyone* as it appears under the scope of *than*. Recall that, in this context, *everyone* has its normal, universal interpretation. This was shown originally with (2.56), but per usual, it
has been repeated in (4.110).

(4.110)a. Orcutt is taller than everyone else  
b. $\not \Rightarrow$ Orcutt is taller than someone else  
c. $\not \Rightarrow$ For someone$_1$ other than Orcutt, Orcutt is taller than them$_1$

Just as in the case of anyone, I adopt universal semantics for everyone. Its lexical entry is shown in (4.111), and unsurprisingly, is identical to that of anyone’s shown in (4.99), except for its phonological component.

(4.111)

$$
\text{Lexical entry –}
\text{everyone else : DET } \lambda P \lambda w \forall x \left( x \neq o \rightarrow P(x)(w) \right)
$$

The semantic representation for (4.110) is given in (4.112a), and its linguist’s tree is shown in (4.112b).
(4.112)a.

Orcutt is taller than everyone else $\sim$

$$\left[ \lambda w \forall x \left( x \neq o \rightarrow \delta (\text{tall}) (o) (x) (w) \right) \right] =$$

$$\{ w \in D_s \mid \forall x \left( x \neq o \rightarrow o >_S x @ w \right) \}$$

b.

The semantic representation in (4.112a) denotes the set of indices in which, for everyone other than Orcutt, Orcutt has more height than them. Unsurprisingly, this set of indices is the same set characterized by the semantic representation in (4.100a), as it and the one in (4.112a) are equivalent.

Finally, the interaction between both sentential and differential negation and the quantifier $everyone$ as it appears under the scope of $than$ is similar to the interaction between negation and $someone$. Consequently, I refer the reader to §2.9.2 for the relevant data points, as well as §4.103 and §4.107 of this chapter for how precisely to account for these data points.
4.6. QUANTIFIERS

4.6.4 Some more quantifiers

In the final section of this chapter, I will consider how my analysis handles a variety of other quantifiers not discussed explicitly in chapter 2.

As pointed out by authors like Schwarzschild and Wilkinson (2002), van Rooij (2008) and Beck (2010) among many more, most semantic analyses of adjectival comparatives, including Larson’s (1988), have difficulties accounting for the meanings of comparatives involving non-monotonic and negative quantifiers like exactly and no one without additional assumptions. I will not review the failures of the various analyses considered in chapter 2 as they relate to these types of quantifiers; this would lead us too far astray, as these analyses deliver counter-intuitive results.

Numerals and Non-monotonic quantifiers

Under my analysis, it is straightforward to capture the semantics of the relevant data. Building on the first-order work of Hodges (1997; p. 31), I assume that the lexical entry for at least two is as in (4.113).12

\[
(4.113)
\]

\[
\text{Lexical entry —}
\]

\[
\text{at least two : DET :} \\
\lambda P' \lambda P \lambda w \exists x_1 \exists x_2 (P'(x_1)(w) \land P'(x_2)(w) \land P(x_1)(w) \land P(x_2)(w) \land x_1 \neq x_2)
\]

Now consider the semantic representation in (4.114). This expression denotes the set of indices such that for exactly two men, Orcutt has more height than them.13

---

12See Hodges (1997; ibid.) for how at least \( n \) can be defined inductively in the first-order setting for all \( n \leq \omega \). Here, I only consider a representative example.

13I forego the derivation of (4.114) and the remainder of examples in this section, trusting I have provided all the resources through the course of this chapter to do so.
4.6. QUANTIFIERS

(4.114)

Orcutt is taller than at least two men \(\sim\)
\[
\lambda w \exists x_1 \exists x_2 (\textit{man} (x_1) (w) \land \textit{man} (x_2) (w) \land \\
\delta (\textit{tall}) (o) (x_1) (w) \land \delta (\textit{tall}) (o) (x_2) (w) \land x_1 \neq x_2)
\]

Similarly, I provide the lexical entry for the quantifier \textit{at most two} in (4.115).

(4.115)

\[
\textbf{Lexical entry —}
\]

at most two : DET :
\[
\lambda P' \lambda P \lambda w \neg \exists x_1 \exists x_2 \exists x_3 (P' (x_1) (w) \land P' (x_2) (w) \land P' (x_3) (w) \land \\
P (x_1) (w) \land P (x_2) (w) \land P (x_3) (w) \land x_1 \neq x_2 \land x_2 \neq x_3 \land x_1 \neq x_3)
\]

The value of the semantic representation in (4.116) is the set of all indices in which for at most two men, Orcutt has more height than them.

(4.116)

Orcutt is taller than at most two men \(\sim\)
\[
\lambda w \neg \exists x_1 \exists x_2 \exists x_3 (\textit{man} (x_1) (w) \land \\
\textit{man} (x_2) (w) \land \textit{man} (x_3) (w) \land \delta (\textit{tall}) (o) (x_1) (w) \land \\
\delta (\textit{tall}) (o) (x_2) (w) \land \delta (\textit{tall}) (o) (x_3) (w) \land x_1 \neq x_2 \land x_2 \neq x_3 \land x_1 \neq x_3)
\]

Finally, putting the insights of (4.113) and (4.115) together, the semantic representation in (4.117) denotes the set of all indices such that for at least but at most two men, Orcutt has more height than them.
4.6. QUANTIFIERS

(4.117) Orcutt is taller than exactly two men \( \sim (4.114) \land (4.116) \)

Stated another way, (4.114) denotes the set of all indices such that for exactly two men, Orcutt has more height than them.

**Negative quantifiers**

von Stechow (1984) claims that negative quantifiers are unacceptable under the scope of `than`. (But see, also, Klein (1991) for an extensive discussion on this point.)

(4.118) ?Orcutt is taller than no one else

However, there does not seem to be a consensus on this felicity judgement in the literature.\(^{14}\)

Authors have struggled to interpret statements like (4.118). However, consider the statement in (4.119a). Not only does it feel perfectly acceptable for me, but it is clear that its intended interpretation is given in (4.119b).

(4.119)a. Orcutt is taller than no one else except Smith

\[ b. \quad \equiv \text{No one but Smith is shorter than Orcutt} \]

It is easy to account for the meaning of (4.119a) under the reading in (4.119b).

\(^{14}\)See, for example, Bhatt and Takahashi (2007), who argue that negative quantifiers are acceptable under the scope of `than` in the context of phrasal but not clausal comparatives.

(i) Orcutt is taller than no one

\( \text{Phrasal comparative} \)

(ii) #Orcutt is taller than no one else is

\( \text{Clausal comparative} \)

I will not discuss clausal comparatives until chapter 5. However, I find negative quantifiers acceptable in both the context of phrasal and clausal comparatives.
4.7. CHAPTER SUMMARY AND NEXT CHAPTER PREVIEW

Orcutt is taller than no one else except Smith

\[ \lambda w (\neg \exists x (x = s \land \delta (tall) (o) (x) (w)) \land \delta (tall) (o) (s) (w)) \]

By De Morgan’s laws and duality (see, again, lemma 1), the representation in (4.120) is equivalent to the one in (4.121).

\[ \lambda w (\neg \exists x (x = s \land \delta (tall) (o) (x) (w)) \land \delta (tall) (o) (s) (w)) \Leftrightarrow \lambda w (\forall x (x \neq s \rightarrow \neg \delta (tall) (o) (x) (w)) \land \delta (tall) (o) (s) (w)) \Leftrightarrow \lambda w (\forall x (x \neq s \rightarrow \delta' (short) (o) (x) (w)) \land \delta (tall) (s) (o) (w)) \]

The set of indices characterized by (4.121) are those in which Orcutt is not taller than anyone but Smith.

The point is that my analysis does not block negative quantifiers as they appear under the scope of than. If, along with von Stechow (1984), believe that such statements are infelicitous, or rather ungrammatical, then my analysis faces an obvious problem of syntactic and semantic over-generation. However, if we find statements like (4.119) acceptable, then my analysis has no problem handling their syntax and semantics.

4.7 Chapter summary and next chapter preview

The purpose of this chapter was to present my analysis of adjectival comparatives, dealing directly with the data discussed in chapter 2 of this dissertation. To do this, I developed a formal theory of scalar dimensions and utilized that theory in the semantic representation I give for the comparative morpheme -er/more. That representation involves a covert operator \( \delta \), which unlike negation (\(\neg\)) and the max-operator, does not play badly at the level of logical form. Instead, utilizing this \( \delta \) operator, I am able to capture

- the disjunctive interpretation of or;
- the conjunctive interpretation of and;
• the universal interpretation of anyone;

• the existential interpretation of someone; and

• the universal interpretation of everyone as this expression appears under the scope of than.

Importantly, I can capture the semantics of these expressions in this particular syntactico-semantic environment because I treat the than phrase, from a semantic perspective at least, as a generalized quantifier that, again, does not play poorly with the $\delta$ operator.

Moreover, I showed that my analysis is able to handle both sentential and differential negation, something Larson’s (1988) revised A-not-A analysis cannot do because it posits tacit negation ($\neg$) at the level of logical form. (See, again, §2.9.3 for an argument as to why Larson (1988) cannot account for the narrow-scope reading of sentential negation, let alone the meaning of differential negation.)

This chapter marks one step in showing my analysis is to be preferred to a degree-based approaches, because of my empirical coverage without the extra ontological assumption of degrees. In the next chapter, I will generalize my analysis to the account for the syntax and semantics clausal comparatives. Specifically, I will how the syntax and semantics of phrasal comparatives can be derived from that of clausal ones. I will also consider a wider range of data than that discussed here and in chapter 2, providing more (implicit) evidence and reasons for adopting my analysis.
5 | Moving farther

5.1 Introduction

In this chapter, I will present my syntactico-semantic analysis of what is often referred to as a clausal comparative like the one in (5.1).

(5.1) Orcutt is taller than Smith is

Contrasted with a phrasal comparative like (5.2), we see, in the case of clausal comparatives, the presence of be under the scope of than.

(5.2) Orcutt is taller than Smith

Many authors have suggested that clausal comparatives involve a form of ellipsis sometimes referred to as **comparative ellipsis** or **comparative deletion** such that, in the case of (5.1), the adjective *tall* has gone missing. This is shown clearly in (5.3).

(5.3) Orcutt is taller than Smith is **tall**

To complicate matters even further, in some instances, clausal comparatives do not seem to involve elided material. This is when the two adjectives are not identical, as in the case of the **inter-adjectival** comparative shown in (5.4).
5.2. CLAUSAL COMPARATIVES

(5.4) Orcutt is more handsome than Smith is intelligent

Inter-adjectival comparative

Ideally, we want a syntax and semantics that is robust enough to handle the cases in (5.1)–(5.4). In this chapter, I want to investigate the syntax and semantics of clausal comparatives in detail, generalizing the ideas presented in chapter 4 to provide a unified analysis of both phrasal and clausal comparatives, including ones that do (and do not) involve comparative ellipsis.

At a high level, my analysis can be thought of as a hybrid approach in the following sense.

• Building on insights from Heim (1985) and Larson (1988), I reject my analysis of phrasal comparatives from chapter 4 and instead propose a novel analysis of clausal comparatives in which phrasal ones can be derived.

• Following Cresswell (1976), I introduce the notion of a trans-world scale, and show how the semantics of clausal comparatives I provide can account for a variety of interesting modal and temporal facts involving such comparatives.

• Following Lassiter (2011), I build a theory of measurement on top of my semantics for clausal comparatives and demonstrate how a variety of measure phrases can be captured under my semantics.

5.2 Clausal comparatives

Beginning with (at least) Hankamer (1973) and continuing on through the work of Hoeksema (1983), Heim (1985), Larson (1988), Hazout (1995), Merchant (2009) and Bhatt and Takahashi (2007) among many more, there has been much debate surrounding whether there is a syntactic and/or semantic difference between the statements in (5.5) and (5.6).
5.2. **CLASUAL COMPARATIVES**

(5.5) Orcutt is taller \([_PP \text{ than } [_{DP} \text{ Smith}]]\)

*Phrasal comparatives*

(5.6) Orcutt is taller \([_PP \text{ than } [_{S} \text{ Smith is}]]\)

*Clausal comparatives*

Hankamer (1973) claims that, in English at least, there exists a syntactic difference between (5.5) and (5.6). Under a Hankamer (1973) style analysis, as both parses make clear, in the case of the former, *than* sub-categorizes for a determiner phrase (DP), and in the case of the latter it sub-categorizes for a clause (S). Consequently, comparatives like (5.5) are referred to as **PHRASAL COMPARATIVES** and comparatives like the one in (5.6) are referred to as **CLASUAL** ones.

From a syntactic perspective, that there exists a separate prepositional *than* in English is supported by the data in (5.7).

(5.7) a. Who is Orcutt taller than?
    b. #Who is Orcutt taller than is?

Merchant (2009; (7) & (8))

Under standard Chomskyan-style assumptions about *wh*-movement, the *who* in (5.7a) is base-generated under the scope of *than* and then ‘moved’ to left-periphery of the statement. That (5.7a) is judged to be grammatical is contrasted with the ungrammaticality of (5.7b). This contrast is supposed to reflect the fact that *who* can only move out from under the scope of PP *than*, not clausal *than*.

The data in (5.8) reflects a fact about the English pronominal system, namely that *himself* is an accusative, not nominative pronoun. It is generally assumed to syntactically function as an object; it cannot appear in the subject position of a clause.

(5.8) a. No one\(_1\) is taller than himself\(_1\)
    b. #No one\(_1\) is taller than himself\(_1\) is

If the *than* in (5.8b) were a PP, then we would expect *himself* to be grammatically licensed, which it is not. That it is not can be explained naturally under the assumption that *than* is in
fact a clausal denoting one. The same point can be made by considering the contrast below.

(5.9)  
\[
\begin{align*}
&\text{a. Orcutt is taller than } \{ \text{her} \} \\
&\text{b. Orcutt is taller than } \{ \text{#her} \text{ she} \} \text{ is}
\end{align*}
\]

There is a cross-linguistic evidence that suggests this distinction is real. Merchant (2009) points out that Greek distinguishes between clausal and phrasal comparatives overtly through the comparative markers \textit{ap’oti} and \textit{apo}, as (5.10) and (5.11) indicate respectively.

(5.10)  
\[
\begin{align*}
&I \text{ Maria pezi kiθara kalitera ap’oti pezi kiθara o the Maria.NOM plays guitar better than.CLAUSAL plays guitar the Giannis Giannis.NOM} \\
&\text{‘Maria plays the guitar better than Giannis plays the guitar’}
\end{align*}
\]

(5.11)  
\[
\begin{align*}
&I \text{ Maria pezi kiθara kalitera apo ton Gianni the Maria.NOM plays guitar better than.PHRASAL the Gianni.ACC} \\
&\text{‘Maria plays the guitar better than Giannis’}
\end{align*}
\]

\text{Merchant (2009; (4) & (9))}

Other cross-linguistic evidence for the clausal/phrasal distinction is cited in the literature. Hoeksema (1983) shows that Dutch seems to have two homophonous \textit{thans}. The contrast between the Dutch sentences in (5.12) and (5.13) parallels the contrast between their English counterparts in (5.8a) and (5.8b) respectively.

(5.12)  
\[
\begin{align*}
&\text{Niemand is sterker dan zichzelf Nobody is stronger than himself}
\end{align*}
\]

(5.13)  
\[
\begin{align*}
&\text{#Niemand is sterker dan zichzelf is Nobody is stronger than himself is}
\end{align*}
\]

\text{Hoeksema (1983; (10a) & (10b))}

From a semantic perspective, the obvious question is this: is there a truth-conditional difference between phrasal and clausal comparatives? Under standard assumptions about truth-conditional meaning, two syntactically alike expressions, including sentences, are
5.2. CLAUSAL COMPARATIVES

said to be identical in meaning just in case they are syntactically substitutable, or rather, interchangeable without affecting the truth-values of the resultant expressions. One standard test for determining whether two statements mean the same thing, where to mean the same thing is possibly distinct from the notion of truth-conditional equivalence, is embedding those statements under NON-FACTIVE ATTITUDE verbs like believe, imagine, and think and determining whether the new, more complex statements are truth-conditionally identical.

\[(5.14)\]
\[
a. \quad \text{Wymann believes that Orcutt is taller than Smith is}
\]
\[
b. \quad \Leftrightarrow \text{Wymann believes that Orcutt is taller than Smith}
\]

Whether be is present or not under the scope of than, seemingly makes no semantic difference. The statements are identical in meaning. For example, Wymann believes that Orcutt is taller than Smith is just in case Wymann believes that Orcutt is taller than Smith: he cannot believe one without believing the other. If we take the data in (5.14) seriously, why not simply leverage the analysis of comparatives presented in chapter 5 and leave the explanation of the syntactic difference between phrasal and clausal comparatives to the syntacticians? I turn, now, to investigating two syntactico-semantic ways in which clausal comparatives differ from their phrasal counterparts.

5.2.1 Inter-adjectival comparatives

From a semantic perspective, there is also a difference between phrasal and clausal comparatives. One important difference between phrasal and clausal comparatives is that the latter, not the former, allow for inter-adjectival comparisons, some examples of which are listed below. (See Kennedy (1997), Kennedy (2001), Büring (2007), Sassoon (2007), Bale (2008), Bale (2011) and van Rooij (2011) for a more complete list.)

\[(5.15)\] Orcutt is more intelligent than Smith is handsome

*Indirect comparative* (Bale 2011; (4a))

\[(5.16)\] The desk is wider than every couch is long

*Comparative sub-deletion* (Heim 2006; (9))
The semantics of inter-adjectival comparatives is quite complicated. And not all authors judge all the statements of the underlying syntactic shapes in (5.15)–(5.17) as being meaningful. However, that inter-adjectival comparatives are treated as being contentful is evidenced by the fact that they are the objects of study across intellectual disciplines. Imagine, now, an ethicist engaged in a general, first-order moral inquiry. A statement like (5.18) is well within the parameters of his or her philosophical consideration.

(5.18) It is much better to give your money to charity than to gamble it on sports

Lassiter (2011; (5.63))

From a truth-conditional perspective, the obvious question in considering statements like (5.18) is: what does the world have to look like for this statement to be true? Do sentences like this have meaning in the way in which a truth-conditional semanticist understands meaning? This seems like a matter ethicists must decide, and I am not an ethicist. However, examples like (5.18) are strikingly similar to the class of sentences characterized by the schema in (5.19), which some political economists and social choice theorists attempt to come to grips with.

(5.19) Action $x$ is more useful for Orcutt than action $y$ is for Smith

van Rooij (2011; (10))

The data in (5.18) and (5.19) suggest that inter-adjectival comparatives are not only attested across intellectual domains like philosophy, social choice theory, and economics, but more importantly, they are central topics of investigation by researchers in these areas. I think, then, that semanticists should have something to say about the meaning of such comparatives.

The above data suggest to me that we cannot leverage my analysis in the previous section to account for inter-adjectival comparatives. Here’s why. Recall that the semantic representation of the comparative morpheme I posited in (4.2) makes reference to only one representative adjectival meaning $P$. It is repeated, below, in (5.20).
(5.20)

Lexical entry —

\[
\text{er : } \Lambda \backslash (A/\text{PP}) : \lambda P \lambda Q \lambda x (Q \lambda y (\delta (P) (x) (y)))
\]

I see no obvious way in which to utilize the lexical entry in (5.20) to capture the syntax and semantics of clausal comparatives. So, there is some work to be done.

5.2.2 Comparisons across worlds

Another important semantic difference between phrasal and clausal comparatives is that the latter, not the former, allows us to make so-called trans-world comparisons. In this chapter, I want to cash out the following idea taken from Cresswell (1976; p. 281).

We take as basic data that we can and do make comparisons, i.e., that comparative sentences can be true or false; further, that we have the ability to make counterfactual comparisons. That is to say, our competence in this area is not limited to judgements involving how things actually are but can encompass judgements about how things might be. This means we can make other-world comparisons and even trans-world comparisons... Consider adjectival comparisons. These can be reduced to sentences of the form

(64) \( x \) is more \( F \) than \( y \) is \( G \)

This schema generates a relation \( \varphi \) between two things. In possible-worlds semantics the things that are compared are in particular possible worlds. E.g., \( x \) may be more beautiful in \( y \) in \( w \) but not in \( w' \); or \( x \) may be more beautiful than \( y \) in \( w' \).

Schwarzschild and Wilkinson (2002), Heim (2006) and Beck (2010) among many more authors all provide natural language data that reify what I take to be Cresswell’s (1976) point, namely, that clausal comparatives allow for trans-world, or in the terminology of this dissertation, trans-index comparisons. (I will move fluidly between talk of worlds and indices.) Some similar examples are provided in (5.15).
5.2. CLAUSAL COMPARATIVES

Figure 5.1: Indices $u$ and $v$ are both accessible from $w$; and $v$ temporally precedes both $w$ and $u$.

(5.21) a. Orcutt is taller than Smith might be
    b. Orcutt was taller than Smith is
    c. Orcutt might be taller than Smith was

These data demonstrate that in clausal comparatives (at least) two (Montagovian) indices can be varied. These examples represent three types of trans-index comparisons.

- **Type 1**: Statements like (5.21a) manipulate the index solely of the ‘clause’ under the scope of *than*.
- **Type 2**: Statements like (5.21b) manipulate the index solely of the ‘clause’ not under the scope of *than*.
- **Type 3**: Statements like (5.21c) manipulate both indices.

Focusing on (5.21c), as it is the most complex of the three, the proposition expressed would be paraphrased as “the set of all indices $w$ such that there exists an index $u$ accessible from $w$ such that Orcutt’s height in $u$ exceeds that of Smith’s in some accessible index $v$ that itself temporally precedes $w$.” Quite a mouthful. But it should be clear that the trans-index comparison is being made across $u$ and $v$; in order to determine the truth of (5.21b) at an index $w$, we must check Orcutt’s height at $u$ and Smith’s at $v$. This is represented graphically in figure 5.1.
5.2. CLAUSAL COMPARATIVES

Observe, now, that phrasal comparatives do not obviously allow us to make such complicated trans-index comparisons.

(5.22)  

a. Orcutt is taller than Smith  
b. Orcutt might be taller than Smith

Focusing on (5.22b), the proposition expressed would be paraphrased as, “the set of all indices \( w \) such there exists an index \( u \) accessible from \( w \) such that Orcutt’s height in \( u \) exceeds Smith’s in \( u \)” Not quite as much a mouthful. However, it should be clear that no such trans-world comparison is being made here: in order to determine the truth of (5.22b) at a world \( w \), we must check only Orcutt’s and Smith’s height at \( u \).

Notice, again, that my lexical entry for the comparative morepheme in (5.20) does not allow us to make the sort of trans-world comparisons we do in (5.15), as the value of \( \delta \) maps adjectival meanings to ternary relations of type \((\text{ees})\). Such relations encode only for a single index parameter, not two. Again, the analysis of phrasal comparatives I presented in §4.2 cannot obviously be leveraged to account for the semantics of clausal comparatives. There is work left to be done.

5.2.3 An analysis

I turn, now, to my syntactico-semantic analysis of clausal comparatives. As before, it will be useful to assume abbreviations for the syntactic categories I will use in this chapter. These abbreviations are repeated in table 5.1. A few notes are in order. Following Hankamer (1973), I have assumed two homophonous forms of than: a phrasal and clausal one. Before presenting my analysis, I will assume the following stock of semantic variables.

- \( x, y, z, x', y', z' \ldots \) are variables of type \( e \);
- \( u, v, w, u', v', w' \ldots \) are variables of type \( s \);
- \( P, P', \ldots \) are variables of type \((\text{es})\);
- \( Q, Q', \ldots \) are variables of type \(((\text{es})s)\); and
- \( C \) is a variable of type \(((\text{ses})\text{es})\).
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<table>
<thead>
<tr>
<th>Category A</th>
<th>Semantic type ( \tau(A) )</th>
<th>Abbreviation</th>
<th>Traditional name</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>(s)</td>
<td>A</td>
<td>Adjective</td>
</tr>
<tr>
<td>N\S</td>
<td>(es)</td>
<td>IV</td>
<td>Intransitive Verb</td>
</tr>
<tr>
<td>S/IV</td>
<td>((es)s)</td>
<td>NP</td>
<td>Noun phrase/Proper name</td>
</tr>
<tr>
<td>NP/CN</td>
<td>((es)(es)s)</td>
<td>DET</td>
<td>Determiner</td>
</tr>
<tr>
<td>A/S</td>
<td>((es)s)</td>
<td>PP</td>
<td>Prepositional phrase</td>
</tr>
<tr>
<td>PP/NP</td>
<td>(((es)s)(es)s)</td>
<td>PP</td>
<td>Preposition (phrasal)</td>
</tr>
<tr>
<td>PP/(S/A)</td>
<td>(((es)s)(es)s)</td>
<td>PC</td>
<td>Preposition (clausal)</td>
</tr>
<tr>
<td>(((I\A)/A)/PP</td>
<td>(((es)s)(es)ses)</td>
<td>CA</td>
<td>Comparative adjective</td>
</tr>
</tbody>
</table>

Table 5.1: Abbreviations for syntactic categories

- \( \kappa \) is a variable of type \((es)(es)s\)
- \( \rho \) is of semantic type \((es)es\):
- \( \sigma \) is of semantic type \(((es)s)(es)s\); and
- \( \varsigma \) is of semantic type \(((es)s)(es)ses\)
- \( \epsilon \) is of semantic type \(((es)((es)s)(es)ses))

Having established some notational conventions, I argue that the syntax and semantics of clausal comparatives can be derived by assuming the lexical entry for the comparative morpheme as in (5.23).

(5.23)

\[
\text{Lexical entry –}
\]

\[
er : A \backslash (((I \backslash A) / A) / PP) : \lambda P' \lambda Q \lambda P \lambda y \lambda x \lambda w (Q \lambda y \lambda u (\Delta (P', P) (x) (w) (y) (u)) (v))
\]
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Of course, the analytic form of the comparative can be given as in (5.24), changing only the linear order in which the comparative morpheme combines with an adjective. This is reflected in the difference between the syntactic type of -er and more.

(5.24)

Lexical entry –

\[
\text{more} : (((\text{I} / \text{A}) / \text{PP}) / \text{A}) : \\
\lambda P' \lambda Q \lambda P \lambda v \lambda x \lambda w (Q \lambda y \lambda u (\Delta (P', P) (x) (w) (y) (u)) (v))
\]

No doubt the lexical entry for the clausal comparative morpheme is more articulated than its phrasal counterpart. From a semantic perspective, and roughly speaking, the comparative morpheme will combine with two adjectival meanings, returning a quaternary relation between an individual and index on the one hand and a (possibly) different individual and index on the other. This will no doubt become clearer throughout this chapter, when I explicate the semantic behavior of the function denoted by \( \Delta \). (In fact, as I will show, this function is essentially a generalization of the \( \delta \) I utilized in the previous chapter). As for the syntactic behavior of the comparative morpheme, I find it easiest to explain it through the use of examples.

**Comparative sub-deletion**

I begin first by deriving a comparative sub-deletion case using the example shown in (5.16) and repeated below in (5.25). I do this because such a derivation does not involve any syntactic mechanisms we have not encountered before.

(5.25) Orcutt is taller than Smith is wide

The lexical entries I will be assuming are shown in table 5.2. Notice that this lexicon differs from the one in table 4.2. Here, I assume two lexical entries for than–consistent
5.2. CLAUSAL COMPARATIVES

<table>
<thead>
<tr>
<th>Lexeme</th>
<th>Category/Type</th>
<th>Semantic Representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orcutt</td>
<td>NP (([es]))</td>
<td>(\lambda P(P(s)))</td>
</tr>
<tr>
<td>is</td>
<td>IV/({[A]})</td>
<td>(\lambda Ck\lambda w(Cwfw))</td>
</tr>
<tr>
<td>tall</td>
<td>A ((es))</td>
<td>tall</td>
</tr>
<tr>
<td>wide</td>
<td>A ((es))</td>
<td>wide</td>
</tr>
<tr>
<td>er</td>
<td>(\lambda {([([A]/A)]/PP)}((es)((es)s)(es)s)) (\lambda P(Q))</td>
<td></td>
</tr>
<tr>
<td>than</td>
<td>(P_P) (((es)s)(es)s)) (\lambda Q(Q))</td>
<td></td>
</tr>
<tr>
<td>Smith</td>
<td>(NP) ((es)s)) (\lambda P(P(s)))</td>
<td></td>
</tr>
<tr>
<td>is</td>
<td>(NP) ({A}/A) ((es)((es)s)(es)s) (\lambda P(Q))</td>
<td></td>
</tr>
<tr>
<td>COMPGAP</td>
<td>({\lambda s}/({\lambda ; (\lambda ; s)}) (((es)(es)s)(es)s) (\lambda k\lambda P(kPP))</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.2: A revised lexicon for clausal comparatives

with Hankamer (1973), as well as two for is. The utility will become clearer through the course of this chapter.

What is relevant is the lexical entry for the comparative morpheme -er. My revised lexical entry for the comparative morpheme follows a suggestion by van Rooij (2008). His idea is a generalization of Larson’s (1988) original proposal for the syntax and semantics of adjectival comparatives. Specifically, van Rooij’s (2008) idea is to allow the semantic representation of the comparative morpheme to take two (possibly distinct) adjectival meanings, as opposed to one. Under my analysis, two adjectival meanings are then mapped to a trans-world scale via the function \(\Delta\), which as I will show in the next section, is a generalization of the function \(\delta\) we encountered in (5.20) (but see, again, §4.3 for an explanation of this function). I have provided the derivation of (5.25) in figure 5.2. As usual, the proof tree in this figure can be inverted and what we are left with is the linguist’s tree in (5.26).
Figure 5.2: Derivation of comparative sub-deletion
From a syntactic perspective, in words, the derivation in can be understood as follows.

1. First, the comparative morpheme -er combines with the adjective tall on its left, yielding the comparative adjective taller.

2. In parallel, the verb is of syntactic type (NP\S) / A combines with with the phonetically null expression ⊥ on its right, yielding the complex expression is ⊥ of category NP/S.

3. The complex expression is ⊥ combines with the noun phrase Smith on its left, yielding the sentence Smith is ⊥.

4. This expression is subsequently lifted via the introduction rule /I, resulting in the expression Smith is of category S/A. At this point, the null expression is discharged and plays no role in the remainder of the derivation.
5. The preposition than combines with the noun-like phrase Smith is on its right, yielding the prepositional phrase than Smith is.\(^1\)

6. The comparative adjective taller combines with the prepositional phrase than Smith is, yielding the (complex) expression is taller than Smith is.

7. This complex expression then combines with the adjective wide on its right, yielding the (complex) intransitive verb is taller than Smith is wide.

8. The verb is combines with this comparative adjective on its right, yielding the intransitive verb phrase is taller than Smith is wide.

9. Finally, the noun phrase Orcutt combines with this (complex) intransitive verb, yielding the sentence Orcutt is taller than Smith is.

The nominalization of a clause

The keys to the above derivation are steps (3)–(5). Importantly, the verb is is applied to the phonetically null expression \(\emptyset\), which is subsequently discharged via the introduction rule /I/. This has the effect of transforming the (complex) sentence Smith is \(\emptyset\) into the (complex) noun-like phrase Smith is, which can subsequently be combined with the preposition than. In this way, I treat the ‘clause’ under the scope of than analogous to the way in which I treated the arguments of than in phrasal comparatives in chapter 4—namely as a noun phrases (or from a semantic perspective, generalized quantifiers).

This modeling decision is based off of Larson’s (1988) analysis of clausal comparatives, which is set in the generative tradition. To better understand what I mean, compare my treatment of the prepositional phrase, shown in (5.27) to Larson’s (1988), which is shown in (5.28). (But see also Heim (2006) for an excellent discussion of Larson’s (1988) analysis.)

\(^1\)I use the expression noun-like because, from a semantic perspective at least, the semantic representation of the expression Smith is is the generalized quantifier \(\lambda P(P(s))\).
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(5.27)

(5.28)

Larson (1988; (19b))

To quote Larson (1988; p. 8), “[In (5.28)], $[\text{AP} \ O_j]$ is an abstract $[+\text{WH}]$ element—an empty operator . . . and $[\text{AP} \ e_j]$ is its trace”. Although there isn’t a structural isomorphism between the syntactic tree in (5.27) and (5.28), their conceptual similarity is highlighted by the fact that their syntactico-semantic interactions and modes of combination result in equivalent meanings—the semantic representation of both the prepositional phrase in (5.27) and (5.28) is the generalized quantifier $\lambda P (P (s))$.

Treating the ‘clause’ under the scope of than as a sort of noun-like phrase is not a new idea. Bartsch and Vennemann (1972) point out that transformational grammarians proposed a similar idea, which itself is shown in figure 5.3 (Bartsch and Vennemann 1972; (B)). This tree also raises another interesting point. In §4.3, I pointed out that a sentence
involving a comparative adjective does not entail a corresponding sentence involving that adjective’s positive form.

(5.29)  
  a. Orcutt is taller than Smith is  
  b. $\not\Rightarrow$ Orcutt is tall  
  c. $\not\Rightarrow$ Smith is tall

This semantic fact has syntactic consequences. As Bartsch and Vennemann (1972; p. 169) point out, “the meanings of sentences with positives are not conceptual constituents of the meanings of the corresponding comparative sentences: one does not have to interpret the positive sentences in order to interpret the comparative sentences.”

I take the authors to mean this. Under standard assumptions about compositionality, namely that the meaning of a sentence is a function of the meanings of its parts and the way in which those meanings are composed, it follows from data like (5.29) that the syntactico-semantic composition of an adjectival comparative should not compose the corresponding sentence containing that adjective’s positive form(s) during any stage of this process. The syntactic tree in figure (5.3) is an example of the type of illicit syntactic analysis that generates two clausal structures involving tall’s positive form through the derivational process. As with Larson (1988), notice that at no point in my analysis in (5.26) do I generate a
sentence involving the positive form of an adjective.

**Comparative deletion**

From a lexical semantic perspective, the entry for the comparative morpheme `-er` in (5.23) is looking for two (possibly distinct) adjectival meanings, and consequently, is general enough to handle cases like (5.25). But what about cases like (5.30) in which the second adjective appears to have been deleted?

\[(5.30) \quad \text{Orcutt is taller than Smith is tall}\]

Examples like (5.30) are thought to involve some form of ellipsis, aptly named **comparative ellipsis**. (See, for example, Bresnan (1973) for the *locus classicus* syntactic analysis of comparative constructions; but also Lechner (2004) for an extensive overview of comparative deletion/ellipsis.) In order to capture the syntax and semantics of statements, I choose not to rely on an elliptical mechanism, but rather utilize Barker’s (2013) continuation mode as laid out in §3.3. The derivation of (5.30) is shown in figure 5.4.
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Figure 5.4: Derivation of a clausal comparative using continuations
Inverting the proof tree, we again have the linguist’s tree represented in (5.31).

(5.31)

From a syntactic perspective, in words, this derivation can be understood as follows.

1. First, the comparative morpheme -er combines with the phonetically null adjectival expression -∅ on its left, yielding the comparative adjective -∅ er.

2. In parallel, the verb is combines with with the phonetically null adjectival expression -∅ on its right, yielding the complex expression is -∅ of category NP/S.

3. The complex expression is -∅ combines with the noun phrase Smith on its left, yielding the sentence Smith is -∅.

4. This expression is subsequently lifted via the introduction rule /I, resulting in the
expression *Smith is* of category S/A. The null expression is discharged and plays no role in the remainder of the derivation.

5. The preposition *than* combines with the noun-like phrase *Smith is* on its right, yielding the prepositional phrase *than Smith is*.

6. The comparative adjective *∅ er* combines with the prepositional phrase *than Smith is*, yielding the (complex) expression *is ∅ er than Smith is*.

7. This complex expression then combines with the phonetically null adjectival expression *∅* on its right, yielding the complex expression *∅ er than Smith is ∅*.

8. The verb *is* combines with this expression on its right, yielding the intransitive verb *is ∅ er than Smith is ∅*.

9. The noun phrase *Orcutt* combines with the the complex expression *is ∅ er than Smith is ∅* to yield the sentence

\[ \text{Orcutt is ∅ er than Smith is ∅}. \]

10. The phonetically null expression introduced in step (3) is continuized. The effect of this is that the expression *Orcutt is ∅ er than Smith is ∅* is subsequently lifted via the introduction rule \( I \), resulting in the expression \( \lambda X (Orcutt is X er than Smith is ∅) \) of category A\( \Lambda \) S.

11. The phonetically null expression introduced in step (8) is continuized. The effect of this is that the expression \( \lambda X (Orcutt is X er than Smith is ∅) \) is subsequently lifted via the introduction rule \( I \), resulting in the expression

\[ \lambda Y (\lambda X (Orcutt is X er than Smith is Y)) \] of category A\( \Lambda \) (A\( \Lambda \) S).

12. The expression COMPGAP combines with the expression

\[ \lambda Y (\lambda X (Orcutt is X er than Smith is Y)) \], yielding the expression

\[ \text{COMPGAP} (\lambda Y (\lambda X (Orcutt is X er than Smith is Y))) \] of category A\( \Lambda \) S.

13. This resultant expression combines with the adjective *tall*, yielding the expression

\[ \text{tall} (\text{COMPGAP} (\lambda Y (\lambda X (Orcutt is X er than Smith is Y)))) \] of category S.
14. In order to capture word order facts, this expression may be $\beta$-reduced as follows. (This $\beta$-reduction is represented graphically in (5.31) via the dashed arrows.)

\[
\text{tall} \left( \text{COMPGAP} (\lambda Y (\lambda X (\text{Orcutt is } X \text{ er than Smith is } Y))) \right) \\
\equiv \text{tall} (\lambda X (\text{Orcutt is } X \text{ er than Smith is COMPGAP})) \\
\equiv \text{Orcutt is taller than Smith is COMPGAP}
\]

From a lexical perspective, the innovation here is assigning COMPGAP the semantics of the reduplicator combinator $\lambda \kappa \lambda P (\kappa PP)$. This combinator has the effect of transforming a comparative meaning looking for two adjectival meanings into a comparative meaning looking for one. The relevant portion of the derivation in figure 5.4 is shown below, making explicit the relevant $\beta$-reductions.

(5.32)

\[
\text{tall} (\lambda \kappa \lambda P (\kappa PP) (\lambda P' \lambda P \lambda w (\Delta (P, P') (o) (w) (s) (w)))) \\
\equiv \text{tall} (\lambda P (\lambda P' \lambda P \lambda w (\Delta (P, P') (o) (w) (s) (w))PP)) \\
\equiv \text{tall} (\lambda P (\lambda w (\Delta (P, P) (o) (w) (s) (w)))) \\
\equiv \lambda w (\Delta (\text{tall, tall}) (o) (w) (s) (w))
\]

Stepping back, my analysis captures comparative deletion by trading in traditional elliptical mechanisms for (i) continuations and (ii) the assumption of the existence of the phonetically null lexical item COMPGAP.

**Limitations to continuations as comparative deletion**

There are inherent limitations to my analysis. As I mentioned previously, comparative deletion is mandatory. However, continuations are not. To better understand what I mean, notice that I have no way to block the generation of the ungrammatical statement in (5.33) by my grammar.

(5.33)  \#Orcutt is taller than Smith is tall
5.2. CLAUSAL COMPARATIVES

To see this, note that, instead of continuizing as in (5.31), it is possible to opt for a direct analysis of comparative deletion analogous to the sub-deletion derivation shown in (5.26). Such an analysis would result in the linguist’s tree in (5.34), which shows that (5.33) is derivable by my grammar.

This is an unwelcome result, as it shows that my grammar over-generates strings; and I see no obvious way to derive the obligatory use of continuations from first principles so that comparative deletion itself becomes obligatory. However, Barker and Shan (2013; chapter 18) observe that continuations in general have the problem of over-generation and offer some potential ways to constrain them. However, such technical innovations outrun the scope of this dissertation; and for now, I will live with the shortcomings of my grammar.
5.3 TRANS-WORLD SCALAR DIMENSIONS

Up until this point, I have explicated how the syntactic mechanisms of my natural and semantic representation language analysis work. However, I have not elucidated what my semantic representations mean, at least not in the Lewisian sense of that term (Lewis 1970).

In this section, I will provide a revision of the model of scalar dimensions first presented in §4.3.2. This revision will ultimately allow me to capture this idea of trans-world comparisons. Throughout the course of this section, it should become clear why I have traded Montague’s (1974c) intensional logic and Gallin’s (1975) TY₂ for Muskens’s (1995) TT₂: modeling a trans-world scale is far more perspicuous when working over relational frames as opposed to functional ones.

As in the case of §4.3.2, it should be understood from the outset that it is my intention to provide a framework for the formalization for scalar dimensions, providing only minimum mathematical requirements on the structure of scalar dimensions generally. Although I will attempt to give a full blown characterization of the scalar dimension of HEIGHT, such characterizations of scalar dimensions in general will necessarily draw from the hard sciences, including the cognitive sciences, as well as psychology. I turn, now, to the exposition of the semantic representation of (5.32), repeated in (5.35).

(5.35) Orcutt is taller than Smith \( \rightsquigarrow \lambda w (\Delta(\text{tall}, \text{tall}) (o) (w) (s) (w)) \)

Here, the value of \( \Delta \) is a function and will play a central role in my semantic analysis of
comparatives over the course of this chapter. Specifically, the value of $\Delta$ is a function that takes, as arguments, the values of two adjectives both of which are semantic type $(es)$. The value of $\Delta$ returns a quaternary relation of type $(eses)$. This is shown pictorially in figure 5.5. Importantly, this relation is to be understood as an ordering over individual/index pairs; or rather, what can be thought of, in many instances at least, as a trans-world scale. This ordering itself is identified, at least in part, with the scalar dimension those adjectives encode for. In definition 26, I revise my model of a scalar dimensions from the original definition given in 22.

Abusing notation, I will often write ‘$x @ w \succ_S y @ u$’, which is to be read $x$ has more $S$ at index $w$ than $y$ does at index $u$. Again, $\succ_S$ and $\prec_S$ are order duals; and $\equiv_S$ will be referred to as the indifference relation of $S$. In fact, none of the basics of scalar dimensions outlined in §4.3 will change in any fundamental way given this revised notion of a scalar dimension. The only difference, here, is that I am now working with quaternary relations as opposed to ternary ones. All previous definitions can be augmented accordingly.

**Definition 26** (Scalar dimension (Revised version 2)). Let a scalar dimension $S$ be the tuple $S = (\mathcal{A}, \mathcal{P})$. Here, $\mathcal{A} \subseteq D_{(es)}$ is a system of sets referred to as $S$’s adjectival component, whose elements correspond to adjectival meanings associated with $S$. $\mathcal{P} \subseteq D_{(eses)}$ is also a system of sets referred to as $S$’s scalar component and whose elements include at least the quaternary relations $\succ_S$, $\prec_S$, $\equiv_S$, and $\preceq_S$ such that for all $x, y, z \in D_e$ and $u, v, w \in D_s$.

**Trans-world axioms:**

1. $\neg (x @ w \prec_S x @ w)$ \hspace{1cm} *Irreflexivity*
2. $(x @ w \succ_S y @ u) \leftrightarrow (y @ u \prec_S x @ w)$ \hspace{1cm} *Duality*
3. $x @ w \equiv_S x @ w$ \hspace{1cm} *Reflexivity*
4. $(x @ w \equiv_S y @ u) \rightarrow (y @ u \equiv_S x @ w)$ \hspace{1cm} *Symmetry*
5. $(x @ w \equiv_S y @ u) \land (y @ u \equiv_S z @ v) \rightarrow (x @ w \equiv_S z @ v)$ \hspace{1cm} *Transitivity*
6. $(x @ w \succ_S y @ u) \land (x @ w \equiv_S z @ u) \rightarrow (z @ v \succ_S y @ w)$
7. $(x @ w \succ_S y @ u) \leftrightarrow (x @ w \succ_S y @ u) \lor (x @ w \equiv_S y @ u)$
8. \((x \ @ \ w \leq_{\Theta} y \ @ \ u) \leftrightarrow (x \ @ \ w \prec_{\Theta} y \ @ \ u) \lor (x \ @ \ w \equiv_{\Theta} y \ @ \ u)\)

To see how such a revision works in practice, let’s consider my definition for the scalar dimension of \textit{HEIGHT} first shown in definition 19 and revised in definition 22. I revise it again in definition 27.

**Definition 27 (Scalar dimension of \textit{HEIGHT} (Revised version 2)).** Let a scalar dimension of \textit{HEIGHT} be the tuple \(H = (\mathcal{A}, \{\succ_{H}, \prec_{H}, \preceq_{S}, \succeq_{S}, \equiv_{S}\})\) such that \(\succ_{H}, \prec_{H}, \preceq_{S}, \succeq_{S}, \equiv_{S}\) obey axioms (1)–(8) as given by definition 26. Moreover, for all \(x, y, z \in D_e\), and \(u, v, w \in D_s\) the relation \(\succ_{H}\) obeys the axioms in (1)–(2).

1. \((x \ @ \ w \succ_{H} y \ @ \ u) \land (y \ @ \ u \succ_{H} z \ @ \ v) \rightarrow (x \ @ \ w \succ_{H} z \ @ \ v)\) \hspace{1cm} \text{Transitivity}

2. \((x \ @ \ w \succ_{H} y \ @ \ u) \lor (y \ @ \ u \succ_{H} x \ @ \ w) \lor (x \ @ \ w \equiv_{H} y \ @ \ u)\) \hspace{1cm} \text{Totality}

Of course, a generalized version of duality with respect to the scalar dimension of \textit{HEIGHT} holds in this setting.

**Lemma 3 (Duality).** The following statements hold for all \(\mathcal{M}\) satisfying axioms (1)–(5) of definition 26 and axioms (1)–(2) of definition 27, individuals \(x, y, z \in D_e\), and indices \(u, v, w \in D_s\)

1. \(\neg (x \ @ \ w \prec_{H} x \ @ \ w)\) \hspace{1cm} \text{Irreflexivity}

2. \((x \ @ \ w \prec_{H} y \ @ \ u) \land (y \ @ \ u \prec_{H} z \ @ \ v) \rightarrow (x \ @ \ u \prec_{H} z \ @ \ v)\) \hspace{1cm} \text{Transitivity}

3. \((x \ @ \ w \prec_{H} y \ @ \ u) \lor (y \ @ \ u \prec_{H} x \ @ \ w) \lor (x \ @ \ w \equiv_{H} y \ @ \ u)\) \hspace{1cm} \text{Totality}

4. \((x \ @ \ w \prec_{\Theta} y \ @ \ u) \land (x \ @ \ w \equiv_{\Theta} z \ @ \ v) \rightarrow (z \ @ \ v \prec_{\Theta} y \ @ \ u)\)

It is easy to show that the relation \(\succ_{H}\), and consequently \(\prec_{H}\), are strict weak orders, thus codifying Cresswell’s (1976) notion of a trans-world scale. (See, again, definition 20 for the idea of a strict weak order.)

**Lemma 4.** For all individuals \(x, y \in D_e\) and indices \(u, v, w \in D_s\), the relation \(\succ_{H}\), as given by definition 27, is a strict weak order over \(D_e \times D_s \times D_e \times D_s\).
5.3. TRANS-WORLD SCALAR DIMENSIONS

Proof. Irreflexivity and transitivity are immediate per definition 27. For the almost connected property, suppose that \( x @ w \succ_{\mathcal{J}} y @ u \). Consider an arbitrary individual \( z \) and index \( v \). By the totality condition, there are three cases to consider.

- If \( x @ w \succ_{\mathcal{J}} z @ v \), then the proof is completed.
- If \( z @ v \succ_{\mathcal{J}} x @ w \), given that \( x @ w \succ_{\mathcal{J}} y @ u \), by transitivity, \( z @ v \succ_{\mathcal{J}} y @ u \), and the proof is again complete.
- If \( x @ w \models_{\mathcal{J}} z @ v \), given that \( x @ w \succ_{\mathcal{J}} y @ u \), by axiom (5) of definition 26, it follows that \( z @ v \succ_{\mathcal{J}} y @ u \). So, the proof is completed.

\[ \square \]

For a visual aid of the a trans-world scale, consider example 2.

Example 2. Let \( M = (\mathcal{R}, I) \) be, at least in part, such that

1. \( D_e = \{o, s, j\} \);
2. \( D_s = \{u, v, w\} \); and
3.

\[ \mathcal{J} = \{\text{[tall]}, \text{[short]}\}, \{\langle o, w, s, u \rangle, \langle s, u, j, v \rangle, \langle o, w, j, v \rangle \}, \{\langle s, u, o, w \rangle, \langle j, v, s, u \rangle, \langle j, v, o, w \rangle \}, \emptyset \} \]

Here,

- \[ \Delta \] will associate the pair \[ \text{[tall, tall]} \] with the set
  \[ \{\langle o, w, s, u \rangle, \langle s, u, j, v \rangle, \langle o, w, j, v \rangle \} \]

- \[ \Delta \] will associate the pair \[ \text{[short, short]} \] with the set
  \[ \{\langle s, u, o, w \rangle, \langle j, v, s, u \rangle, \langle j, v, o, w \rangle \} \]
So, in this model,

- Orcutt at index \( w \) has more height than Smith at index \( u \), who in turn has more height than Jones at index \( v \).

- Similarly, Jones at index \( v \) has less height than Smith at index \( u \), who in turn has less height than Orcutt at index \( w \).

- And no one at any world is as tall as anyone else anywhere.

The scalar dimension of \( \mathsf{HEIGHT} \), as shown in (3) can be represented graphically as in figure 5.6. Here, we have have two scales, whose edges represent the relations \( \mathsf{HAVING \ MORE \ OR \ LESS \ HEIGHT \ THAN} \) and whose elements are (equivalence classes of) entity/index pairs, not degrees.

\[
\langle o, w \rangle \leftrightarrow \langle s, u \rangle \leftrightarrow \langle j, v \rangle \quad \langle j, v \rangle \rightarrow \langle s, u \rangle \rightarrow \langle o, w \rangle
\]

\( \mathsf{has \ more \ height \ than} \) \quad \mathsf{has \ less \ height \ than} \n
Figure 5.6: The scalar dimension of \( \mathsf{HEIGHT} \) as a trans-world scale

Of course, revising the notion of a scalar dimension is going to cause not only the sort of formal revisions given in §5.2.3 of this chapter, but also revisions to the formal theory laid out in chapter 4. I will address this point in more detail in the next section.

5.4 Deriving phrasal comparatives from clausal ones

Now we can begin to make some headway in understanding the semantic representations in (5.36a) and (5.36b).
5.4. DERIVING PHRASAL COMPARATIVES FROM CLAUSAL ONES

(5.36)  a. Orcutt is taller than Smith is \[\sim\]
\[\hat{\lambda}w(\Delta(\text{tall}, \text{tall}) (o)(w)(s)(w)) = \{w \in D_s | o \@ w \succ \_s @ w\}\]

b. Orcutt is shorter than Smith is \[\sim\]
\[\hat{\lambda}w(\Delta(\text{short}, \text{short}) (o)(w)(s)(w)) = \{w \in D_s | o \@ w \prec \_s @ w\}\]

In order to come to terms with what the semantic representations in (5.36a) and (5.36b) mean in the Lewisian sense, I need a theory of the behavior of the function denoted by \(\Delta\): I need to be precise in regard to the sorts of orders \(\Delta\) maps adjectival meanings to. This is because \(\Delta\) takes two parameters \(P\) and \(P'\), which as we saw in the case of comparative sub-deletion in figure 5.2, are not necessarily identical to each other. I will leave cases where \(P\) and \(P'\) differ from each other, i.e., inter-adjectival comparatives, as they are quite complicated and involve instances of cross-polar (a)nomaly, comparisons of deviation, and indirect comparatives, examples of which are shown in (5.15)–(5.17), for future work (Kennedy (1997), Kennedy (2001), Sassoon (2007), Büring (2007), Bale (2008), Bale (2011) and van Rooij (2011). But see also Cresswell (1976) and Bierwisch (1989) for historical perspectives on these issues.)

In example 2, I stipulated that the value \(\Delta\) associates the pair \([\text{tall}, \text{tall}]\) with the quaternary relation \(\succ \_s\), which intuitively is read as has more height than. (Similar reasoning applies in the case where the value of \(\Delta\) takes \([\text{short}, \text{short}]\).) Consequently, (5.36a) denotes the set of indices \(w\) in which Orcutt has more height in \(w\) than Smith does in \(w\). Similarly, (5.36b) denotes the set of all indices \(w\) in which Orcutt has less height in \(w\) than Smith does in \(w\).

This seems to capture the meaning of these statements. However, it leads to both a formal and conceptual problem. The formal problem is this. The data in (5.37) suggest that the phrasal comparative in (5.37a) and clausal one in (5.37b) are logically equivalent.

(5.37)  a. Orcutt is taller than Smith
b. \(\Leftrightarrow\) Orcutt is taller than Smith is
However, my analysis currently has no way of capturing this fact. To see this, reconsider the set of indices in (4.18) repeated below in (5.38), which itself corresponds to the meaning that my original analysis given in §4.3.3 predicts for the phrasal comparative in (5.37a).

(5.38)

Orcutt is taller than Smith \rightarrow

\[ \delta(tall)(o)(s) = \{ w \in D_s \mid o >_s s @ w \} \]

There is nothing about the TT\textsubscript{2} models I am considering that force the set of indices in (5.38) to be identical with the one in (5.36a). However, if we take the data in (5.37) seriously, they should. Stated another way, in order to formally capture the validity of (5.37), the ternary relation $>_s$ HAS MORE HEIGHT THAN, which is the relation the value of $\delta$ associates the value of tall with, should be related to the quaternary relation $\succ>_s$ HAS MORE HEIGHT THAN, which is relation the value of $\Delta$ associates the values of tall and tall with.

Of course, this can be done by brute force by adopting the following axiom in (5.39).

(5.39)

$\delta(P)(x)(y)(w) \leftrightarrow \Delta(P,P)(x)(w)(y)(w)$

for all $x, y \in D_e, w \in D_s$ and $P \in D_{(es)}$

The second-order statement in (5.39) forces the statements in (5.37) to come out as being logically equivalent under my analysis. However, in adopting such a axiom, there would be a certain level of redundancy in my analysis. Why assume the existence of two delta functions and two relations HAS MORE HEIGHT THAN—a ternary $>_s$ and quaternary one $\succ>_s$?

Perhaps more importantly, from a linguistic perspective, why assume the existence of two homophonous comparative morphemes—one that derives phrasal comparatives as in (5.20), and one that derives clausal ones as in (5.23)? In the Gricean spirit, I think it is good practice to avoid multiplying “senses beyond necessity”. (Even though I have assumed homophonous forms be and than!) In fact, this is the worry Heim (2006; p. 9) has in regard to generalizing Larson’s (1988) original proposal to account for clausal comparatives: “The difficulty I see (just to hint at it) is that one seems to need entries for comparative adjectives . . . which manipulate not just one time/world parameter but two.”
We can do better. How exactly? By rejecting my analysis of phrasal comparatives outlined in chapter 4, thus replacing the value of $\delta$ and the ternary relations it associates adjectival meanings with by the value of $\Delta$ and the quaternary relations it associates adjectival meanings with. Specifically, I will follow a program sketched in Heim (1985) and derive phrasal comparatives from clausal ones.

My new analysis of a phrasal comparative like (5.37a) now mirrors my analysis of their clausal kin like (5.37b). The only difference is that I fall back on the use of the preposition than of syntactic category $p_P$ instead of the homophonous preposition than of syntactic category $p_C$. The semantic representation of a phrasal comparative is shown in (5.40a); its linguist’s tree in (5.40b); and its Gentzen/Prawitz-style derivation is shown in figure 5.7.
Figure 5.7: Derivation of a phrasal comparative from a clausal one
5.4. DERIVING PHRASAL COMPARATIVES FROM CLAUSAL ONES

(5.40) a. Orcutt is taller than Smith $\sim$

\[ \lambda w (\Delta \text{tall}, \text{tall}) (o) (w) (s) (w)) = \{ w \in D_s | o \uparrow w \succ \downarrow s \uparrow w \} \]

b. Notice that the \textit{than} of syntactic category \textsc{pp} can combine directly with \textit{Smith} as in (5.40b), resulting in a phrasal comparative. On the other hand, \textit{than} of syntactic category \textsc{pc} combines with the nominal-like expression \textit{Smith is}, resulting in a clausal comparative. There is no need to assume the existence of two comparative morphemes. I only need the one defined in (5.23), not the one defined in (5.20).
We might take pause: as it stands, I have assumed three homophonous forms of the verb *be*—two in this chapter alone. This is ultimately a function of the fact that I have generalized my definition of the comparative morpheme, which itself has important syntactico-semantic consequences. To see what I mean, consider, first, the three forms of *be* I have assumed through the course of this dissertation, all of which are shown in (5.41)–(5.43).

\[(5.41)\] is: \(\ IV / A : \lambda P (P)\)

*Chapter 4 form of 'be'*

\[(5.42)\] is: \(\ IV / (I\ A) : \lambda C \lambda x \lambda w (Cwxw)\)

*Chapter 5 form of 'be'*

\[(5.43)\] is: \(\ (NP \ S) / A : \lambda P \lambda Q \lambda w (QP (w))\)

*Chapter 5 form of 'be'*

Although (5.41) and (5.43) appear to be different, they are really nothing more than the predicative form of *be*—the latter being a sort of ‘lifted’ representation of it. So, we may as well reduce them to one instance, say the latter. A little inspection will reveal substituting (5.43) for (5.41) throughout the course of chapter 4 would change nothing.

The real issue is the form of *be* shown in (5.42). I have assumed this form of *be* to account for the revised lexical entry of the comparative morpheme *-er*. This is because, from a semantic perspective, this revised form, at least in part, denotes a quaternary relation between an individual and index on the one hand and a (possibly) different individual and index on the other; this is different from my original entry for *-er*, which only included a single index. Consequently, I cannot combine the complex expressions the morpheme is involved in with the form of *be* as it is shown in (5.43).

From a syntactic perspective, the ramifications of this modeling decision are made clearer by investigating (partial) derivations of a phrasal comparative per the theory laid out in chapter 4 versus the one laid out in chapter 5. These derivations are shown in (5.44) and (5.45) respectively.
Observe that, in (5.44), the expression *taller than Smith* is an adjective of syntactic type $A$. This is as it should be, as the following coordination evidence suggests that this expression can be coordinated with the adjective *happy*.

(5.46) \[
[IV \{IV/A \text{ is } ] \left[ A \left[ _A \text{ taller than Smith } \right] \text{ and } [ A \text{ happy } \right] ] \]
\]
However, in (5.45), the expression ∅ er than Smith ∅ of syntactic type I\A cannot obviously combine with the be as it is defined (5.41).

Observe that the syntactic category I\A has an adjectival component to it; however, it itself is not adjectival. Thus, if we were to adopt the theory of phrasal comparatives in this chapter, rejecting the analysis laid out in chapter 4, we would seemingly not be able account for basic coordination facts like (5.46), which rely on treating the complex expression taller than . . . as adjectival.

However, there is a way out, and I can account for the coordination facts in (5.46). I do this by deploying the now familiar type-logical trick I have utilized throughout the course of this dissertation. To see this, consider the (partial) derivation in (5.47).

(5.47)

So, while the theory laid out, here, does require of us to assume an extra form of be, it is general enough to account for the coordination facts in (5.46). It’s important to point out, however, that if we are to adopt this analysis of phrasal comparatives, the theory laid out in chapter 4, namely the lexical entry for differential negation in 4.88 is going to have to be revised to account for the type-change of the comparative morpheme -er/more reflected in table 5.1 and examples (5.23) and (5.24). However, this can be accomplished without much ado.
5.4.1 Certain syntactico-semantic limitations

Again, there are inherent limitations to my analysis: my grammar over-generates. The linguist’s tree in (5.48c) shows that my grammar derives the ungrammatical string in (5.48a) with predicted meaning in (5.48b).

(5.48)  

\begin{enumerate} 
\item #Orcutt is taller than Smith wide
\item $\Leftrightarrow$ Orcutt is taller than Smith is wide
\item 
\end{enumerate}

This is an unwelcome result. Notice that continuations themselves are not to blame for the inadequacies of my grammar. Rather, it is due to my lexical entry for the comparative morpheme in (5.23): the fact that it is looking to combine with two adjectives independently, as opposed to one, results in the generation of strings like (5.48a). As of now, there is no way to force be to be present when the adjective of the than clause is realized. So, there is an empirical cost in the quest for complete generality: in an attempt to derive phrasal comparatives from clausal ones, my grammar over-generates.
5.5 Modality

I turn back to the sort of clausal comparatives that make overt trans-world comparisons like ones in (5.15), repeated below in (5.49).

(5.49)  
   a. Orcutt is taller than Smith might be  
   b. Orcutt was taller than Smith is  
   c. Orcutt might be taller than Smith was

Recall that the three examples are representative of three types of trans-indice comparisons.

- **Type 1**: Statements like (5.49a) manipulate the index solely of the ‘clause’ under the scope of *than*.
- **Type 2**: Statements like (5.49b) manipulate the index solely of the ‘clause’ not under the scope of *than*.
- **Type 3**: Statements like (5.49c) manipulate both indices.

5.5.1 Type 1 trans-world comparisons

I begin, first, with my analysis of type 1 modal statements. I will assume the existence of a non-logical constant $R$ of semantic type $(ss)$, whose value is understood as being a binary accessibility relation over $D_s$. $R$ itself is assumed to be reflexive, symmetric, and transitive.$^2$ It is natural to understand $R$ as being a METAPHYSICAL accessibility relation. Atomic statements of the form ‘$Rwu$’ will be read *index u is R*, or rather *metaphysically accessible from w*. Now, I am simplifying things a bit by treating *might be* as a complex lexical expression. I assume its lexical entry to be as defined (5.50).

(5.50)  

\[
\text{Lexical entry — } \quad \text{might be : } (\text{NP} \setminus S) / \lambda : \lambda P \lambda Q \lambda w \exists u (Rwu \land QPu)
\]

$^2$Specifically, $R$ satisfies the standard S5 axioms and denotes an equivalence relation over $D_s$. 
Figure 5.8: Derivation of a type 1 clausal comparative involving a modal under the scope of *than*
The derivation of (5.51a) is shown in figure 5.8; and its semantic representation and linguist’s tree is given in (5.51a) and (5.51b) respectively.

\[(5.51)\]

a. Orcutt is taller than Smith might be
\[\sim \lambda w \exists u (Rwu \land \Delta (\text{tall, tall})(w)(s)(w)) = \left\{ w \in D_s \mid \exists u (Rwu \land o \land w \succ_s s \land u) \right\}\]
To begin with, this statement involves the tensed expressions *was* and *will be*. As it stands, I have no formal theory of time. So far, I have been treated Montagovian indices as structureless, basic entities. In order to account for the meaning of examples like (5.52), I will have to remedy that. Following Muskens (1995; p. 11), I introduce the non-logical constants ‘≈’ and ‘≺’ of type (ss). The statement ‘\(w \approx u\)’ will be read *w and u have the same world component* and the statement ‘\(w ≺ u\)’ will read *w’s temporal component precedes u’s*. We can now make \(D_s\) behave as the Cartesian product of two sets, the second of which is a linear order—the sort of structure necessary for a (rudimentary) theory of time.

**Definition 28** (Temporal behavior of Montagovian indices). Let \(≈\) and \(≺\) be non-logical constants of type (ss) such that for all \(w, u, v \in D_s\):

**World/time axioms:**

1. \(w \approx w\) \hspace{1cm} \text{Reflexivity}
2. \(w \approx u \rightarrow u \approx w\) \hspace{1cm} \text{Symmetry}
3. \((w \approx u) \land (u \approx v) \rightarrow (w \approx v)\) \hspace{1cm} \text{Transitivity}
4. \(\neg(w ≺ w)\) \hspace{1cm} \text{Irreflexivity}
5. \((w ≺ u) \land (u ≺ v) \rightarrow (w ≺ v)\) \hspace{1cm} \text{Transitivity}
6. \((w ≺ u) \rightarrow (w ≺ v) \lor (v ≺ u)\) \hspace{1cm} \text{Almost connected}
7. \(\exists v ((w \approx v) \land \neg(u ≺ v) \land \neg(v ≺ u))\)
8. \((w \approx u) \land \neg(w ≺ u) \land \neg(u ≺ w) \rightarrow (w = u)\)

Now, let me assume the lexical entries for *was* and *will be* as they are defined in (5.53) and (5.54) respectively.
Importantly, the lexical entry for *will be* will manipulate the index of the ‘clause’ under the scope of *than*. Using the conjunction rule laid out in definition 24, it is now easy to account for the syntax and semantics of (5.55). The statement’s semantic and syntactic representations are shown in (5.55a) and (5.55b) respectively. A partial derivation is shown in figure 5.9.
Figure 5.9: Partial derivation of a type 1 clausal comparative with conjoined tensed clauses under the scope of than
5.5. MODALITY

(5.55)  a.

Orcutt is taller than Smith was and Jones will be

\[ \lambda w (\exists u (u < w \land \Delta (\text{tall, tall}) (o) (w) (s) (u)) \land \exists v (w < v \land \Delta (\text{tall, tall}) (o) (w) (j) (v))) \] =
\{ w \in D_s \mid \exists u (u < w \land o @ w \succ \delta s @ u) \land \exists v (w < v \land o @ w \succ \delta s @ v) \}

b.

The semantic representation in (5.55a) denotes the set of all indices \( w \) such that there exists an index \( u \) that temporally precedes \( w \) and Orcutt’s height in \( w \) exceeds that of Smith’s in \( u \), and there exists an index \( v \) that temporally proceeds \( w \) and Orcutt’s height in \( w \) exceeds that of Jones’ in \( v \).
5.5. MODALITY

5.5.2 Type 2 trans-world comparisons

As mentioned, I assumed the existence of two homophonous forms of be. I will now show the utility of that decision. In order to account for a type 2 statement like 5.56, I assume the existence of homophonous forms of its tensed variants.\(^3\)

(5.56) Orcutt was taller than Smith is

In order to capture the semantics of (5.56), I assume the existence of the tensed form of be in (5.57).

(5.57)

\[
\text{Lexical entry —}
\]

\[
\begin{align*}
\text{was} & : IV / (I \Downarrow A) : \lambda C \lambda x \lambda w \exists u (u \prec w \land C u x w)
\end{align*}
\]

The lexical entry in (5.57) will manipulate the index of the ‘clause’ not under the scope of than. To better understand this, consider the semantic representation of (5.56) in (5.58a); its linguist’s tree in (5.58b); and its Gentzen/Prawitz-style derivation in figure 5.10.

\(^3\)Again, this, of course, is not the only strategy here. Partee (2002), for example, assumes a base form of be and derives its various meanings by the additional assumptions of type-shifters. For purposes of this chapter, I will trade formal and empirical parsimony for conceptual clarity.
Figure 5.10: Derivation of a type 2 clausal comparative using continuations
(5.58)  a.

Orcutt was taller than Smith is \(\sim\)

\[
\left[ \lambda w \exists v (v < w \land \Delta(\text{tall, tall})(o)(v)(s)(w)) \right] = \\
\{ w \in D_s \mid v < w \land o \bowtie v \succ s \bowtie w \}
\]

b.

The semantic representation in (5.58a) denotes the set of all indices \(w\) such that there exists an index \(v\) that temporally precedes \(w\) and Orcutt’s height in \(v\) exceeds Smith’s height in \(w\).

5.5.3 Type 3 trans-world comparisons

As in the case of chapter 4, derivations have become mechanical. It is unsurprising, then, that accounting for type 3 trans-world comparisons is merely a matter of combining the insights of §5.5.2 and §5.5.1. Consider, again, the type 3 comparative in (5.59).
(5.59) Orcutt was taller than Smith might be

Utilizing the lexical entries for *was* found in (5.57) and *might be* in (5.50), the semantic representation of (5.59) is given in (5.60a); its linguist’s tree in (5.60b). (I forgo the Gentzen/Prawitz derivation, as it is simply a matter of combining the relevant sub-proofs shown in figures 5.8 and 5.10.)
5.5. MODALITY

(5.60) a.

Orcutt was taller than Smith might be ~

\[ \lambda w \exists v \exists u (v < w \wedge Rw u \wedge \Delta (\text{tall, tall})(o)(v)(s)(u)) \]

\[ \{ w \in D_s \mid \exists v \exists u (v < w \wedge Rw u \wedge o \otimes v \gg s \otimes s \otimes u) \} \]

b.

The semantic representation in (5.60a) denotes the set of all indices \( w \) such that there exists an index \( u \) accessible from \( w \) such that Orcutt’s height in \( u \) exceeds that of Smith’s height in some accessible index \( v \) that itself temporally precedes \( w \).
5.5. MODALITY

5.5.4 The Russell ambiguity

Introducing modal and temporal claims into the system laid out here introduces interesting problems. Let me begin this section by considering the statement in (5.61)—a variant of a statement considered by Russell (1905).

(5.61) Orcutt believes Smith₁ is taller than he is

*Russell ambiguity*

Observe that this statement is ambiguous between (at least) two readings. On the one hand, (5.61) asserts that Orcutt believes a (physical) contradiction, namely the one in (5.62).

(5.62) In the world Orcutt₁ believes himself₁ to be, Smith₂ is taller in that (same) world than he₂ is in that (same) world

*Russell ambiguity: contradictory reading*

Of course Smith cannot be taller than himself in the same world—no one can be. It is a physical impossibility. This fact is appropriately modeled in my formal theory of the scalar dimension of HEIGHT presented in definition 27, namely by inheriting the irreflexivity axiom given in definition 26.

However, there is a non-contradictory reading of (5.61), namely the one in (5.63).

(5.63) In the world Orcutt₁ believes himself₁ to be, Smith₂ is taller in that (same) world than he₂ is in (a possibly different) world

*Russell ambiguity: non-contradictory reading*

It is easy to come up with a situation in which (5.63) is non-contradictory. Suppose, for example, that, in the world Orcut takes himself (and Smith to be), Smith is 5′10″, when in reality Smith is only 5′9″. In such a situation, Orcutt would simply be confused, and (5.63) would be true.

In the logic TT₂, it is easy to represent both readings. I begin, first, with the contradictory reading in (5.62).
5.5. MODALITY

(5.64)

Orcutt believes Smith$_1$ is taller than he is $\sim$

\[
[\lambda w \forall v (B (o) (w) (v) \to \Delta (\text{tall, tall}) (v) (s) (v))]
\]

\[
\{ w \in D_s \mid \forall v (B (o) (w) (v) \to s \circ v \succ s \circ v) \}
\]

*Russell ambiguity: contradictory reading*

In (5.64), $B$ is understood as being a DOXASTIC accessibility relation. Atomic statements of the form ‘$Bxwu$’ will be read for individual $x$, index $u$ is $B$, or rather doxastically accessible from $w$. So, the statement in (5.64) denotes the set of indices $w$ such that for all indices $v$ doxastically accessible from $w$ per Orcutt’s belief state, Smith is taller in $v$ than Smith is in $v$. So, Orcutt believes a (physical) contradiction.

The question becomes: can I compositionally derive this semantic representation using the formal theory laid out so far? The answer is in the affirmative. To do this, I will fall back on the use of continuations and the adoption of the the lexical entry for *he* in (5.65).

(5.65)

**Lexical entry**

*he* : $IV/((NP) \downarrow IV) : \lambda \vartheta \lambda x \lambda w (\vartheta \lambda P \lambda u (P (x) (u)) xw)$

Note, here, that the semantic representation for *he* is taken from Barker (2013) and reflects the reduplicator semantics over individuals. It will provide our theory of anaphora. And finally, I must adopt the entry for *believe* as in (5.66).

(5.66)

**Lexical entry**

*believe* : $(N/S) / S : \lambda p \lambda z \lambda w \forall v (B (z) (w) (v) \to pv)$
I now have everything at my disposal to derive the semantic representation of the contradictory reading of the Russell ambiguity shown in (5.64). The Gentzen/Prawitz-style derivation for the contradictory reading of the Russell ambiguity is shown in figure 5.11.

Now, I want to turn to the reading which is much more difficult to derive compositionally: the non-contradictory reading shown in (5.63). It is clear what the semantic representation of this reading should be. It is given in (5.67).

(5.67)

\[
\text{Orcutt believes Smith}_1 \text{ is taller than he}_1 \text{ is } \sim \\
\left[ \lambda w \forall v (B(o)(w) \to \Delta(tall, tall)(s)(w)(s)(v)) \right] = \\
\{ w \in D_s \mid \forall v (B(o)(w) \to s @ v \succ_s s @ w) \}
\]


Russell ambiguity: non-contradictory reading

The statement in (5.67) denotes the set of indices \( w \) such that for all indices \( v \) doxastically accessible from \( w \) per Orcutt’s belief state, Smith is taller in \( v \) than Smith is in \( w \). In this case, Orcutt does not believe a contradiction; there will be \( \text{TT}_2 \) models that validate the atomic statement ‘\( s @ v \succ_s s @ w \)’, namely those in which \( w \) is not included in Orcutt’s doxastic belief worlds. It is much more difficult to derive this representation, but it can be done. A Gentzen/Prawitz-style derivation is shown figure 5.12.

5.6 Measures

Historically, one of the main motivations for moving to a degree-based semantics is data like that in (5.68).

(5.68)  
\begin{align*}
\text{a.} & \text{ John is six feet tall} \\
\text{b.} & \text{ He is that tall} \\
\text{c.} & \text{ How tall is he?} \\
\text{d.} & \text{ However tall he is, } \ldots
\end{align*}

Heim (2000; (1a)–(1d))
Figure 5.11: The contradictory reading of the Russell ambiguity
Figure 5.12: The non-contradictory reading of the Russell ambiguity
Assuming that the meaning of an expression just is the object to which it refers, the underlined expressions in (5.68) are taken as evidence that natural language makes reference to degrees. Given that languages like English do in fact make reference to degrees, many authors have concluded that degrees exist, at least in a theoretically useful sense. Reading off one’s ontology from natural language à la Lewis (1973) is a move very much rooted in the so-called Fregean linguistic turn in which traditional philosophical problems of say epistemology or ontology are transformed into linguistic ones so that they may be more easily solved (Heck 1999).

However, there are those like Klein (1980) that exhibit a sort of Quinean-skepticism in regard to the question of the existence of degrees and, if pressed, would undoubtedly admit that a bloated ontology which includes degrees “offends the aesthetic sense of us who have a taste for desert landscapes” (Quine 1953a). Whether degrees exist is a theory external question, in the sense of Carnap (1950). No doubt the existential burden of proof, here, lies squarely on the shoulders of the degree semanticist. I think a hefty dose of skepticism should be levied against the assumption of any extra bit of ontology; after all, it is the degree semanticist who is positing a new class of abstract objects into our ontology that we have little-to-no empirical intuitions about. However, the degree semanticist can defend the existence of degrees much in the same way Lewis (1973) defends the existence of possible worlds, namely by citing the fact that we, as competent speakers of a natural language like English, talk as if degrees existed.

A Quinean may retort to the degree semanticist, “[degrees] reside in the way in which we say things, and not in the things we talk about” (Quine 1953b; replacing ‘degrees’ for ‘necessity’). The Quinean will then try and find a suitable natural language paraphrase of the sentences in (5.68) and provide a semantic analysis thereof that doesn’t make reference to degrees. I think, though, at this point the (non-)degree semanticists find themselves at an impasse: the philosopher is worried about colloquial speech dictating ontological decisions, whereas the linguist is concerned about getting the linguistic facts right. I think the linguist can retort, “If language says degrees exist, then let’s meet it on its own terms and provide an analysis thereof.” This is the sentiment of Cresswell (1976; p. 281).

Must we postulate the kalon as a degree of beauty or the andron as a degree of manliness? Degrees of beauty may be all right for the purposes of illustration
but may seem objectionable in hard-core metaphysics . . . It is not, in my opinion, the business of logic or linguistics (at least syntax) to explain how it is that we make the comparisons that we do make or what the principles are by which we make them. But it is the business of linguistics and logic when used in the service of linguistics, to tell us how we put the comparisons we make into the linguistic forms into which we put them.

However, what is important, here, is the way in which measures are introduced and what exactly they’re being used for. As I will show, measures will only be relevant for accounting for the semantics of comparative constructions that involve explicit reference to measures, like the ones in (5.68)—no more and no less. Moreover, I will show how measures can be built on top of the semantics laid out in this chapter.

From a mathematical perspective, a measurement system can be understood, at least abstractly, as a mapping from a structure—an ordering or scale—to the real numbers $\mathbb{R}$. Measure theorists like Krantz et al. (1971) distinguish between (at least) three types of underlying scales: (i) ordinal scales; (ii) interval scales; and (iii) ratio scales. Again, from a mathematical perspective, what differentiates these types of scales is their structure, or rather what types of axioms those scales satisfy. Intuitively, the difference between these scale types is a matter of how articulate of a structure those scales reflect—ordinal scales being the least articulate and ratio scales being the most.

Authors like Sassoon (2007), Sassoon (2010), Lassiter (2011) and van Rooij (2011) have all argued that all three scale types are reflected at the level of natural language inference. That is to say, through the investigation of the types of (in)valid inferences comparative adjectives are involved in, we can be keyed into the type of scale a particular scalar dimension encodes for *qua* that comparative adjective. For example, Lassiter (2011), following Krantz et al. (1971), argues that the scalar dimension of *temperature* is to be understood as an interval scale; whereas *height* should be understood as a ratio scale. (I refer the reader to Lassiter (2011) for an excellent discussion of these types of scales, including their definitions, as well as their relation to natural language, particularly natural language inference.) Specifically, I will show how those properties can be incorporated naturally into the theory of scalar dimensions developed in §4.3 and subsequently revised in §5.3.
5.6. MEASURES

In this section, I will continue my investigation of the scalar dimension of HEIGHT, extending my formal theory of it by characterizing it as a ratio scale. So, this section should be seen as an extension of §4.3 and §5.3 in the sense that I show how one can extend the formal notion of a scalar dimension by adding a measurement system to it. Again, it should be understood that I am not making claims about scalar dimensions generally. Many scalar dimensions will be formalized differently: some might simply encode for an ordinal scale, while others might encode for an interval scale. And as authors like Bartsch and Vennemann (1972) and Bierwisch (1989) point out, many scalar dimensions do not have measurement systems attached to them. (But see Sassoon (2010) for an opposing view.)

5.6.1 Measure phrases

In this section, I will give a compositional semantics involving explicit reference to measures as in the following.

(5.69) a. Orcutt is five feet tall
     b. Orcutt is taller than five feet

Measure phrases

(5.70) Orcutt is two inches taller than Smith

Differential phrases

(5.71) Orcutt is two times taller than Smith

Factor phrases

As van Rooij (2011) points out, in order to capture the semantics of statements like (5.69)–(5.71), we need a formal theory of measurement that allows us to add (+) and multiply (×). Ratio scales provide such an articulate structure.

Definition 29 (Ratio scale). A triple \((X, R, \circ, \mu)\) is a ratio scale just in case \(X\) is a non-empty set, \(R\) is a binary relation over \(X\) such that for all \(x, x', y, y', z \in X\)

1. \(\neg (xRx)\)  \hspace{1cm} Irreflexivity
2. \((xRy) \land (yRz) \Rightarrow (xRz)\) \hspace{1cm} \text{Transitivity}

3. \((xRy) \lor (yRx) \lor (x = y)\) \hspace{1cm} \text{Connected}

And \(\circ\) is the \textbf{concatenation operation} such that

4. \(x \circ (y \circ z) \Leftrightarrow (x \circ y) \circ z\) \hspace{1cm} \text{Associativity}

5. \(x \circ (y \circ z) \Leftrightarrow (x \circ y) \circ z\) \hspace{1cm} \text{Monotonicity}

6. \(xRy \Rightarrow \exists n (nx \circ y' \circ Rny \circ x')\) \hspace{1cm} \text{Archimedean}

where \(nx\) is defined inductively as \(1x = x\) and \((n + 1)x = nx \circ x\). And \(g, f \in \mu\) are \textbf{associated measure function of} \(\mathcal{S}\) just in case

7. \(f : X \mapsto \mathbb{R}\)

8. \(f(x) > f(y) \Leftrightarrow xRy\) and ‘\(>\)’ is understood as the greater-than ordering on \(\mathbb{R}\)

9. \(g(x) = (\alpha \times f(x)) + \beta\) for some numbers \(\alpha, \beta \in \mathbb{R}\) such that \(\alpha \neq 0\) and \(\beta = 0\)

10. \(f(x \circ y) = f(x) \circ f(y)\) \hspace{1cm} \text{Additivity}

Intuitively, the concatenation operation \((\circ)\) is best understood as the process of laying two rods, for example, end-to-end, thus forming a new, longer, concatenated rod. In the case of a ratio scale, it is taken to be 0 which reflects the fact that measurement occurs from an absolute 0 point. In the case of an interval scale, no such 0 point is assumed. Compare, for example, the difference between the measurement systems attached to the scalar dimension of \textsc{temperature}, for example, versus that of \textsc{height}. In the case of the latter, measurement occurs from an absolute 0 point, whereas in the case of the former, it does not, disregarding, of course 0° Kelvin. (See Sassoon (2010) and van Rooij (2011) for more on this point and its relation to various linguistic examples not considered in this dissertation.)

As axioms (1)–(3) of definition 29 would indicate, ratio scales are built on top of a linear order. Stated another way, ratio scales are constructed by beginning with an ordering over a set and attaching a measurement system to it so-to-speak. It is not difficult to show that the scalar dimension of \textsc{height} as it is defined formally in definition 27 forms a
linear order modulo the indifference relation \( \simeq_\mathcal{H} \). So, I will revise my theory of the scalar dimension of \textsc{Height} one last time to transform it into a proper ratio scale. I do this by adding, what I will call, a \textbf{measure component} to the scalar dimension of \textsc{Height}. This is given explicitly by definition (30).

\textbf{Definition 30} (Scalar dimension of \textsc{Height} (Final version)). Let a scalar dimension of \textsc{Height} be the triple \( \mathcal{H} = (\mathcal{A}, \{\succ_\mathcal{H}, \prec_\mathcal{H}, \succeq_\mathcal{H}, \preceq_\mathcal{H}, \simeq_\mathcal{H}\}, \{\mu_\mathcal{H}, \circ\}) \). As before, ‘\( \mathcal{A} \)’ will be referred to as \( \mathcal{H} \)'s \textbf{adjectival component}, ‘\( \{\succ_\mathcal{H}, \prec_\mathcal{H}, \succeq_\mathcal{H}, \preceq_\mathcal{H}, \simeq_\mathcal{H}\} \)’ will be referred to as \( \mathcal{H} \)'s \textbf{scalar component}, and ‘\( \{\mu_\mathcal{H}, \circ\} \)’ will be referred to as \( \mathcal{H} \)'s \textbf{measure component}. The relations \( \succ_\mathcal{H}, \prec_\mathcal{H}, \succeq_\mathcal{H}, \preceq_\mathcal{H}, \simeq_\mathcal{H} \) obey axioms (1)–(8) as given by definition 26 and axioms (1) and (2) as given by definition 27. Now, for all \( x, x', y, y', z \in D_e \) and \( w, w', u, u', v \in D_s \), let \( \circ \) be the \textbf{concatenation operation} that obeys the axioms in (4)–(6).

1. \((x, w) \circ ((y, u), \circ (z, v)) \leftrightarrow ((x, w) \circ (y, u)) \circ (z, v)\) \hspace{1cm} \text{Associativity}

2. 

\[(x, w) \succeq_\mathcal{H} (y, u) \leftrightarrow (x, w) \circ (z, v) \succeq_\mathcal{H} \circ (y, u) \circ (z, v) \]

\[\leftrightarrow (z, v) \circ (x, w) \succeq_\mathcal{H} \circ (z, v) \circ (y, u)\]

\hspace{1cm} \text{Monotonicity}

3. \(x \circ w \succ_\mathcal{H} y \circ v \rightarrow \exists n ((x, w) \circ (y', u') \succ_\mathcal{H} n (y, v) \circ (x', w'))\)

\hspace{1cm} \text{Archimedean}

where \( nx \) is defined inductively as \( 1 (x, w) = (x, w) \) and \((n + 1) (x, w) = n (x, w) \circ (x, w)\). Finally, \( g, f \in \mu_\mathcal{H} \) are \textbf{associated measure functions of} \( \mathcal{H} \) just in case the obey axioms (7)–(10).

4. \( f : D_e \times D_s \rightarrow \mathbb{R}\)

5. \( f ((x, w)) > f ((y, v)) \leftrightarrow x \circ w \succ_\mathcal{H} y \circ v \), where ‘\( > \)’ is understood as the greater-than ordering on \( \mathbb{R} \)

6. \( f ((x, w)) = f ((y, v)) \leftrightarrow x \circ w \simeq_\mathcal{H} y \circ v \), where ‘\( = \)’ is understood as the equality relation on \( \mathbb{R} \)
7. \( g((x, w)) = (\alpha \times f((x, w))) + \beta \) for some numbers \( \alpha, \beta \in \mathbb{R} \) such that \( \alpha \neq 0 \) and \( \beta = 0 \)

8. \( f((x, w) \circ (y, u)) = f((x, w)) \circ f((y, u)) \)  

**Additivity**

Stepping back, observe that the scalar dimension of \( \text{HEIGHT} \) now has three components:

- an **adjectival component** encoding for the adjectival meanings that lexically encode for that scalar dimension;

- a **scalar component** encoding for the orderings those adjectival meanings are associated with; and

- a **measure component** encoding for the types of measures those adjectival meanings are associated with.

What is most important are the ideas of a **measure component** and its associated **measure function**. As I will argue, many scalar dimensions like \( \text{HEIGHT} \) will have several measure functions attached to them—\( \text{meters, feet, yards, etc.} \) This is important because some measure functions will be associated with certain scalar dimensions and not others, e.g., \( \text{meters} \) will be associated with \( \text{HEIGHT} \) and not \( \text{WEIGHT} \). This will allow us to explain the obvious contrast in felicity between (5.72a) and (5.72b).

(5.72)  

a. Orcutt is five feet tall  
b. #Orcutt is five feet heavy

What will differentiate measure functions, from a mathematical perspective, will be the properties they have, or rather the axioms they satisfy, and what scalar dimension they are tied to. Putting everything together, the revised picture of a scalar dimension I have in mind is shown pictorially in figure 5.13.

**Predicative measure phrases**

Let me now show the full power of the theory of scalar dimensions laid out in this section. To begin with, I will give the semantics of the statement in (5.69a), which is repeated in (5.73).
5.6. MEASURES

Figure 5.13: The scalar dimension of \textsc{height} as a ratio scale

(5.73) Orcutt is five feet tall

\textit{Predicative measure phrase}

In order to capture the meaning of this statement, I assume the lexical entry for \textit{five feet} in (5.74).\footnote{The semantic representation in (5.74) assumes an ‘exactly’ semantics. Assuming that \textit{exactly}-readings of statements involving numerals are pragmatically enriched readings the ‘\(=\)’ would be replaced by ‘\(\geq\)’ in this representation and all of the proceeding ones involving numerals.}

\begin{itemize}
  \item \textbf{Lexical entry —}
  \begin{align*}
    \text{five feet} : \Lambda \Lambda : \lambda P \lambda x \lambda w (\text{feet} (P) (x) (w) = 5)
  \end{align*}
\end{itemize}

A few notes on this entry are in order. For the purposes of simplification, I have disregarded that \textit{five feet} should not in fact be treated as a single expression—to do so ignores that there is inherent compositionality in expressions like \textit{one, two, three, four, five feet}, etc. I make this oversimplification, as otherwise I would have to augment chapter 3 by introducing a theory of the real numbers (\(\mathbb{R}\)) at the level of the logic \(\text{TT}_2\), and also in the type-logical
system itself. This would lead me too far afield, but I am certain that it can be done in a relatively straightforward way. So, I will simply assume that I have the power of the reals at my disposal.

Having said that, from a syntactic perspective, the expression five feet will be understood as an adjectival modifier. From a semantic perspective, the value of the non-logical constant ‘feet’ will be understood as a function of type \((es)esn\), where ‘\(n\)’ is of type real \((\mathbb{R})\). Specifically, this function takes an adjectival meaning as an input, say \([\text{tall}]\), and returns an associated measure function \(f\) of the scalar dimension that that adjective encodes for, which in this case at least, would be HEIGHT. This is depicted in figure 5.14. As axiomatized in definition 30, such a measure function \(f\) will take an entity/index pair \((x, w)\) as an input and return a real value \(n\) as an output. Intuitively, this can be understood as the measure of that entity \(e\) under that measure function \(f\) with respect to that particular index \(s\). (See, again, figure 5.13.)

The semantic representation of (5.73) is given in (5.75a); its linguist’s tree is shown in (5.75b); and its Gentzen/Prawitz-style derivation is shown in figure 5.15.
5.6. MEASURES

(5.75)  

\[
\lambda w (\text{feet}(\text{tall})(o)(w) = 5) \rightarrow \\
\{ w \in D_s \mid f((o,w)) = 5 \text{ such that } f \in \mu_S \}
\]

b.

The set denoted by (5.75a) includes all and only those indices in which Orcutt’s height measured in feet is 5. It is now easy to account for the difference in felicity of the statements (5.76).

(5.76)  

\[
\text{Orcutt is five feet tall}
\]
Specifically, if we adopt a partial version of the TT₁ logic à la Muskens (1995), it is now possible to say that the function $[\text{feet}]$ is undefined for adjectival meanings not encoded for by the scalar dimension of HEIGHT. (See, again, figure 5.14.) Under this view, statements like (5.76b) would simply lack a truth-value, as $[\text{heavy}]$ would not be encoded by the scalar dimension for HEIGHT, whereas $[\text{tall}]$ would be. In this way, I can account for the apparent oddity of (5.76b), while still providing the requisite truth-conditions for statements like (5.76a).

Measure phrases

I will turn, now, to giving the truth-conditions of statements like (5.69b), repeated in (5.77).

(5.77) Orcutt is taller than five feet

Measure phrase under the scope of ‘than’

Degree semanticists like von Stechow (1984) and Heim (2006) have no problem accounting for the meaning of statements like (5.77), as they treat its denotation as a relation between the maximal degree of height Orcutt possesses and the degree referred to be the expression five feet. In the system laid out here, (5.77) is problematic, if only because, first, I have been treating the arguments of than, from a semantic perspective at least, as generalized quantifiers. This clashes with my desire to maintain the lexical entry for five feet, defined in (5.74), whose semantic component is obviously not a generalized quantifier meaning. And second, I have claimed repeatedly that the meaning of adjectival comparatives involve orderings over individuals, not degrees (or measures for that matter). Thus, I cannot treat the meaning of (5.77) as a relation between measures. How, then, to proceed? Observe, first, that the statements in (5.78) are equivalent.

(5.78) a. Orcutt is taller than five feet

b. $\Leftrightarrow$ Orcutt is taller than something that is five feet tall

In fact, the equivalence in (5.78) is trivially true in our world, at least, (and ones like it). Take, for example, the left-to-right direction ($\Rightarrow$). If (5.78a) is true, that is to say, if Orcutt
Orcutt’s height: 6’1”

Initial segment: 5’

Figure 5.16: The initial segment of Orcutt corresponding to 5’

is, say, 6’1”, then there will be an initial segment of him himself that will constitute that ‘thing’ so-to-speak which is 5’, thus satisfying (5.78b). This is shown pictorially in figure 5.16.

Obviously, I have not developed a mereological semantics and to do so would extend beyond the scope of this dissertation. (See Krifka (1989), Krifka (1992b), Krifka (1998) and Link (2002) among many more for how this is done in practice). However, I can sketch this intuition rather straightforwardly. I extend my lexicon by adding the silent lexical expression INITSEG, as given by (5.79).

(5.79)

<table>
<thead>
<tr>
<th>Lexical entry —</th>
</tr>
</thead>
<tbody>
<tr>
<td>INITSEG : ((pP \ (CA \ (1\λ\ A))) / A) / (A / A) :</td>
</tr>
<tr>
<td>( \lambda P \lambda \varphi \lambda \varsigma \lambda \nu \lambda \nu \lambda \nu \nu \nu \exists y (\rho P y w \land \varsigma \varphi \lambda P (P (y)) P v w) )</td>
</tr>
</tbody>
</table>

This lexical entry looks extremely daunting, when in reality it is only a sort of ‘lifted’ form of the existential quantifier; and it is this silent element that will allow me to model
the intuition brought out by the inference in (5.78). To see how the entry defined in (5.79) works in practice, let us look at some examples. I have given the semantic representation of (5.78a) in (5.80a) and its linguist’s tree in (5.80b).

(5.80)  
a. Orcutt is taller than five feet $\rightarrow$  
$$[\lambda w \exists y (\text{feet}(\text{tall})(y)(w) = 5 \land \Delta (\text{tall}, \text{tall})(o)(y)(w))] =$$  
$$\{w \in D_s | \exists y (f((y,w)) = 5 \land o \ @ \ w \succ \Delta_y y @ w) \text{ such that } f \in \mu_{\gamma} \}$$  

b. 

![Diagram showing the semantic representation of the sentence Orcutt is taller than five feet and its associated tree structure.]
Figure 5.17: Derivation of a measure phrase under the scope of than
5.6. MEASURES

The representation in (5.80a) denotes the set of indices such that there exists something whose height measured in feet is 5, and Orcutt is taller than that thing—whatever it may be. Again, given a proper mereological axiomatization, the witness for this existential claim will be the initial segment of Orcutt himself.

The axiomatic theory laid out, here, has nice consequences. In particular, it can capture inferences like the one in (5.81) via standard equational reasoning.\(^5\)

\[(5.81)\]
\begin{align*}
a. & \text{ Orcutt is taller than Smith} \\
b. & \text{ Smith is taller than five feet} \\
c. & \Rightarrow \text{ Orcutt is taller than five feet}
\end{align*}

Proof. It suffices to show that \((5.81a) \land (5.81b) \Rightarrow (5.81c)\).

- \(\Rightarrow\) By (5.81a) and (5.40a), it follows that \(o \join w \succ_s y \join w\) for arbitrary \(w\). By axiom (5) of definition 30, \(f((o, w)) > f((s, w))\), where \(f\) is the associated measure function of the scalar dimension \(S\) as it is returned by the function \([\text{feet}]\) applied to \([\text{tall}]\).

By (5.81b) and (5.80a), there exists a \(y\) such that \(s \join w \succ_y y \join w\). Again, by axiom (5) of definition 30, \(f((s, w)) > f((y, w))\).

But then, by the transitivity of \(>\), the standard ordering on the reals \((\mathbb{R})\), \(f((o, w)) > f((y, w))\). Substituting identical for identical, conclude \(f((o, w)) > 5\). Thus (5.81c) is valid.

Differential phrases

I consider, now, the semantics of differential phrases like *five inches*, shown first in (5.70), and repeated for convenience in (5.82).

\[(5.82)\] Orcutt is five inches taller than Smith

---

\(^5\)This proof tacitly relies coordinating the premises (5.81a) and (5.81b) via the conjunction rule given by definition 24 so that the index parameters \(w\) are identical.
In this context, *five feet* seems to mean something different than in examples like (5.73) or (5.77). The idea, here, is that in (5.82), the difference between the measure of Orcutt’s height and the measure of Smith’s height is 5′′. To capture this fact, I would like my semantic system to deliver the representation for (5.82) as it is shown (5.83).

(5.83)

Orcutt is five inches taller than Smith

\[
\lambda w \exists y (\text{inch'} (\text{tall, tall}) (o) (w) - \text{inch'} (\text{tall, tall}) (s) (w) = 5 \land \Delta (\text{tall, tall}) (o) (w) (s) (w)) =
\{ w \in D_t \mid g ((o, w)) - g ((s, w)) = 5 \land o \preceq s \preceq w \text{ such that } g \in \mu_{\partial} \}
\]

First, let me comment on the value of the non-logical constant ‘inch’. It is understood as a function of type \((es) (es) esn\), where ‘n’ is of type real \((\mathbb{R})\). Specifically, this function takes two adjectival meanings as an input, in this case \(\left[ \text{tall} \right], \left[ \text{tall} \right]\), and returns an associated measure function \(g\) of the scalar dimension that that adjective encodes for, which in this case at least, would be HEIGHT. Although I will not do so here, I want this kind of generality in order to account for the semantics of statements like (5.83).

(5.84) Orcutt is five inches taller than Smith is wide

Having established this, the representation in (5.83) denotes the set of indices such that the difference between the measure of Orcutt’s height in inches and Smith’s height in inches is 5.

I see no obvious way of transforming the semantic representation of *five feet* in (5.79), using only the proof rules laid out in §3.3, to derive such a representation. I see (at least) two other options afforded to me: (i) define a differential operator so-to-speak, similar to the operator INIT, that takes as an input the semantic representation defined in (5.79) and returns a representation that will allow me to compositionally derive the representation shown in (5.83); or (ii) posit a homophonous form of *five feet* that gives me such a representation directly. There is a precedent for option (i). Alrenge and Kennedy (2013), for example, opt for and utilize a silent differential operator in their semantics for adjectival comparatives in a variety of interesting ways. For the sake of perspicuity, I will choose
to pursue option (ii) with the knowledge that it is rather straightforward to augment my analysis in accordance with option (i).  

(5.85)  

\[ \begin{align*}
\text{Lexical entry} & - \\
\text-five inches} : & ((((\text{\footnotesize{I}}\text{\footnotesize{\Lambda}}) / \text{\footnotesize{A}}) / \text{\footnotesize{PP}}) / \text{\footnotesize{CA}}) / \text{\footnotesize{A}} : \\
& \lambda P' \lambda Q \lambda P \lambda v \lambda x \lambda w (\text{inch}' (P', P) (x) (w)) - \\
& Q \lambda y \lambda u (\text{inch}' (P', P) (y) (u)) (v) = 5 \land \varepsilon PQP' v x w
\end{align*} \]

Using the lexical entry in (5.85), it is now easy to derive the semantic representation in (5.83). I have provided the linguist’s tree in (5.86), and its Gentzen/Prawitz derivation is shown in figure 5.19.

\footnote{Pursuing option (i) simply amounts to positing a phonetically silent differential operator, say, $\partial$ that takes as an input the semantic representation in (5.79) and returns the one in (5.85).}
It is now easy to account for valid inference like the one in (5.87) using basic arithmetical reasoning.

(5.87)  

a. Orcutt is two inches taller than Smith  
b. Smith is sixty inches  
c. \( \Rightarrow \) Orcutt is sixty two inches
Figure 5.19: Derivation of a comparative containing a differential phrase
5.6. MEASURES

**Proof.** It suffices to show that $(5.87a) \land (5.87b) \Rightarrow (5.87c)$.

\[
g((o, w)) - g((s, w)) = 2 \\
g((o, w)) - 60 = 2 \\
g((o, w)) = 62
\]

where $g \in \mu_{\beta}$.

**Factor phrases**

Lastly, I consider factor phrases like *two times* as exemplified in the statement (5.70), which again, is repeated in

(5.88) Orcutt is two times taller than Smith

Interestingly enough, the choice of measurement is immaterial to the truth of statements like (5.88). Suppose, for example, that Orcutt is 6’, whereas Smith is 3’. Clearly, Orcutt is two times taller than Smith. However, we know independently that the conversion from feet to inches is given by (5.89)

(5.89) 1 foot = 12 inches

So, even if we were to measure Orcutt and Smith in inches (72”), he would still be twice as tall as Smith (36”). This idea is reflected formally in axiom (6) of definition 30. Specifically, this axiom guarantees that the various measure functions associated with a particular interval scale will be related, or rather, can be translated into each other. Stated another way, it guarantees the unit of measurement on a ratio scale is, in some sense, immaterial. (See Lassiter (2011) and van Rooij (2011) for more on this point.) In regard to axiom (6), it is natural to identify ‘$g$’ with *inches*, ‘$f$’ with *feet*, ‘$\alpha$’ with ‘12’, and ‘0’ with ‘$\beta$’. Thus, we can move fluidly between inches and centimeters, as in the case of (5.90).
(5.90) 6 feet = (12 × 6) + 0 = 72 inches

I will build on that insight of particular measurements being immaterial so-to-speak in defining the semantics of factor phrases. In (5.91), I give the lexical entry for the expression \textit{two times}, again disregarding its inherent compositionality.

(5.91)

\begin{verbatim}
Lexical entry —

two times : (((((f\wedge A) \wedge A) / PP) / CA) / A : \\
\lambda P'\lambda \epsilon \lambda Q \lambda P \lambda \nu \lambda \lambda w \forall F \left( \frac{F (P', P) (x) (w)}{Q \lambda y \lambda u (F (P', P) (y) (u)) (y)} = 2 \wedge \epsilon P' Q P \nu \lambda x \lambda w \right)
\end{verbatim}

The semantic representation I deliver for (5.88) is shown in (5.92a); its linguist’s tree is shown in (5.92b); and its Gentzen/Prawitz-style derivation is shown in figure 5.20. Here, the variable ‘$F$’ is of type $(es) (es) esn$, where ‘$n$’ is of type real ($\mathbb{R}$). (This again assumes that I have augmented the logic TT$_2$ with a theory of the reals ($\mathbb{R}$).)
5.6. MEASURES

(5.92)  a. Orcutt is two times taller than Smith →

\[ \lambda w \forall F \left( \frac{F(tall, tall)(o)(w)}{F(tall, tall)(s)(w)} = 2 \land \Delta(tall, tall)(o)(s)(w) \right) = \{ w \in D_s \mid \frac{f(((o, w))}{f((s, w))} = 2 \land o @ w >_f s @ w \text{ for all } f \in \mu_f \} \]

b. To make real sense of the semantic representation in (5.92a), we must think of it in terms of a partial logic. To interpret this representation, the variable ‘F’ ranges over all and
only those measure functions defined for the scalar dimension encoded by $\langle \text{[tall]}, \text{[tall]} \rangle$. This would include, for example, the functions denoted by \text{feet}' or \text{inch}'. Stated another way, in evaluating the truth of (5.92a), I consider all and only those measure functions associated with the scalar dimension of \text{HEIGHT}. So, 5.92 denotes the set of all indices such that, under all measurements associated with the scalar dimension of \text{HEIGHT}, the ratio of Orcutt’s measured height as compared to Smith’s measured height is 2, and Orcutt is taller than Smith. In this sense, the choice of measurement is immaterial.

Again, it is now easy to account for valid inference like the one in (5.87) using basic arithmetical reasoning.

\begin{align*}
\text{(5.93) } & \quad \text{a. Orcutt is two times taller than Smith} \\
& \quad \text{b. Smith is 30 inches} \\
& \quad \text{c. } \Rightarrow \text{Orcutt is sixty two inches}
\end{align*}

\textit{Proof.} It suffices to show that $(5.93a) \land (5.93b) \Rightarrow (5.93c)$.

\[
\begin{align*}
\frac{g((o,w))}{g((s,w))} &= 2 \\
\frac{g((o,w))}{30} &= 2 \\
g((o,w)) &= 60
\end{align*}
\]

where $g \in \mu_\delta$.

5.6.2 Permission modals

Now that I have developed a theory of modality and measures, I want to consider one final construction that has proven problematic in the literature. As has been pointed out, what I am calling Type 2 constructions present certain theoretical difficulties. To see what I mean, imagine, now, that Orcutt, who is 6'1", is in line to ride a ride at the state fair. Suppose, now, that there is a sign that says, “You are required to be 5'0" tall to ride the ride". Now
Figure 5.20: Derivation of an adjectival comparative containing a factor phrase
suppose that McX is operating the ride and utters (5.94) to Orcutt.

(5.94) \textbf{McX}: You\textsubscript{1} (=Orcutt\textsubscript{1}) are taller than you\textsubscript{1} are required to be

It has been pointed out by authors like Heim (2006) that (5.94) is consistent with (at least) two distinct situations.

(5.95) a. \textbf{Situation 1}: Orcutt is tall enough to ride the ride, and consequently, \textit{can} ride the ride.

b. \textbf{Situation 2}: Orcutt is too tall to ride the ride, and consequently, \textit{cannot} ride the ride.

Given the context as I’ve outlined it, (5.95a) is clearly consistent with (5.94). Orcutt need only be 5’0” to ride the ride; and he is in fact 6’1”. So, Orcutt meets the \textit{minimum} height requirement to ride the ride, and consequently, is taller than is required but can still ride the ride. On the other hand, the consistency of (5.95b) with (5.94) is a little bit trickier to get: it requires of us the assumption that there exists a \textit{maximum} height requirement to ride the ride. If, for example, such a maximum is 6’0”, then clearly Orcutt is too tall to ride the ride.

The example in (5.94) is interesting if only because this ambiguity is robust across all permission attitudes including, but not limited to \textit{than he had to be} . . ., \textit{than it was necessary} . . ., \textit{than it was required} . . ., etc.. Moreover, a similar ambiguity cannot be detected in statements involving other attitudes like \textit{than Jones believes} . . .

(5.96) Orcutt\textsubscript{1} is taller than McX believes he\textsubscript{1} is

In fact, it is unclear what an ambiguity of (5.96) analogous to (5.95) would look like. Heim (2006) is able to account for the above ambiguity by falling back on the \Pi operator that was discussed in chapter 2 of this dissertation. The particulars of her analysis are unimportant for the present purposes, as they pattern analogously to her treatment of the Booleans and quantifiers discussed in detail in the same chapter. What is important, however, is that her analysis assumes that the ambiguity of (5.94) is a \textit{structural one}, and as such, should be handled at the level of logical form. However, I think there is good reason to believe that this ambiguity is at least a \textit{contextual one}, if not partly lexically determined, that has to do with the (under-)specificity of requirements generally.
5.6. MEASURES

I think it is uncontroversial that requirements, by their very nature, are scalar. They always seem to have minimums; however, they may or may not have maximums. Here’s what I mean. Returning, now, to the example in (5.94), the sign describes a minimum height requirement to ride the ride—5’0”, not a maximum. So, the context itself is under-specified: does a maximum exist or not? To push this point a bit farther, consider the situation in which Orcutt is donating to his favorite church. As Protestant customs generally have it, one is required give to 10% of their income to the church. Suppose Orcutt does this and more; and McX utters the statement in (5.97) to Orcutt.

(5.97)  **McX**: You₁ (=Orcutt₁) are more generous than you₁ are required to be

Is (5.97) really ambiguous between the two readings in (5.98)?

(5.98)  a.  **Situation 1**: Orcutt has given at least 10% of his income and is in the church’s good graces.

   b.  **Situation 2**: Orcutt has given more than 10% of his income, and in fact so much more, he’s given too much and fallen out of the church’s good graces.

I am not convinced (5.98b) is an actual reading of (5.97): it seems like giving, and consequently being generous, has no upper bound. This is not to say that laws and regulations could not be written to restrict the amount given by a particular person or other—but what funny laws they would be! The point is, though, that if such a maximum exists, it would have to be contextually specified, suggesting that the ambiguity observed by authors like Heim (2006) is not a function of the meaning of the comparative morpheme -er and its interaction with so-called modal quantifiers like required.

In general, when considering type 2 constructions that contain permission modals, there seems to me (at least) four possible situations that can obtain in regard to requirements.

(5.99)  a.  A situation in which the existence of a maximal requirement exists.

   b.  A situation in which a maximal requirement does not exist.

   c.  A situation in which a minimal requirement does not exist.

   d.  A situation in which an individual does not know whether a maximal requirement exists.
I am particularly interested in situations (5.99a) and (5.99b) and will not have anything to say about (5.99c) and (5.99d). I assume (5.99c) to be analogous to (5.99b), and I take it that (5.99d) involves complex theories of epistemology and its relation to the law. Such topics far outrun the scope of this dissertation.

The existence of a maximal requirement

Returning to the above thought experiment, suppose, now, that the sign described above instead reads, “You are required to be 5′ tall to ride the ride but no taller than 6′”. Notice that, in such a situation, both a minimum and maximum height requirement have been contextually specified. Assuming Orcutt is 6′1″, the ambiguity of (5.94) is obliterated and the statement is consistent only with the situation described in (5.95b), namely the one in which Orcutt cannot ride the ride.

To model this situation, let me assume the existence of a non-logical constant $R$ of semantic type ($ss$), whose value is understood as being a binary deontic accessibility relation over $D_s$. Atomic statements of the form ‘$Rwu$’ will be read index $u$ is $R$, or rather deontically accessible from $w$. Moreover, I will assume the existence of a deontic modal base $\Phi_D$, which itself is understood of a set of TT sentences in which the truth of a statement involving a deontic modal is judged against. That is to say, $\Phi_D$ is the set of laws, etc., individuals are required to follow in a particular world.

Now, to model the above law in the object language I have developed through the course of this dissertation, let me assume the lexical entry for required as it is specified in (5.100).

(5.100)

```
Lexical entry —
required : (NP\S)/S : λpλw∀u (R (w) (u) ∧ Pu)
```

Now, let us now adopt the following law in (5.101) by adding its semantic representation to our modal base.
It is required that Orcutt be taller than 5′0″ but no taller than 6′0″:

\[ \forall w \forall u (R(w)(u) \rightarrow \exists y \text{feet(tall)}(y)(u)(y)(u) = 5 \land \Delta(tall,tall)(x)(u)(y)(u)) \]

Now, let me adopt one more axiom, which will allow for sound modal and counterfactual reasoning.

\[ \forall w \forall n \forall x (6 \geq n > 5 \rightarrow \exists v (R(w)(v) \land \text{feet(tall)}(x)(v) = n)) \]

The axiom in (5.102) guarantees that, for every individual, and for every permissible measurement of the scalar dimension \text{HEIGHT}, as it is measured in feet, there is a world in the deontic modal base such that that person instantiates that measurement. This makes sense, at least intuitively, as anyone can imagine themselves instantiating any permissible height under the measurement of feet (or any measurement for that matter—but I do not need that kind of generality here).

From a formal perspective, adopting (5.101) and (5.102) as laws amounts to the assumption that our deontic modal base has those statements as elements.

\[ (5.103) \Phi_D = \{(5.101),(5.102)\} \]

Consider, now, the semantic representation of the type 2 statement in (5.104), applied to the world Orcutt actually inhabits, which I will call ‘a’.

\[ (5.104) \quad \text{Orcutt}_1 \text{ is taller than he}_1 \text{ is required to be} \rightarrow \]

\[ \forall u (R(a)(u) \rightarrow \Delta(tall,tall)(o)(a)(o)(u)) \]

Now, I will show that the modal base in (5.103), when combined with the statement in (5.104), delivers the situation described in ((5.95b), which I have repeated below.)
(5.105) **Situation 2**: Orcutt is too tall to ride the ride, and consequently, **cannot** ride the ride.

Namely, I will show that the statement in (5.105) is true, evaluated of course, with respect to our deontic modal base.

(5.106) Orcutt is taller than six feet

**Proof.** It suffices to show that \((5.101) \land (5.102) \land (5.104) \Rightarrow (5.106)\). I prove by contradiction. There are two cases to consider.

- Suppose the measure of Orcutt’s height in feet is less than 5: \(f((o, a)) < 5\) for \(f \in \mu_\delta\). Take an arbitrary world \(u\) in the deontic modal base. By (5.101), we know that the measure of everyone in feet at \(u\)—Orcutt in particular—is greater than 5. So, by (5.104), \(f((o, a)) > f((o, u)) > 5\). Of course this is a contradiction.

- Suppose the measure of Orcutt’s height in feet is greater than 5 but less than or equal to 6: \(6 \geq f((o, a)) > 5\) for \(f \in \mu_\delta\). By (5.102), there exists a world \(u\) in the deontic modal base such that Orcutt’s height in feet is equal to 6. But by (5.104), we know that \(f((o, a)) > f((o, u)) = 6\), which is a contradiction.

\(\Box\)

**The non-existence of a maximal requirement**

Now let me consider a situation in which a maximal requirement does not exist. Suppose that the sign from our thought experiment “You are required to be 5’ tall to ride”. Here, only a minimum height requirement has been contextually specified. Assuming Orcutt is 6’1”, the ambiguity of (5.94) is obliterated and the statement is consistent only with the situation described in (5.95a), repeated in (5.107), namely the one in which Orcutt cannot ride the ride.

(5.107) **Situation 1**: Orcutt is tall enough to ride the ride, and consequently, **can** ride the ride.

I model this situation by adopting the law in (5.108) to our deontic modal base.
(5.108)

It is required that everyone be $5'0''$:

$$\forall w \forall u \exists x (R(w)(u) \rightarrow \text{feet}(x)(v) = 5)$$

It is now easy to show that the statements in (5.108) and (5.104) jointly imply the one in (5.109).

(5.109) Orcutt is taller than five feet

So the apparent ambiguity can be resolved by understanding the nature of requirements as a problem of contextual specificity.

5.7 Chapter summary

In this chapter, I gave a semantics for clausal comparatives. I did this by revising my lexical entry for the comparative morpheme -er/more and utilizing the continuation mode of combination. As such, I was able to account for the syntax and semantics of

- comparative sub-deletion;
- comparative deletion;
- trans-world comparisons of all types;
- the Russell ambiguity;
- measure phrases of all types; and
- permission modals.

Importantly, in generalizing the semantics of chapter 4, I lose no empirical coverage, but rather gain it.
6 | Looking ahead

6.1 A brief summary

In this dissertation, I provided a syntax and semantics for adjectival comparatives that does not rely on degrees. In making such a move, I showed that one gets a better and more general treatment of the syntax and semantics of adjectival comparatives. My Montagovian-style analysis allowed me to capture semantic facts like the disjunctive interpretation of or and the semantics of permission modals as they appear in clausal comparatives. I did this without stipulating the existence of misbehaved covert operators at the level of logical form.

In fact, my move away from a degree semantics was not without empirical justification. In chapter 4, I showed that, in many instances, one should not conceive of the semantics of adjectival comparatives as involving scales at all, as the scalar dimensions many adjectives encode are not transitive in general. However, my analysis was not without problems, as I was unable to provide insight into puzzling facts involving indefinites under the scope of than, as well as the seemingly bizarre behavior of negative quantifiers. Moreover, I did not consider a wide range of interesting constructions including so-called meta-linguistic comparatives and nominal and verbal comparatives. I turn now to discuss issues surrounding these constructions and sketch possible solutions for them.

6.2 Meta-linguistic comparatives

Meta-linguistic comparatives (McCawley 1998; Giannakidou and Yoon 2008; Giannakidou and Stavrou 2009; Giannakidou and Yoon 2010; Morzycki 2011) are an interesting sort of comparative and are shown in (6.1a).
6.2. META-LINGUISTIC COMPARATIVES

(6.1)  

a. Orcutt is more dumb than he is crazy  
b. ⇔ Orcutt₁ is more dumb than he₁ is crazy

Morzycki (2011; (2a))

Meta-linguistic comparatives differ from ordinary comparatives like the ones considered throughout this dissertation. According to Morzycki (2011), meta-linguistic comparatives cannot involve the synthetic form of the comparative -er; they require more. To account for this fact, Morzycki (2011) assumes two homophonous lexical entries for the analytic form of the comparative more. For him, a meta-linguistic comparative interpretation of an adjectival comparative is due to a lexical ambiguity. In the formalism of this paper, this amounts to a lexicon that includes (at least) the additional entry (6.2).

(6.2)

Lexical entry (Meta-linguistic) –

\[ \text{more} : \text{CA/A} : ? \]

The reason I leave the meaning of meta-linguistic more as a question (?) is because there is no true consensus in the literature as to what statements like (6.1) actually mean.

However, under my analysis, I do not need to resort to elliptical means to generate the string in (6.1a); rather, I can fall back on continuations and an expansion of the lexicon. Specifically, building on Barker’s (2013) analysis of the anaphoric pronouns (s)he, I introduce the phonetically null expression SUBGAP in (6.3).

(6.3)

Lexical entry –

\[ \text{SUBGAP} : \text{IV} \land \left( \left( \text{NP} \land \text{IV} \right) \right) : \]

\[ \lambda \vartheta \lambda x \lambda w (\vartheta \lambda P \lambda u (P(x)(u))xw) \]

where \( \vartheta \) is a variable of type \(((es)s)es\)
From a semantic perspective, SUBGAP is a sort of reduplicator for semantic expressions of type \( e \) in the same way COMPGAP is a reduplicator for semantic expressions of type \((es)\). (See, again, table 5.2 for they syntax and semantics of COMPGAP.) To see what I mean, the semantic representation I predict for (6.1a) is shown in (6.4a), and it’s linguist’s tree is in (6.4).

(6.4) a.

Orcutt is more dumb than crazy \(\sim\)

\[\lambda w (\Delta (\text{dumb, crazy})(o)(w)(o)(w))\]
6.2. META-LINGUISTIC COMPARATIVES

For the time being, I will remain silent about what the semantic representation in (6.4a) means, as it again relies on the sort of order the value of \( \Delta \) associates the adjectival meanings of **dumb** and **crazy** with. I do want to point out, however, that the combination of continuations with the lexical entries for **SUBGAP** and **COMPGAP** allow for massive over-generation. The most intolerable result is in (6.5). Specifically, my grammar generates the string in (6.5a) with the predicted meaning in (6.5b).

(6.5)  
   a.  \#Orcutt is taller than **SUBGAP** **COMPGAP**  
   b.  \( \Leftrightarrow \) Orcutt\(_1\) is taller than he\(_1\) is tall

Perhaps less intolerable is the fact that I can derive (6.6a) with the predicted meaning in (6.6b).

(6.6)  
   a.  ??Orcutt is taller than **SUBGAP** **wide**  
   b.  \( \Leftrightarrow \) Orcutt\(_1\) is taller than he\(_1\) is wide

I don’t find the statement in (6.6a) particularly felicitous; however, consider the one in (6.7a) with predicted meaning in (6.7b).

(6.7)  
   a.  ?Orcutt is happier than **SUBGAP** **sad**  
   b.  \( \Leftrightarrow \) Orcutt\(_1\) is happier than he\(_1\) is sad

This statement seems markedly better to me—assuming felicity judgements themselves are gradable (Wasow 2008)—which begs an interesting question: are statements like (6.7a) true instances of meta-linguistic comparatives? Authors like Morzycki (2011) think not. For him, a sentence like (6.7) would be ruled ungrammatical; and the statement in (6.7b) would presumably be understood as a comparative of deviation in the sense of Kennedy (1997, 2001), not a meta-linguistic comparative.

Turning now to the semantics of meta-linguistic comparatives, authors like McCawley (1998) and Giannakidou and Stavrou (2009) treat statements like (6.1) as being meta-linguistic, because they claim that their meaning makes reference to actual linguistic forms and their use. The analogy, here, is with so-called **meta-linguistic negation** constructions, some examples of which are shown in (6.8).
6.2. META-LINGUISTIC COMPARATIVES

(6.8)  
  a. He didn’t order ‘[eI]pricots’; he ordered ‘[æ]pricots’
  b. He didn’t call the POlice; he called the poLICE

Horn (1985)

In the statements above, what is being taken issue with is not the meaning of the English expressions *apricot* or *police* but rather their pronunciations in a particular context. From a truth-conditional perspective, the two pairs of statements in (6.8) are identical. For example, consider (6.8a): the sentences denote the same proposition, as ‘[eI]pricots’ and ‘[æ]pricots’ refer to the same objects in the world. However, there is a meaning difference between the two clauses. Consequently, negation cannot be targeting the sentences *qua* sentences, but rather something else, namely their pronunciations. In this sense, metalinguistic negation has little, if anything to do with the meaning of the sentences falling under the scope of negation in the sense in which the meaning of a sentence is understood truth-conditionally, but rather, their phonetic realization.

With this in mind, we can make initial sense of of the claim that meta-linguistic comparatives compare the relative appropriateness of uttering two expressions in a particular context. Consider the following mini-dialogue between Wymann and McX.

(6.9)  
  a. Wymann: Orcutt is crazy
  b. McX: No. Orcutt is more DUMB than crazy
  c. Wymann: I think you’re wrong. It’s more appropriate to say that Orcutt is ...

If, in (6.9b), we understand SMALL CAPS as indicating prosodic emphasis, then it appears that what McX and Wymann are disagreeing about is the appropriate way to describe Orcutt: to McX’s mind, it’s more appropriate to describe Orcutt using the English expression *dumb* than it is to describe him using the expression *crazy*. For Wymann, it’s a better choice in terms to use *crazy* to describe Orcutt as opposed to using the term *dumb*. Understood in this light the semantics of meta-linguistic comparatives, at least in part, have to do with an individual’s choice in using one English expression over another in a particular situation to describe (a possibly different) individual.

Along with Morzycki (2011), Giannakidou and Stavrou (2009), assume the existence of a separate lexical entry for meta-linguistic *more*, differing from the normal analytic form
of the comparative in both its syntax and semantics. Its semantic representation is shown in (6.10)

(6.10)

\[ \text{more}_{ML} \rightarrow \lambda p \lambda q \exists d \left( (R(a)(p)(d)) \land d > \max (\lambda d' \left( R(a)(q)(d') \right)) \right) \]

where \( p, q \) are variables of type \((s)\), \( d \) is a variable ranging over degrees, \( R \) is a "gradable propositional attitude [of type \((e(s)d)\)] supplied by the context: either an epistemic attitude meaning approximately "appropriate to say", or an attitude expressing preference (desiderative or volitional); \( a \) is the individual anchor [of type \( e \)]" (Giannakidou and Stavrou 2009; (40))

The basic idea behind the representation in (6.10) is that meta-linguistic \textit{more} expresses a relation between the degree to which one utterance \( p \) that encodes for a sentence is appropriate to be said in a particular context by a particular individual \( a \) versus the degree to which a (possibly) different utterance \( q \) that encodes for a sentence is appropriate to be said in the same context by the same speaker. That's a mouthful. Stated another way, for Giannakidou and Stavrou (2009), meta-linguistic \textit{more} is a relation between the degrees of appropriateness of the (potential) utterances of two sentences in a particular context by a particular speaker.

That meta-linguistic comparatives involve the comparison of utterances of sentences already seems problematic, for consider the statement in (6.11).

(6.11) Orcutt is more DUMB than anything else

The question, here, is what the domain of (higher-order) quantifier denoted by \textit{anything else} is—the set of English utterances that encode for sentences? I think not. Rather, the

\[^{1}\text{As it is formulated, the representation in (6.10) presents certain philosophical difficulties, as propositions themselves cannot be said. General philosophical wisdom tells us that propositions are abstract entities that do not enter into causal relations with the material world. Rather, speakers utter sentences, which themselves encode for propositions. It makes more sense for the variables \( p \) and \( q \), as they appear in (6.10), to range over utterances that encode sentences, not propositions. Utterances can be said; and as such, one utterance can be more appropriate to say in a particular context by a particular speaker than another utterance. For the remainder of this section, I will treat \( p \) and \( q \) as being variables ranging over utterances that encode sentences.}\]
domain of this quantifier seems to contain the sort of things utterances that encode for the sorts of things that adjectives in their predicative form are taken to encode for—properties. (Contrary to Giannakidou and Stavrou (2009), this is effectively the view of Morzycki (2011).)

Moreover, notice that statements of the form in (6.12) do not always receive a meta-linguistic interpretation.

(6.12) NP is more AP than AP′

Take, as an example, so-called comparisons of contrast categories like (6.13).

(6.13) This rod is more red than it is blue

Comparisons of contrast categories (Sassoon 2007; (24a))

What is being compared, here, is, at least not obviously, the relative appropriateness of the applicability of using the terms red versus blue to describe the rod, but rather whether or not the rod actually is more red than blue as a fact about the material world, not a fact about how people use language to describe the material world. To better understand this point, consider the red-to-blue color spectrum as shown in figure (6.1). When the rod and the spectrum are juxtaposed, it’s clear that the rod is more red than blue independent of the terms which we use to describe it.

Evidently, all comparatives of the syntactic shape in (6.12) do not receive a meta-linguistic interpretation. (At least they don’t have to: adding prosodic emphasis to the expressions red in (6.13) seems to allow for a meta-linguistic interpretation.) The data in
(6.13) is enough to show that having the form of the schema in (6.12) is not a sufficient condition on an adjectival comparative receiving a meta-linguistic interpretation. And to complicate matters even further, some adjectival comparatives not of the syntactic shape in (6.12) can receive a meta-linguistic interpretation, as the following dialogue would suggest.

(6.14)  
\begin{itemize}
  \item a. Wymann: Orcutt is more DUMB than crazy
  \item b. McX: True. But nobody is more DUMB than Smith (is)
\end{itemize}

It doesn’t seem like standard Roothian alternative semantics will get us all the way in interpreting (6.14b), as it traffics in meanings of linguistic expressions, not utterances of linguistic expressions (Rooth (1985) but see also Hamblin (1973) and Krifka (1992a) for more on alternative-style semantics, as well as Morzycki (2011) for a semantics of meta-linguistic comparatives in an alternative framework). Under a meta-linguistic interpretation, what McX means is roughly that, while it is in fact true that Orcutt is more dumb than crazy, the expression *dumb* is a more fitting description of Smith than it is of anyone else. Similarly, consider the statement in 6.15, with prosodic emphasis placed on the comparative adjective *happier*.

(6.15)  Orcutt is HAPPIER than sad

As far as I can tell at least, the statement can receive a meta-linguistic interpretation. If I am correct, the data in (6.14) and (6.15) suggest that having the form of the schema in (6.12) is not a necessary condition for predicting whether a comparative receives a meta-linguistic interpretation. From my perspective, it’s beginning to seem less and less likely a meta-linguistic interpretation of an adjectival comparative, in English at least, is a function of some structural or lexical ambiguity. Rather, a meta-linguistic interpretation of an adjectival comparative is a function of how people use language in context with the aid, for example, of prosodic emphasis. (But see Giannakidou and Yoon (2008) and Giannakidou and Stavrou (2009) for typological evidence of how languages like Greek and Korean seem to overtly lexicalize a meta-linguistic comparative morpheme.)

It is not entirely obvious to me the way in which these empirical insights could be incorporated into the syntactico-semantic system laid out here. Consequently, I leave such a project for future work.
6.3 Nominal and verbal comparatives

The semantics of nominal comparatives have been studied in depth by Hendriks (1995), Nerbonne (1995), Hackl (2001) and Wellwood et al. (2011). Although I will not go into too much depth here, the semantics of these comparative types touch on the research in the areas of generalized quantifier theory (Keenan 1996, 2002; Peters and Westerstahl 2006) and the mass/count distinction (Quine 1960; Krifka 1989, 1992b, 1998; Link 2002), among other things. To give the reader a taste of the relevant issues in this domain, as Wellwood et al. (2011) point out that there’s a semantic difference between nominal comparatives that involve count nouns like apples and mass nouns like water, as shown in (6.16a) and (6.16b) respectively.

(6.16) a. Orcutt ate more apples than Smith
    b. Orcutt drank more water than Smith

The meaning of a count noun like apples is said to be a set of distinguishable or countable (in the non-set-theoretic sense of the term) objects. However, the referents of a mass noun like water tend to be a set of indistinguishable or non-countable objects. With this in mind, the statement in (6.16a) involves a comparison of the number of apples Orcutt ate versus the number of apples Smith did, while (6.16b) involves the comparison of, say, the volume of water Orcutt drank versus the volume Smith did.

At a high-level, and focusing only on the semantics of nominal comparatives involving count nouns, in (6.17), we see that more is modifying the subject NP: the sentence means roughly that the number of boys that slept outnumbered the number of girls that slept.

(6.17) More boys slept than girls did

Roughly, the truth-conditions of (6.17) are as in (6.18).

(6.18) \( | \{ x \mid \text{boy}(x) \wedge \text{slept}(x) \} | > | \{ x \mid \text{girl}(x) \wedge \text{slept}(x) \} | \)

The statement in (6.18) is true just in case the cardinality of the set of boy-sleepers is greater than the cardinality of the set of girl-sleepers. (See Hendriks (1995) for how the logical form in (6.18) can be derived compositionally in a categorial-style grammar.) Importantly,
6.3. NOMINAL AND VERBAL COMPARATIVES

set-theoretic cardinality \(|\cdot|\) is a type of measure: so, what (6.18) is saying is that the measure of the set of boy-sleepers is greater than the set of girl-sleepers. I will refer to such nominal comparatives like the one above as **subject comparatives**. It is natural to understand (6.19) as the underlying form that all subject comparatives share.

(6.19) More NP₁ VP₁ than NP₂ VP₂

In (6.20a), on the other hand, *more* is modifying the object NP, and the sentence means that the number of apples that the boys ate outnumbered the number of apples that the girls did. The logical representation of this sentence are reflected in (6.20b).

(6.20) a. Boys ate more apples than girls did

b. \[ \{y | \exists x (\text{boy}(x) \land \text{apple}(y) \land \text{eat}(x,y))\} > \{y | \exists x (\text{girl}(x) \land \text{apple}(y) \land \text{eat}(x,y))\} \]

Similarly, in (6.21a), *more* is modifying the indirect object. Here, the sentence means something like the number of girls that boys gave apples to exceeds the number of boys that girls gave apples to. The sentence’s logical form is given in (6.21b).

(6.21) a. Boys gave apples to more girls than girls gave to boys

b. \[ \{z | \exists x \exists y (\text{boy}(x) \land \text{apple}(y) \land \text{girl}(z) \land \text{give}(x,y,z))\} > \{z | \exists x \exists y (\text{girl}(x) \land \text{apple}(y) \land \text{boy}(z) \land \text{give}(x,y,z))\} \]

I will refer to sentences like (6.20a) as **object comparatives** and sentences like (6.21a) as **indirect object comparatives**. I claim that the underlying forms of object and indirect object comparatives are given in (6.22a) and (6.22b) respectively.

(6.22) a. NP₁ [VP V₁ more NP₂] than NP₃ [VP V₂ NP₄]

b. NP₁ [VP V₁ NP₂ to more NP₃] than NP₄ [VP V₂ NP₅ to NP₆]
Depending on what the NPs and VPs refer to and whether we draw a distinction between mass and count nouns, nominal comparatives have quite a few sub-types. No doubt many of these will be semantically anomalous, depending, of course, on how the above schemas are instantiated, as the example below demonstrates. (Obviously, boys cannot eat more apples than themselves.)

(6.23) \#Boys\textsubscript{1} ate more apples than boys\textsubscript{1} ate apples

However, the semantics of many of these sub-types is quite complicated as the following example would suggest.

(6.24) Boys drank more water than girls ate apples

In this example, we’re asked to compare the amount of water boys drank to the number of apples girls ate. This begs the question: is there a common measure in which the amount of such water can be compared to the amount of such apples? I leave this question open, saying only that Nerbonne (1995), building Krifka’s (1989) measure-theoretic work on the algebraic structure of mass nouns, provides a semantics for nominal comparatives of all types. I imagine that his analysis can be leveraged to account for examples like (6.24).

However, matters are more complicated when considering verbal comparatives like 6.25.

(6.25) Orcutt ran more than Smith did

The sentence in (6.25) is under-specified: did Orcutt run a greater distance than Smith did; did he run for a longer time than Smith; or did he run more often than Smith? Given such under-specificity, the obvious next question becomes what should a semantic theory of statements like (6.25) look like? No doubt such an analysis will have to draw on work on aspect and tense (Vendler 1967; Tenny 1987; Krifka 1989; Brinton 1988; Krifka 1992b; Olsen 1994; Smith 1991; von Stechow 1995; Jackendoff 1996; Krifka 1998). And given the tight-knit relationship between the semantics of the nominal and verbal domain (Krifka 1989, 1992b, 1998), I suspect that a semantic account of (6.25) will be closely related to analyses of nominal comparatives like the ones proposed by Nerbonne (1995) and Hackl (2001).
I don’t have much more to say on the semantics of nominal and verbal comparatives—my intent is only to make the reader aware of the comparative landscape and the various semantic complications that come with it. To conclude, take what I will refer to as a mixed comparative, an example of which is in (6.26).

(6.26) Some men more hurriedly gave more apples in a shorter amount of time to more children than every woman gave water to them. I call these examples ‘mixed comparatives’ because they involve both nominal, verbal, and adverbial comparasions. I find examples like these to be mind-bending; and I doubt a single aforementioned semantic theory of nominal and verbal comparatives that could generate the set of situations (6.26) is consistent with. What examples like this demonstrate is how little we, as semanticists, actually know about the meaning of comparatives. It is my hope that such a range of constructions can be analyzed and better understood using the formal theory I have laid out in this dissertation.
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